Our first script

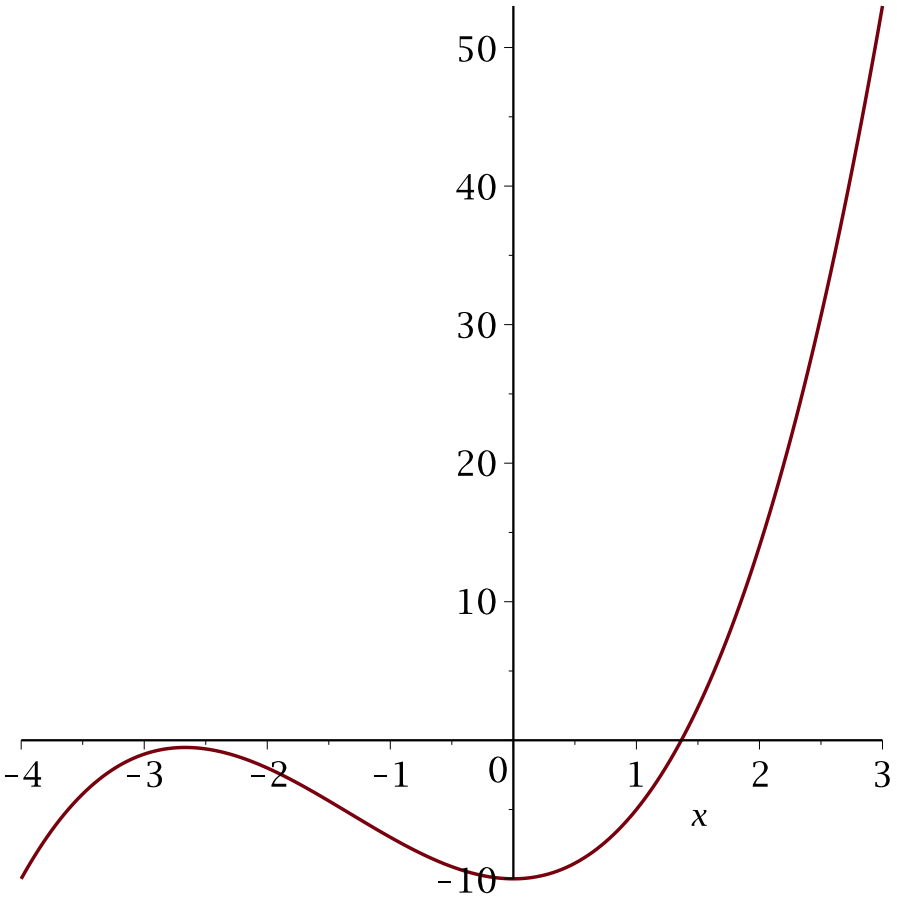
```
> restart;
  Digits:=10;
```

$$Digits := 10 \quad (1)$$

```
> f:=x->x^3+4*x^2-10;
```

$$f := x \to x^3 + 4\,x^2 - 10 \quad (2)$$

```
> plot(f(x),x=-4..3);
```



The Algorithm

```
> a:=1.0;
```

$$a := 1.0 \quad (3)$$

```
> b:=2.0;
```

$$b := 2.0 \quad (4)$$

```
> FA:=f(a); f(b):=f(b); iter:=0;
```

$$FA := -5.000$$
$$f(2.0) := 14.000$$

(5)

$$iter := 0 \quad (5)$$

```
> p:=a+(b-a)/2; FP:=f(p); FA*FP;
```

$$p := 1.500000000$$
$$FP := 2.37500000$$
$$-11.87500000 \quad (6)$$

since FA*FP<0, the next interval becomes [1,1.5]

```
> b:=1.5; p:=a+(b-a)/2; FP:=f(p); iter:=1;
```

$$b := 1.5$$
$$p := 1.250000000$$
$$FP := -1.796875000$$
$$iter := 1 \quad (7)$$

Now we check the sign of FA*FP

```
> FA*FP;
```

$$8.984375000 \quad (8)$$

and a=p=1.25 while b=1.5. Next, the new p and FA are

```
> a:=p; p:=a+(b-a)/2; FA:=f(p); iter:=2;
```

$$a := 1.250000000$$
$$p := 1.375000000$$
$$FA := 0.16210938$$
$$iter := 2 \quad (9)$$

Next, since FP*FA<0, b becomes p=1.375 and a is as before 1.25. The new p becomes

```
> b:=p; p:=a+(b-a)/2; iter:=3;
```

$$b := 1.375000000$$
$$p := 1.312500000$$
$$iter := 3 \quad (10)$$

and so on.

Here is the full script:

```
> f:=x->x^3+4*x^2-10; a:=1.0; b:=2.0; FA:=f(a); FP:=f(a); iter:=0;
```

$$f := x \rightarrow x^3 + 4x^2 - 10$$
$$a := 1.0$$
$$b := 2.0$$
$$FA := -5.000$$
$$FP := -5.000$$
$$iter := 0 \quad (11)$$

```
> while  FP<>0 and (b-a)>=10^(-4) do
  p:=a+(b-a)/2;
  FP:=f(p);
  iter:=iter+1;
```

```
if FA*FP>0 then a:=p;
FA:=FP;
else
b:=p
end if
end do;
```

$$p := 1.500000000$$
$$FP := 2.37500000$$
$$iter := 1$$
$$p := 1.250000000$$
$$FP := -1.796875000$$
$$iter := 2$$
$$p := 1.375000000$$
$$FP := 0.16210938$$
$$iter := 3$$
$$p := 1.312500000$$
$$FP := -0.848388672$$
$$iter := 4$$
$$p := 1.343750000$$
$$FP := -0.350982668$$
$$iter := 5$$
$$p := 1.359375000$$
$$FP := -0.096408842$$
$$iter := 6$$
$$p := 1.367187500$$
$$FP := 0.03235578$$
$$iter := 7$$
$$p := 1.363281250$$
$$FP := -0.032149969$$
$$iter := 8$$
$$p := 1.365234375$$
$$FP := 0.00007203$$
$$iter := 9$$
$$p := 1.364257812$$
$$FP := -0.016046697$$
$$iter := 10$$
$$p := 1.364746094$$
$$FP := -0.007989259$$
$$iter := 11$$
$$p := 1.364990234$$
$$FP := -0.003959107$$
$$iter := 12$$
$$p := 1.365112304$$
$$FP := -0.001943668$$
$$iter := 13$$
$$p := 1.365173340$$
$$FP := -0.000935846$$
$$iter := 14$$

Slightly more elegant script

```
> restart;
> Digits:=10;
```

$$Digits := 10 \tag{13}$$

```
> f:=x->x^3+4*x^2-10; a:=1.0; b:=2.0; FA:=f(a); FP:=f(a);
```

$$f := x \to x^3 + 4\,x^2 - 10$$
$$a := 1.0$$
$$b := 2.0$$
$$FA := -5.000$$
$$FP := -5.000 \tag{14}$$

```
> for n from 1 to 20
  while FP<>0 and (b-a)>=10^(-4) do
  p[n]:=a+(b-a)/2;
  FP:=f(p[n]);
  if FA*FP>0 then a:=p[n];
  FA:=FP;
  else
  b:=p[n]
  end if
  end do;
```

$$p_1 := 1.500000000$$
$$FP := 2.37500000$$
$$p_2 := 1.250000000$$
$$FP := -1.796875000$$
$$p_3 := 1.375000000$$
$$FP := 0.16210938$$
$$p_4 := 1.312500000$$
$$FP := -0.848388672$$
$$p_5 := 1.343750000$$
$$FP := -0.350982668$$
$$p_6 := 1.359375000$$
$$FP := -0.096408842$$
$$p_7 := 1.367187500$$
$$FP := 0.03235578$$
$$p_8 := 1.363281250$$
$$FP := -0.032149969$$
$$p_9 := 1.365234375$$
$$FP := 0.00007203$$
$$p_{10} := 1.364257812$$
$$FP := -0.016046697$$
$$p_{11} := 1.364746094$$

$$FP := -0.007989259$$
$$p_{12} := 1.364990234$$
$$FP := -0.003959107$$
$$p_{13} := 1.365112304$$
$$FP := -0.001943668$$
$$p_{14} := 1.365173340$$
$$FP := -0.000935846 \quad (15)$$

```
> solve({f(x)=0},{x});
```

$$\left\{x = \frac{1}{3}\left(71 + 3\sqrt{105}\right)^{1/3} + \frac{16}{3\left(71 + 3\sqrt{105}\right)^{1/3}} - \frac{4}{3}\right\}, \left\{x = -\frac{1}{6}\left(71 + 3\sqrt{105}\right)^{1/3} - \frac{8}{3\left(71 + 3\sqrt{105}\right)^{1/3}} - \frac{4}{3} + \frac{1}{2}\,\mathrm{I}\sqrt{3}\left(\frac{1}{3}\left(71 + 3\sqrt{105}\right)^{1/3} - \frac{16}{3\left(71 + 3\sqrt{105}\right)^{1/3}}\right)\right\}, \left\{x = -\frac{1}{6}\left(71 + 3\sqrt{105}\right)^{1/3} - \frac{8}{3\left(71 + 3\sqrt{105}\right)^{1/3}} - \frac{4}{3} - \frac{1}{2}\,\mathrm{I}\sqrt{3}\left(\frac{1}{3}\left(71 + 3\sqrt{105}\right)^{1/3} - \frac{16}{3\left(71 + 3\sqrt{105}\right)^{1/3}}\right)\right\} \quad (16)$$

```
> evalf(%);
```

$$\{x = 1.365230013\}, \{x = -2.682615007 + 0.3582593602\,\mathrm{I}\}, \{x = -2.682615007 - 0.3582593602\,\mathrm{I}\} \quad (17)$$

And here is another stopping criterion:

```
> restart; f:=x->x^3+4*x^2-10; a:=1.0; b:=2.0; FA:=f(a); FP:=f(a);
  iter:=0;
```

$$f := x \to x^3 + 4x^2 - 10$$
$$a := 1.0$$
$$b := 2.0$$
$$FA := -5.000$$
$$FP := -5.000$$
$$iter := 0 \quad (18)$$

```
> while  FP<>0 and abs(FP)>=10^(-4) do
  p:=a+(b-a)/2;
  FP:=f(p);
  iter:=iter+1;
  if FA*FP>0 then a:=p;
  FA:=FP;
  else
  b:=p
  end if
  end do;
```

$$p := 1.500000000$$
$$FP := 2.37500000$$

$$iter := 1$$
$$p := 1.250000000$$
$$FP := -1.796875000$$
$$iter := 2$$
$$p := 1.375000000$$
$$FP := 0.16210938$$
$$iter := 3$$
$$p := 1.312500000$$
$$FP := -0.848388672$$
$$iter := 4$$
$$p := 1.343750000$$
$$FP := -0.350982668$$
$$iter := 5$$
$$p := 1.359375000$$
$$FP := -0.096408842$$
$$iter := 6$$
$$p := 1.367187500$$
$$FP := 0.03235578$$
$$iter := 7$$
$$p := 1.363281250$$
$$FP := -0.032149969$$
$$iter := 8$$
$$p := 1.365234375$$
$$FP := 0.00007203$$
$$iter := 9 \quad \textbf{(19)}$$

And still another stopping criterion:

```
> restart; f:=x->x^3+4*x^2-10; a:=1.0; b:=2.0; FA:=f(a); FP:=f(a);
  iter:=0;
```

$$f := x \to x^3 + 4x^2 - 10$$
$$a := 1.0$$
$$b := 2.0$$
$$FA := -5.000$$
$$FP := -5.000$$
$$iter := 0 \quad \textbf{(20)}$$

```
> while  FP<>0 and (b-a)/b>=10^(-4) do
  p:=a+(b-a)/2;
  FP:=f(p);
  iter:=iter+1;
  if FA*FP>0 then a:=p;
  FA:=FP;
  else
  b:=p
  end if
  end do;
```

$$p := 1.500000000$$
$$FP := 2.37500000$$
$$iter := 1$$
$$p := 1.250000000$$

$$
\begin{aligned}
FP &:= -1.796875000 \\
iter &:= 2 \\
p &:= 1.375000000 \\
FP &:= 0.16210938 \\
iter &:= 3 \\
p &:= 1.312500000 \\
FP &:= -0.848388672 \\
iter &:= 4 \\
p &:= 1.343750000 \\
FP &:= -0.350982668 \\
iter &:= 5 \\
p &:= 1.359375000 \\
FP &:= -0.096408842 \\
iter &:= 6 \\
p &:= 1.367187500 \\
FP &:= 0.03235578 \\
iter &:= 7 \\
p &:= 1.363281250 \\
FP &:= -0.032149969 \\
iter &:= 8 \\
p &:= 1.365234375 \\
FP &:= 0.00007203 \\
iter &:= 9 \\
p &:= 1.364257812 \\
FP &:= -0.016046697 \\
iter &:= 10 \\
p &:= 1.364746094 \\
FP &:= -0.007989259 \\
iter &:= 11 \\
p &:= 1.364990234 \\
FP &:= -0.003959107 \\
iter &:= 12 \\
p &:= 1.365112304 \\
FP &:= -0.001943668 \\
iter &:= 13
\end{aligned}
\tag{21}
$$