```
> restart;
> p:=n->1/n;
```

$$p := n \mapsto \frac{1}{n} \tag{1}$$

```
> P[n]:=p(n)-(p(n+1)-p(n))^2/(p(n+2)-2*p(n+1)+p(n));
```

$$P_n := \frac{1}{n} - \frac{\left(\frac{1}{n+1} - \frac{1}{n}\right)^2}{\frac{1}{n+2} - \frac{2}{n+1} + \frac{1}{n}} \tag{2}$$

```
> simplify(%);
```

$$\frac{1}{2\,n+2} \tag{3}$$

```
> p:=n->1/n^2;
```

$$p := n \mapsto \frac{1}{n^2} \tag{4}$$

```
> P[n]:=p(n)-(p(n+1)-p(n))^2/(p(n+2)-2*p(n+1)+p(n));
```

$$P_n := \frac{1}{n^2} - \frac{\left(\frac{1}{(n+1)^2} - \frac{1}{n^2}\right)^2}{\frac{1}{(n+2)^2} - \frac{2}{(n+1)^2} + \frac{1}{n^2}} \tag{5}$$

```
> simplify(%);
```

$$\frac{2\,n^2+4\,n+1}{6\left(n^2+2\,n+\frac{2}{3}\right)(n+1)^2} \tag{6}$$

```
> p:=n->1/n^3;
```

$$p := n \mapsto \frac{1}{n^3} \tag{7}$$

```
> P[n]:=p(n)-(p(n+1)-p(n))^2/(p(n+2)-2*p(n+1)+p(n));
```

$$P_n := \frac{1}{n^3} - \frac{\left(\frac{1}{(n+1)^3} - \frac{1}{n^3}\right)^2}{\frac{1}{(n+2)^3} - \frac{2}{(n+1)^3} + \frac{1}{n^3}} \tag{8}$$

```
> simplify(%);
```

$$\frac{3\,n^4+12\,n^3+15\,n^2+6\,n+1}{2\left(6\,n^4+24\,n^3+33\,n^2+18\,n+4\right)(n+1)^3} \tag{9}$$

```
> P:=n->cos(1/n)-(cos(1/(n+1))-cos(1/n))^2/(cos(1/(n+2))-2*cos(1/
  (n+1))+cos(1/n));
```

$$P := n \mapsto \cos\left(\frac{1}{n}\right) - \frac{\left(\cos\left(\frac{1}{n+1}\right) - \cos\left(\frac{1}{n}\right)\right)^2}{\cos\left(\frac{1}{n+2}\right) - 2\cos\left(\frac{1}{n+1}\right) + \cos\left(\frac{1}{n}\right)} \tag{10}$$

```
> for n from 1 to 10 do
  [evalf(cos(1/n)),evalf(P(n))]
  end do;
```

$$\begin{aligned}
&[0.5403023059, 0.9617750599]\\
&[0.8775825619, 0.9821293535]\\
&[0.9449569463, 0.9897855148]\\
&[0.9689124217, 0.9934156481]\\
&[0.9800665778, 0.9954099422]\\
&[0.9861432316, 0.9966199575]\\
&[0.9898132604, 0.9974083190]\\
&[0.9921976672, 0.9979501735]\\
&[0.9938335085, 0.9983384319]\\
&[0.9950041653, 0.9986260822]
\end{aligned} \tag{11}$$

```
> P:=n->1/ln(n)-(1/ln(n+1)-1/ln(n))^2/(1/ln(n+2)-2/ln(n+1)+1/ln(n))
  ;
```

$$P := n \mapsto \frac{1}{\ln(n)} - \frac{\left(\frac{1}{\ln(n+1)} - \frac{1}{\ln(n)}\right)^2}{\frac{1}{\ln(n+2)} - \frac{2}{\ln(n+1)} + \frac{1}{\ln(n)}} \tag{12}$$

```
> for n from 990 to 1000 do
  [evalf(1/ln(n)),evalf(P(n))]
  end do;
```

$$\begin{aligned}
&[0.1449757576, 0.1286071893]\\
&[0.1449545412, 0.1286875309]\\
&[0.1449333523, 0.1285902641]\\
&[0.1449121910, 0.1286706736]\\
&[0.1448910571, 0.1285731040]\\
&[0.1448699507, 0.1285941830]\\
&[0.1448488716, 0.1285555524]\\
&[0.1448278198, 0.1285166093]\\
&[0.1448067952, 0.1285376068]\\
&[0.1447857977, 0.1285585772]\\
&[0.1447648273, 0.1283983417]
\end{aligned} \tag{13}$$

```
> restart;
> taylor(arctan(x),x,15);
```

$$x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \frac{1}{9}x^9 - \frac{1}{11}x^{11} + \frac{1}{13}x^{13} + O(x^{15}) \tag{14}$$

```
> sum((-1)^i*1/(2*i+1),i=0..infinity);
```

$$\frac{\pi}{4} \tag{15}$$

```
> p:=n->4*(sum((-1)^i*1/(2*i+1),i=0..n));
```

$$p := n \mapsto 4\left(\sum_{i=0}^{n} \frac{(-1)^i}{2i+1}\right) \tag{16}$$

```
> P:=n->p(n)-(p(n+1)-p(n))^2/(p(n+2)-2*p(n+1)+p(n));
```

$$P := n \mapsto p(n) - \frac{(p(n+1)-p(n))^2}{p(n+2)-2\,p(n+1)+p(n)} \quad (17)$$

```
> for n from 1 to 10 do
  [evalf(p(n)),evalf(P(n)),evalf(Pi)]
  end do;
```

[2.666666667, 3.133333333, 3.141592654]
[3.466666667, 3.145238095, 3.141592654]
[2.895238095, 3.139682540, 3.141592654]
[3.339682540, 3.142712843, 3.141592654]
[2.976046176, 3.140881341, 3.141592654]
[3.283738484, 3.142071817, 3.141592654]
[3.017071817, 3.141254824, 3.141592654]
[3.252365935, 3.141839619, 3.141592654]
[3.041839619, 3.141406718, 3.141592654]
[3.232315809, 3.141736099, 3.141592654] (18)

```
> for n from 90 to 100 do
  [evalf(p(n)),evalf(P(n)),evalf(Pi)]
  end do;
```

[3.152581333, 3.141592975, 3.141592654]
[3.130723409, 3.141592343, 3.141592654]
[3.152345031, 3.141592955, 3.141592654]
[3.130954657, 3.141592362, 3.141592654]
[3.152118678, 3.141592936, 3.141592654]
[3.131176269, 3.141592380, 3.141592654]
[3.151901658, 3.141592919, 3.141592654]
[3.131388838, 3.141592396, 3.141592654]
[3.151693406, 3.141592904, 3.141592654]
[3.131592904, 3.141592411, 3.141592654]
[3.151493401, 3.141592889, 3.141592654] (19)

```
> restart;
> taylor(ln(x+1),x,10);
```

$$x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5 - \frac{1}{6}x^6 + \frac{1}{7}x^7 - \frac{1}{8}x^8 + \frac{1}{9}x^9 + O(x^{10}) \quad (20)$$

```
> sum((-1)^(i+1)*1/i,i=1..infinity);
```

$$\ln(2) \quad (21)$$

```
> p:=n->sum((-1)^(i+1)*1/i,i=1..n);
```

$$p := n \mapsto \sum_{i=1}^{n} \frac{(-1)^{i+1}}{i} \quad (22)$$

```
> P:=n->p(n)-(p(n+1)-p(n))^2/(p(n+2)-2*p(n+1)+p(n));
```

$$P := n \mapsto p(n) - \frac{(p(n+1)-p(n))^2}{p(n+2)-2\,p(n+1)+p(n)} \quad (23)$$

```
> for n from 2 to 10 do
  [evalf(p(n)),evalf(P(n)),evalf(ln(2))]
  end do;
```

[0.5000000000, 0.6904761905, 0.6931471806]

```
                [0.8333333333, 0.6944444444, 0.6931471806]
                [0.5833333333, 0.6924242424, 0.6931471806]
                [0.7833333333, 0.6935897436, 0.6931471806]
                [0.6166666667, 0.6928571429, 0.6931471806]
                [0.7595238095, 0.6933473389, 0.6931471806]
                [0.6345238095, 0.6930033417, 0.6931471806]
                [0.7456349206, 0.6932539683, 0.6931471806]
                [0.6456349206, 0.6930657507, 0.6931471806]                (24)
> for n from 90 to 100 do
  [evalf(p(n)),evalf(P(n)),evalf(ln(2))]
  end do;
                [0.6876224873, 0.6931470174, 0.6931471806]
                [0.6986114983, 0.6931473385, 0.6931471806]
                [0.6877419331, 0.6931470277, 0.6931471806]
                [0.6984946212, 0.6931473287, 0.6931471806]
                [0.6878563234, 0.6931470371, 0.6931471806]
                [0.6983826392, 0.6931473196, 0.6931471806]
                [0.6879659725, 0.6931470457, 0.6931471806]
                [0.6982752508, 0.6931473113, 0.6931471806]
                [0.6880711692, 0.6931470537, 0.6931471806]
                [0.6981721793, 0.6931473037, 0.6931471806]
                [0.6881721793, 0.6931470610, 0.6931471806]                (25)
```