Homework 4

Math. 481a, Spring 2026

SHOW ALL YOUR WORK

***IMPORTANT*: Please do all your work in space provided.**
If needed, you can use backspaces. No additional sheets of paper will be accepted.
Check that your homework has a total of 8 pages—there are no blank pages.

Problem 1. (3+3 points)
Construct the Lagrange interpolating polynomial for the following functions, and find a bound for the absolute error on the interval $[x_0, x_n]$.
(a) $f(x) = \exp(2x)\cos(3x)$, $x_0 = 0, x_1 = 0.3, x_2 = 0.6, n = 2$;
(b) $f(x) = \ln(x)$, $x_0 = 1, x_1 = 1.1, x_2 = 1.3, x_3 = 1.4, n = 3$.

Problem 2. (3 points)
Use the Lagrange interpolating polynomial of degree three or less and four-digit chopping arithmetic to approximate $\cos 0.75$ using the following values. Find an error bound for the approximation.

$$\cos 0.698 = 0.7661, \quad \cos 0.733 = 0.7432, \quad \cos 0.768 = 0.7193, \quad \text{and} \quad \cos 0.803 = 0.6946.$$

Problem 3. (3 points)
Let $\omega(x) = \prod_{k=0}^{n}(x - x_k)$. Show that the interpolating polynomial on $x_0, \ldots, x_n$ can be written as

$$P(x) = \omega(x) \sum_{k=0}^{n} \frac{f(x_k)}{(x - x_k)\omega'(x_k)}.$$

Problem 4. (3 points)
Theorem 3.3 (page 109 of the textbook) states that for $n \geq 1$, $x_0, x_1, \ldots, x_n \in [a,b]$ distinct numbers, $f \in C^{n+1}[a,b]$, and for each $x \in [a,b]$, there exists $\xi(x)$ between $x_0, x_1, \ldots, x_n$ such that

$$f(x) = P_n(x) + \frac{f^{(n+1)}(\xi(x))}{(n+1)!}(x-x_0)(x-x_1)\cdots(x-x_n). \tag{1}$$

where

$$P_n(x) = f(x_0)L_{n,0}(x) + \cdots + f(x_n)L_{n,n}(x) = \sum_{k=0}^{n} f(x_k)L_{n,k}(x),$$

with

$$L_{n,k}(x) = \frac{(x-x_0)\cdots(x-x_{k-1})(x-x_{k+1})\cdots(x-x_n)}{(x_k-x_0)\cdots(x_k-x_{k-1})(x_k-x_{k+1})\cdots(x_k-x_n)} = \prod_{\substack{i=0\\ i\neq k}}^{n} \frac{(x-x_i)}{(x_k-x_i)}, \qquad k = 0, 1, \ldots n.$$

Assume $n \geq 2$ and use the above representation to show that

$$\sum_{k=0}^{n} x_k L_{n,k}(x) = x, \qquad \text{for all } x \in \mathbb{R}$$

Problem 5. (3 points)
The population of the United States from 1940 to 1990 is given by the following table:

Year	Population (in thousands)
1940	$132,165$
1950	$151,326$
1960	$179,323$
1970	$203,302$
1980	$226,542$
1990	$249,633$

Use Newton's interpolary divided-difference formula to approximate the population in the years 1930, 1965, and 2000.
Note: *Provide explicit expression for this interpolating polynomial.*

Problem 6. (2+2 points)
(a) Let $f(x) = 3x\exp(x) - \exp(2x)$. Approximate $f(1.03)$ by the Hermite interpolating polynomial of degree at most three using $x_0 = 1$ and $x_1 = 1.05$.

(b) Find the actual error and the error bound.

Problem 7. (4 points)

Use the most appropriate three-point formula to determine approximations that will complete the following table:

x	$f(x)$	$f'(x)$
1.1	1.949477	
1.2	2.199796	
1.3	2.439189	
1.4	2.670324	

The data in the above table were taken for the function $f(x) = \ln(e^{2x} - 2)$. Compute the actual errors and find error bounds using the error formulas for $x = 1.1$, $x = 1.2$, $x = 1.3$, and $x = 1.4$.

Problem 8. (4 points)

Let $f(x) = 3xe^x - \cos x$. Using the data below and formula (4.9) on page 178 to approximate $f''(1.3)$ with $h = 0.1$ and $h = 0.01$. Compare your results to $f''(1.3)$. In other words, find the errors in approximating $f''(1.3)$ for $h = 0.1$ and $h = 0.01$.

x	$f(x)$
1.20	11.59006
1.29	13.78176
1.30	14.04276
1.31	14.30741
1.40	16.86187