Homework 1

Math. 481A, Spring 2026

SHOW ALL YOUR WORK

***IMPORTANT*: Please do all your work in space provided.**
If needed, you can use backspaces. No additional sheets of paper will be accepted.
Check that your homework has a total of 6 pages—there are no blank pages.

Problem 1. (2 points)
Use the error term of a Taylor polynomial to estimate the error involved in using $\sin^2(x) \approx x^2$ to approximate $\sin^2(1°)$.

Problem 2. (3 points)
Use the Mean Value Theorem to show that $|\sin^2(x) - \sin^2(y)| \leq 2|x - y|$ for all x, y real numbers.

Problem 3. (4 points)
Let $f(x) = (2-x)^{-1}$ and $x_0 = 0$. Find the nth Taylor polynomial $P_n(x)$ of $f(x)$ expanded about x_0. Find the value of n necessary for $P_n(x)$ to approximate $f(x)$ to within 10^{-6} on $[0,1]$.

Hint: $\dfrac{1}{2-x} = \left(\dfrac{1}{2}\right)\dfrac{1}{1-(x/2)}$ *and use the formula for the geometric series.* (*Provide all details!*)

Problem 4. (3+3 points)
(a) Using the four-digit rounding arithmetic, perform the following calculations. With the exact value determined to at least five digits, compute the absolute error and relative error.

$$133 + 0.921 \qquad\qquad - 10\pi + 6e - \frac{3}{62}$$

(b) Repeat part (a) using the four-digit chopping arithmetic.

Problem 5. (2+3 points)
The Taylor polynomial of degree n of $\exp(x)$ is

$$\sum_{i=0}^{n} \frac{x^i}{i!}.$$

Use the Taylor polynomial of degree 9 and three-digit chopping arithmetic to find an approximation to $\exp(-5)$ by (a) $\exp(-5) \approx \sum_{i=0}^{9} \frac{(-1)^i 5^i}{i!}$

and by (b) $\exp(-5) = \dfrac{1}{\exp(5)} \approx \left(\sum_{i=0}^{9} \dfrac{5^i}{i!}\right)^{-1}$.

Note: An approximate value of $\exp(-5)$ correct to three digits is 6.74×10^{-3}. Which formula, (a) or (b), gives the most accuracy, and why?

Problem 6. (2+2+2+2+2 points)
Find the (optimal) rates of convergence of the following sequences as $n \to \infty$:

(a) $\lim\limits_{n\to\infty} \sin^2(1/n) = 0$

(b) $\lim\limits_{n\to\infty} n^4[1 - \cos(1/n^2)] = \dfrac{1}{2}$

(c) $\lim_{n\to\infty} \sin(1/n^3) = 0$

(d) $\lim_{n\to\infty} [\sin(1/n^2)]^2 = 0$

(e) $\lim_{n\to\infty} [\ln[(n+1)^2] - \ln(n^2)] = 0$