MATH 481A- SPRING 2026

Falling object problem I

A stone was thrown into air from a roof of a building 100 feet high, with the speed of $20\frac{\text{ft}}{\text{s}}$ and $30°$ angle (from the ground level). While neglecting air resistance, find:

(a) the system of differential equations describing the motion of the stone together with the corresponding initial conditions,

(b) the solution to the system in part (a),

(c) stone's highest altitude,

(d) the time needed for the stone to reach the ground,

(e) stone's position (from the building) when it hits the ground,

(f) stones' velocity (it's a vector !) and speed when it hits the ground.

NOTE:

Choose upward direction as the positive vertical axis and the acceleration due to gravity equal to $32\frac{ft}{s^2}$.

Solution

We place the origin of the coordinate system at the point where building meets the ground level, with the vertical axis (y) up as the positive direction and with the horizontal axis (x) in the direction of the stone movement.

(a) Using Newton's second law, one obtains the system of differential equations

$$\frac{d^2x}{dt^2} = 0, \qquad x(0) = 0, \quad x'(0) = 10\sqrt{3} \tag{1}$$

$$\frac{d^2y}{dt^2} = -32, \quad y(0) = 100, \quad y'(0) = 10. \tag{2}$$

(b) By integrating (1)-(2) twice and using the initial conditions we obtain

$$v_x(t) = 10\sqrt{3}, \qquad v_y(t) = -32 + 10, \qquad x(t) = (10\sqrt{3})t, \qquad y(t) = -16t^2 + 10t + 100, \quad t \geq 0 \tag{3}$$

(c) If t_h denotes time when the stone reaches its highest altitude, then at this moment y's component of stone's velocity $\mathbf{v}(t_h) = (v_x(t_h), v_y(t_h)) = (x'(t_h), y'(t_h))$ must be equal zero, i.e., $v_y(t_h) = 0$. From (3), $v_y(t) = -32t + 10$, and thus $v_y(t_h) = 0$ when $t_h = \dfrac{10}{32} = \dfrac{5}{16} = 0.3125$.

Stone's highest altitude is equal to $y(t_h) = -16(t_h)^2 + 10t_h + 100 = 101\frac{9}{16} = 1625/16 = 101.5625$ feet.

(d) The stone hits the ground when $y(t_g) = 0$, for some $t_g > 0$.

Solving the quadratic equation $y(t) = -16t^2+10t+100 = 0$ for its positive root, we obtain $t_g = \dfrac{5+\sqrt{1625}}{16} = \dfrac{5}{16}\left(\sqrt{65}+1\right) \approx 2.831955546$ seconds.

(e) Stone's x-coordinate when it hits the ground is equal to $x(t_g) = (10\sqrt{3})t_g = \dfrac{25}{8}\left(\sqrt{195}+\sqrt{3}\right) \approx 49.05090891$ feet.

(f) Velocity at $t = t_g$ is equal to $\mathbf{v}(t_g) = (v_x(t_g), v_y(t_g)) = 10(\sqrt{3}, -\sqrt{65})$.

Its speed, $|\mathbf{v}(t_g)| = \sqrt{v_x^2(t_g) + v_y^2(t_g)^2} = 10\sqrt{3+65} = 10\sqrt{68} = 20\sqrt{17}\,\dfrac{\text{ft}}{\text{s}} \approx 82.4621125\dfrac{\text{ft}}{\text{s}}$