Math. 481a, Spring 2026

Neville's method

Let $Q_{i,j}$, for $0 \le j \le i$, denote the interpolating polynomial of degree j on the numbers $x_{i-j}, x_{i-j+1}, \ldots, x_{i-1}, x_i$; that is,

$$Q_{i,j} = P_{i-j,i-j+1,\ldots,i-1,i}$$

We start with

$$\begin{array}{lll} x_0 & f(x_0) = P_0 = Q_{0,0} & \\ x_1 & f(x_1) = P_1 = Q_{1,0} & P_{0,1} = Q_{1,1}. \end{array}$$

where

$$P_{0,1}(x) = \frac{(x - x_0)P_1 - (x - x_1)P_0}{x_1 - x_0}.$$

Next,

$$\begin{array}{llll} x_0 & f(x_0) = P_0 = Q_{0,0} & & \\ x_1 & f(x_1) = P_1 = Q_{1,0} & P_{0,1} = Q_{1,1} & \\ x_2 & f(x_2) = P_2 = Q_{2,0} & P_{1,2} = Q_{2,1} & P_{0,1,2} = Q_{2,2}, \end{array}$$

where

$$P_{1,2}(x) = \frac{(x - x_1)P_2 - (x - x_2)P_1}{x_2 - x_1}, \quad P_{0,1,2}(x) = \frac{(x - x_0)P_{1,2}(x) - (x - x_2)P_{0,1}(x)}{x_2 - x_0}.$$

Next,

$$\begin{array}{lllll} x_0 & f(x_0) = P_0 = Q_{0,0} & & & \\ x_1 & f(x_1) = P_1 = Q_{1,0} & P_{0,1} = Q_{1,1} & & \\ x_2 & f(x_2) = P_2 = Q_{2,0} & P_{1,2} = Q_{2,1} & P_{0,1,2} = Q_{2,2} & \\ x_3 & f(x_3) = P_3 = Q_{3,0} & P_{2,3} = Q_{3,1} & P_{1,2,3} = Q_{3,2} & P_{0,1,2,3} = Q_{3,3}, \end{array}$$

where

$$P_{2,3}(x) = \frac{(x - x_2)P_3 - (x - x_3)P_2}{x_3 - x_2}, \quad P_{1,2,3}(x) = \frac{(x - x_1)P_{2,3}(x) - (x - x_3)P_{1,2}(x)}{x_3 - x_1},$$

$$P_{0,1,2,3}(x) = \frac{(x - x_0)P_{1,2,3}(x) - (x - x_3)P_{0,1,2}(x)}{x_3 - x_0}.$$

Next,

$$\begin{array}{llllll} x_0 & f(x_0) = P_0 = Q_{0,0} & & & & \\ x_1 & f(x_1) = P_1 = Q_{1,0} & P_{0,1} = Q_{1,1} & & & \\ x_2 & f(x_2) = P_2 = Q_{2,0} & P_{1,2} = Q_{2,1} & P_{0,1,2} = Q_{2,2} & & \\ x_3 & f(x_3) = P_3 = Q_{3,0} & P_{2,3} = Q_{3,1} & P_{1,2,3} = Q_{3,2} & P_{0,1,2,3} = Q_{3,3} & \\ x_4 & f(x_4) = P_4 = Q_{4,0} & P_{3,4} = Q_{4,1} & P_{2,3,4} = Q_{4,2} & P_{1,2,3,4} = Q_{4,3} & P_{0,1,2,3,4} = Q_{4,4}, \end{array}$$

where

$$P_{3,4}(x) = \frac{(x - x_3)P_4 - (x - x_4)P_3}{x_4 - x_3}, \quad P_{2,3,4}(x) = \frac{(x - x_2)P_{3,4}(x) - (x - x_4)P_{2,3}(x)}{x_4 - x_2},$$

$$P_{1,2,3,4}(x) = \frac{(x - x_1)P_{2,3,4}(x) - (x - x_4)P_{1,2,3}(x)}{x_4 - x_1}, \quad P_{0,1,2,3,4}(x) = \frac{(x - x_0)P_{1,2,3,4}(x) - (x - x_4)P_{0,1,2,3}(x)}{x_4 - x_0}.$$

Next,

$$
\begin{array}{llllll}
x_0 & f(x_0) = P_0 = Q_{0,0} & & & & \\
x_1 & f(x_1) = P_1 = Q_{1,0} & P_{0,1} = Q_{1,1} & & & \\
x_2 & f(x_2) = P_2 = Q_{2,0} & P_{1,2} = Q_{2,1} & P_{0,1,2} = Q_{2,2} & & \\
x_3 & f(x_3) = P_3 = Q_{3,0} & P_{2,3} = Q_{3,1} & P_{1,2,3} = Q_{3,2} & P_{0,1,2,3} = Q_{3,3} & \\
x_4 & f(x_4) = P_4 = Q_{4,0} & P_{3,4} = Q_{4,1} & P_{2,3,4} = Q_{4,2} & P_{1,2,3,4} = Q_{4,3} & P_{0,1,2,3,4} = Q_{4,4} \\
x_5 & f(x_5) = P_5 = Q_{5,0} & P_{4,5} = Q_{5,1} & P_{3,4,5} = Q_{5,2} & P_{2,3,4,5} = Q_{5,3} & P_{1,2,3,4,5} = Q_{5,4} \quad P_{0,1,2,3,4,5} = Q_{5,5},
\end{array}
$$

where

$$P_{4,5}(x) = \frac{(x - x_4)P_5 - (x - x_5)P_4}{x_5 - x_4}, \quad P_{3,4,5}(x) = \frac{(x - x_3)P_{4,5}(x) - (x - x_5)P_{3,4}(x)}{x_5 - x_3},$$

$$P_{2,3,4,5}(x) = \frac{(x - x_2)P_{3,4,5}(x) - (x - x_5)P_{2,3,4}(x)}{x_5 - x_2}, \quad P_{1,2,3,4,5}(x) = \frac{(x - x_1)P_{2,3,4,5}(x) - (x - x_5)P_{1,2,3,4}(x)}{x_5 - x_1},$$

$$P_{0,1,2,3,4,5}(x) = \frac{(x - x_0)P_{1,2,3,4,5}(x) - (x - x_5)P_{0,1,2,3,4}(x)}{x_5 - x_0}.$$

And so on...