

Neville method Another example

> restart;

> x[0]:=0.0; x[1]:=0.25; x[2]:=0.5;

$$x_0 := 0.$$

$$x_1 := 0.25$$

$$x_2 := 0.5$$

(1)

> P[0]:=cos(0.0); P[1]:=cos(0.25); P[2]:=cos(.5);

$$P_0 := 1.$$

$$P_1 := 0.9689124217$$

$$P_2 := 0.8775825619$$

(2)

> P[0,1]:=x->((x-x[0])*P[1]-(x-x[1])*P[0])/(x[1]-x[0]); P[0,1]:=apply(P[0,1],x); P[0,1]:=unapply(P[0,1],x);

$$P_{0,1} := x \mapsto \frac{(x-x_0) \cdot P_1 - (x-x_1) \cdot P_0}{x_1 - x_0}$$

$$P_{0,1} := -0.1243503132x + 1.000000000$$

$$P_{0,1} := x \mapsto -0.1243503132 \cdot x + 1.000000000$$

(3)

> P[1,2]:=x->((x-x[1])*P[2]-(x-x[2])*P[1])/(x[2]-x[1]); P[1,2]:=apply(P[1,2],x); P[1,2]:=unapply(P[1,2],x);

$$P_{1,2} := x \mapsto \frac{(x-x_1) \cdot P_2 - (x-x_2) \cdot P_1}{x_2 - x_1}$$

$$P_{1,2} := -0.3653194392x + 1.060242281$$

$$P_{1,2} := x \mapsto -0.3653194392 \cdot x + 1.060242281$$

(4)

> P[0,1,2]:=x->((x-x[0])*P[1,2](x)-(x-x[2])*P[0,1](x))/(x[2]-x[0]); P[0,1,2]:=apply(P[0,1,2],x); P[0,1,2]:=unapply(P[0,1,2],x);

$$P_{0,1,2} := x \mapsto \frac{(x-x_0) \cdot P_{1,2}(x) - (x-x_2) \cdot P_{0,1}(x)}{x_2 - x_0}$$

$$P_{0,1,2} := 2.000000000x(-0.3653194392x + 1.060242281) - 2.000000000(x - 0.5)(-0.1243503132x + 1.000000000)$$

$$P_{0,1,2} := x \mapsto 2.000000000 \cdot x \cdot (-0.3653194392 \cdot x + 1.060242281)$$

$$- 2.000000000 \cdot (x - 0.5) \cdot (-0.1243503132 \cdot x + 1.000000000)$$

(5)

> expand(2.000000000*x*(-.3653194392*x+1.060242281)-(2.000000000*(x-.5))*(-.1243503132*x+1.000000000));

$$-0.4819382520x^2 - 0.003865752x + 1.000000000$$

(6)

> P[0,1,2](0.35); cos(0.35); abs(P[0,1,2](0.35)-cos(0.35));

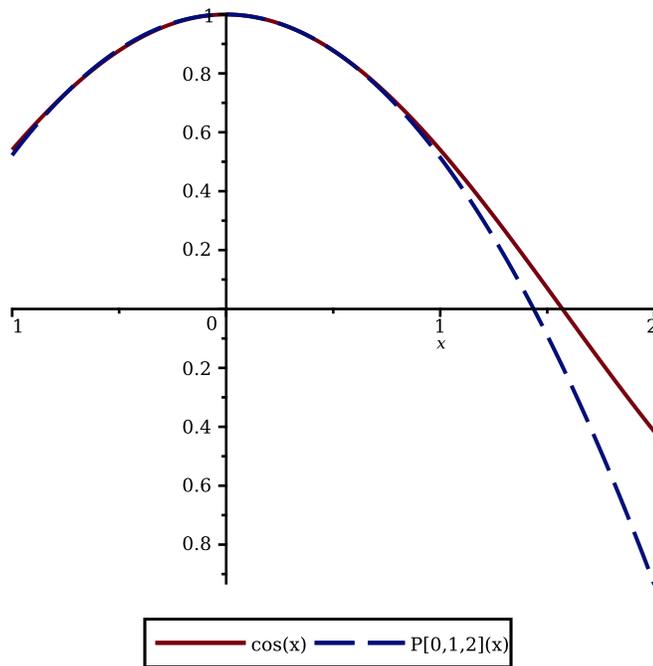
$$0.9396095514$$

$$0.9393727128$$

$$0.0002368386$$

(7)

> plot([cos(x),P[0,1,2](x)],x=-1.0..2.0,linestyle=[solid,dash], legend=["cos(x)", "P[0,1,2](x)"]);



```
> P[0,1,2](0.45); cos(0.45); abs(P[0,1,2](0.45)-cos(0.45));
0.9006679159
0.9004471024
0.0002208135
```

(8)

Adding a node

```
> x[3]:=0.4; P[3]:=cos(0.4);
x3 := 0.4
P3 := 0.9210609940
```

(9)

```
> P[2,3]:=x->((x-x[2])*P[3]-(x-x[3])*P[2])/(x[3]-x[2]); P[2,3]:=
apply(P[2,3],x); P[2,3]:=unapply(P[2,3],x);
P2,3 := x ↦  $\frac{(x-x_2) \cdot P_3 - (x-x_3) \cdot P_2}{x_3 - x_2}$ 
P2,3 := 0.434784321 x + 1.094974722
P2,3 := x ↦ 0.434784321 · x + 1.094974722
```

(10)

```
> P[1,2,3]:=x->((x-x[1])*P[2,3](x)-(x-x[3])*P[1,2](x))/(x[3]-x[1]);
P[1,2,3]:=apply(P[1,2,3],x); P[1,2,3]:=unapply(P[1,2,3],x);
P1,2,3 := x ↦  $\frac{(x-x_1) \cdot P_{2,3}(x) - (x-x_3) \cdot P_{1,2}(x)}{x_3 - x_1}$ 
P1,2,3 := 6.666666667 (x - 0.25) ( 0.434784321 x + 1.094974722)
6.666666667 (x - 0.4) ( 0.3653194392 x + 1.060242281)
P1,2,3 := x ↦ 6.666666667 · (x - 0.25) · ( 0.434784321 · x + 1.094974722)
6.666666667 · (x - 0.4) · ( 0.3653194392 · x + 1.060242281)
```

(11)

```
> P[0,1,2,3]:=x->((x-x[0])*P[1,2,3](x)-(x-x[3])*P[0,1,2](x))/(x[3]-
x[0]); P[0,1,2,3]:=apply(P[0,1,2,3],x); P[0,1,2,3]:=unapply(P[0,
1,2,3],x);
```

$$P_{0,1,2,3} := x \mapsto \frac{(x - x_0) \cdot P_{1,2,3}(x) - (x - x_3) \cdot P_{0,1,2}(x)}{x_3 - x_0}$$

$$P_{0,1,2,3} := 2.500000000 \cdot x \cdot (6.666666667 \cdot (x - 0.25) \cdot (-0.434784321 \cdot x + 1.094974722) - 6.666666667 \cdot (x - 0.4) \cdot (-0.3653194392 \cdot x + 1.060242281)) - 2.500000000 \cdot (x - 0.4) \cdot (2.000000000 \cdot x \cdot (-0.3653194392 \cdot x + 1.060242281) - 2.000000000 \cdot (x - 0.5) \cdot (-0.1243503132 \cdot x + 1.000000000))$$

$$P_{0,1,2,3} := x \mapsto 2.500000000 \cdot x \cdot (6.666666667 \cdot (x - 0.25) \cdot (-0.434784321 \cdot x + 1.094974722) - 6.666666667 \cdot (x - 0.4) \cdot (-0.3653194392 \cdot x + 1.060242281)) - 2.500000000 \cdot (x - 0.4) \cdot (2.000000000 \cdot x \cdot (-0.3653194392 \cdot x + 1.060242281) - 2.000000000 \cdot (x - 0.5) \cdot (-0.1243503132 \cdot x + 1.000000000)) \quad (12)$$

```
> expand(2.500000000*x*((6.666666667*(x-.25))*(-.434784321*x+1.094974722)-(6.666666667*(x-.4))*(-.3653194392*x+1.060242281))-(2.500000000*(x-.4))*(2.000000000*x*(-.3653194392*x+1.060242281)-(2.000000000*(x-.5))*(-.1243503132*x+1.000000000)));
```

$$0.047097600 x^3 - 0.5172614545 x^2 + 0.002021446 x + 1.000000000 \quad (13)$$

```
> plot([cos(x),P[0,1,2,3](x)],x=-1.0..2.0,linestyle=[solid,dash],legend=["cos(x)","P[0,1,2,3](x)"]);
```

