Problem 8 of Section 3.2, page 121 in 10th edition

Suppose x_j=j, for j=0,1,2,3 and it is known that P_01(x)=x+1, P_12(x)=3x-1, and P_123(1.5)=4.

Find P_0123(1.5).

```
> restart;
```

```
> x[0]:=0; x[1]:=1; x[2]:=2; x[3]:=3;
```

$$x_0 := 0$$
$$x_1 := 1$$
$$x_2 := 2$$
$$x_3 := 3 \qquad (1)$$

```
>  P[0]:=P_0; P[1]:=P_1; P[2]:=P_2; P[3]:=P_3;
```

$$P_0 := P\_0$$
$$P_1 := P\_1$$
$$P_2 := P\_2$$
$$P_3 := P\_3 \qquad (2)$$

```
> P[0,1]:=x->((x-x[0])*P[1]-(x-x[1])*P[0])/(x[1]-x[0]); P[0,1]:=
  apply(P[0,1],x): P[0,1]:=unapply(P[0,1],x);
```

$$P_{0,1} := x \mapsto \frac{(x-x_0)\cdot P_1 - (x-x_1)\cdot P_0}{x_1 - x_0}$$
$$P_{0,1} := x \mapsto x\cdot P\_1 - (x-1)\cdot P\_0 \qquad (3)$$

```
> expand(P[0,1](x));
```

$$-P\_0\,x + x\,P\_1 + P\_0 \qquad (4)$$

Since P_01(x)=x+1, we obtain that P_0=1 and P_1=2

```
> P[0]:=1; P[1]:=2;
```

$$P_0 := 1$$
$$P_1 := 2 \qquad (5)$$

```
> P[0,1]:=x->((x-x[0])*P[1]-(x-x[1])*P[0])/(x[1]-x[0]); P[0,1]:=
  apply(P[0,1],x): P[0,1]:=unapply(P[0,1],x);
```

$$P_{0,1} := x \mapsto \frac{(x-x_0)\cdot P_1 - (x-x_1)\cdot P_0}{x_1 - x_0}$$

$$P_{0,1} := x \mapsto x+1 \quad (6)$$

```
> P[1,2]:=x->((x-x[1])*P[2]-(x-x[2])*P[1])/(x[2]-x[1]); P[1,2]:=
  apply(P[1,2],x): P[1,2]:=unapply(P[1,2],x);
```

$$P_{1,2} := x \mapsto \frac{(x-x_1)\cdot P_2 - (x-x_2)\cdot P_1}{x_2 - x_1}$$

$$P_{1,2} := x \mapsto (x-1)\cdot P\_2 - 2\cdot x + 4 \quad (7)$$

```
> expand(P[1,2](x));
```

$$P\_2\,x - P\_2 - 2\,x + 4 \quad (8)$$

Since P_12(x)=3x-1, we obtain that P_2=5

```
> P[0]:=1; P[1]:=2: P[2]:=5;
```

$$P_0 := 1$$

$$P_2 := 5 \quad (9)$$

```
> P[0,1]:=x->((x-x[0])*P[1]-(x-x[1])*P[0])/(x[1]-x[0]); P[0,1]:=
  apply(P[0,1],x): P[0,1]:=unapply(P[0,1],x);
```

$$P_{0,1} := x \mapsto \frac{(x-x_0)\cdot P_1 - (x-x_1)\cdot P_0}{x_1 - x_0}$$

$$P_{0,1} := x \mapsto x+1 \quad (10)$$

```
> P[1,2]:=x->((x-x[1])*P[2]-(x-x[2])*P[1])/(x[2]-x[1]); P[1,2]:=
  apply(P[1,2],x): P[1,2]:=unapply(P[1,2],x);
```

$$P_{1,2} := x \mapsto \frac{(x-x_1)\cdot P_2 - (x-x_2)\cdot P_1}{x_2 - x_1}$$

$$P_{1,2} := x \mapsto 3\cdot x - 1 \quad (11)$$

```
> P[2,3]:=x->((x-x[2])*P[3]-(x-x[3])*P[2])/(x[3]-x[2]); P[2,3]:=
  apply(P[2,3],x): P[2,3]:=unapply(P[2,3],x);
```

$$P_{2,3} := x \mapsto \frac{(x-x_2)\cdot P_3 - (x-x_3)\cdot P_2}{x_3 - x_2}$$

$$P_{2,3} := x \mapsto (x-2)\cdot P\_3 - 5\cdot x + 15 \quad (12)$$

```
> expand(P[2,3](x));
```

$$P\_3\,x - 2\,P\_3 - 5\,x + 15 \quad (13)$$

```
> P[0,1,2]:=x->((x-x[0])*P[1,2](x)-(x-x[2])*P[0,1](x))/(x[2]-x[0]);
  P[0,1,2]:=apply(P[0,1,2],x): P[0,1,2]:=unapply(P[0,1,2],x);
```

$$P_{0,1,2} := x \mapsto \frac{(x-x_0)\cdot P_{1,2}(x) - (x-x_2)\cdot P_{0,1}(x)}{x_2 - x_0}$$

$$P_{0,1,2} := x \mapsto \frac{x\cdot(3\cdot x-1)}{2} - \frac{(x-2)\cdot(x+1)}{2} \quad (14)$$

```
> expand(P[0,1,2](x));
```

$$x^2+1 \quad (15)$$

```
> P[1,2,3]:=x->((x-x[1])*P[2,3](x)-(x-x[3])*P[1,2](x))/(x[3]-x[1]);
  P[1,2,3]:=apply(P[1,2,3],x): P[1,2,3]:=unapply(P[1,2,3],x);
```

$$P_{1,2,3} := x \mapsto \frac{(x-x_1)\cdot P_{2,3}(x)-(x-x_3)\cdot P_{1,2}(x)}{x_3-x_1}$$

$$P_{1,2,3} := x \mapsto \frac{(x-1)\cdot((x-2)\cdot P\_3-5\cdot x+15)}{2}-\frac{(x-3)\cdot(3\cdot x-1)}{2} \quad (16)$$

```
> expand(P[1,2,3](x));
```

$$\frac{1}{2}\,P\_3\,x^2-\frac{3}{2}\,P\_3\,x-4\,x^2+P\_3+15\,x-9 \quad (17)$$

```
> P[1,2,3](1.5);
```

$$-0.1250000000\,P\_3+4.500000000 \quad (18)$$

```
> solve(-0.1250000000*P_3 + 4.500000000=4,P_3);
```

$$4. \quad (19)$$

Since P_123(1.5)=4, we obtain that P_3=4

```
> P[0]:=1; P[1]:=2; P[2]:=5; P[3]:=4;
```

$$P_0 := 1$$

$$P_1 := 2$$

$$P_2 := 5$$

$$P_3 := 4 \quad (20)$$

```
> P[2,3]:=x->((x-x[2])*P[3]-(x-x[3])*P[2])/(x[3]-x[2]); P[2,3]:=
  apply(P[2,3],x): P[2,3]:=unapply(P[2,3],x);
```

$$P_{2,3} := x \mapsto \frac{(x-x_2)\cdot P_3-(x-x_3)\cdot P_2}{x_3-x_2}$$

$$P_{2,3} := x \mapsto -x+7 \quad (21)$$

```
> P[1,2,3]:=x->((x-x[1])*P[2,3](x)-(x-x[3])*P[1,2](x))/(x[3]-x[1]);
  P[1,2,3]:=apply(P[1,2,3],x): P[1,2,3]:=unapply(P[1,2,3],x);
```

$$P_{1,2,3} := x \mapsto \frac{(x-x_1)\cdot P_{2,3}(x)-(x-x_3)\cdot P_{1,2}(x)}{x_3-x_1}$$

$$P_{1,2,3} := x \mapsto \frac{(x-1)\cdot(-x+7)}{2}-\frac{(x-3)\cdot(3\cdot x-1)}{2} \quad (22)$$

```
> P[0,1,2,3]:=x->((x-x[0])*P[1,2,3](x)-(x-x[3])*P[0,1,2](x))/(x[3]-
  x[0]); P[0,1,2,3]:=apply(P[0,1,2,3],x): P[0,1,2,3]:=unapply(P[0,
  1,2,3],x);
```

$$P_{0,1,2,3} := x \mapsto \frac{(x-x_0)\cdot P_{1,2,3}(x)-(x-x_3)\cdot P_{0,1,2}(x)}{x_3-x_0}$$

(23)

$$P_{0,1,2,3} := x \mapsto \frac{x \cdot \left(\frac{(x-1)\cdot(-x+7)}{2} - \frac{(x-3)\cdot(3\cdot x-1)}{2} \right)}{3} - \frac{(x-3)\cdot\left(\frac{x\cdot(3\cdot x-1)}{2} - \frac{(x-2)\cdot(x+1)}{2} \right)}{3} \quad (23)$$

```
> expand(P[0,1,2,3](x));
```

$$-x^3 + 4x^2 - 2x + 1 \quad (24)$$

For simplicity, we call this last Langrange interpolating polynomial f(x):

```
> f:=x->-x^3+4*x^2-2*x+1;
```

$$f := x \mapsto -x^3 + 4\cdot x^2 - 2\cdot x + 1 \quad (25)$$

and the answer is

```
> f(3/2);
```

$$\frac{29}{8} \quad (26)$$

```
> evalf(29/8);
```

$$3.625000000 \quad (27)$$