

Problem 7 of Section 3.2, page 121 in 10th edition

Suppose  $x_j=j$ , for  $j=0,1,2,3$  and it is known that  $P_{01}(x)=2x+1$ ,  $P_{02}(x)=x+1$ , and  $P_{123}(2.5)=3$ .

Find  $P_{0123}(2.5)$ .

**> restart;**

**> x[0]:=0; x[1]:=1; x[2]:=2; x[3]:=3;**

$$x_0 := 0$$

$$x_1 := 1$$

$$x_2 := 2$$

$$x_3 := 3$$

(1)

**> P[0]:=P\_0; P[1]:=P\_1; P[2]:=P\_2; P[3]:=P\_3;**

$$P_0 := P_0$$

$$P_1 := P_1$$

$$P_2 := P_2$$

$$P_3 := P_3$$

(2)

**> P[0,1]:=x->((x-x[0])\*P[1]-(x-x[1])\*P[0])/(x[1]-x[0]); P[0,1]:=apply(P[0,1],x): P[0,1]:=unapply(P[0,1],x);**

$$P_{0,1} := x \mapsto \frac{(x-x_0) \cdot P_1 - (x-x_1) \cdot P_0}{x_1 - x_0}$$

$$P_{0,1} := x \mapsto x \cdot P_1 - (x-1) \cdot P_0$$

(3)

**> expand(P[0,1](x));**

$$-P_0 x + x P_1 + P_0$$

(4)

Since  $P_{01}(x)=2x+1$ , we obtain that  $P_0=1$  and  $P_1=3$

**> P[0]:=1; P[1]:=3;**

$$P_0 := 1$$

$$P_1 := 3$$

(5)

**> P[0,1]:=x->((x-x[0])\*P[1]-(x-x[1])\*P[0])/(x[1]-x[0]); P[0,1]:=apply(P[0,1],x): P[0,1]:=unapply(P[0,1],x);**

$$P_{0,1} := x \mapsto \frac{(x-x_0) \cdot P_1 - (x-x_1) \cdot P_0}{x_1 - x_0}$$

$$P_{0,1} := x \mapsto 2 \cdot x + 1$$

(6)

**> P[0,2]:=x->((x-x[0])\*P[2]-(x-x[2])\*P[0])/(x[2]-x[0]); P[0,2]:=apply(P[0,2],x): P[0,2]:=unapply(P[0,2],x);**

$$P_{0,2} := x \mapsto \frac{(x-x_0) \cdot P_2 - (x-x_2) \cdot P_0}{x_2 - x_0}$$

$$P_{0,2} := x \mapsto \frac{1}{2} \cdot x \cdot P_2 - \frac{1}{2} \cdot x + 1 \quad (7)$$

Since  $P_{02}(x)=x+1$ , we obtain  $P_2=3$

**> P[0]:=1; P[1]:=3; P[2]:=3;**

$$P_0 := 1$$

$$P_1 := 3$$

$$P_2 := 3 \quad (8)$$

**> P[1,2]:=x->((x-x[1])\*P[2]-(x-x[2])\*P[1])/(x[2]-x[1]); P[1,2]:=apply(P[1,2],x): P[1,2]:=unapply(P[1,2],x);**

$$P_{1,2} := x \mapsto \frac{(x-x_1) \cdot P_2 - (x-x_2) \cdot P_1}{x_2 - x_1}$$

$$P_{1,2} := x \mapsto 3 \quad (9)$$

**> expand(P[1,2](x));**

$$3 \quad (10)$$

**> P[0]:=1; P[1]:=3; P[2]:=3;**

$$P_0 := 1$$

$$P_1 := 3$$

$$P_2 := 3 \quad (11)$$

**> P[0,1]:=x->((x-x[0])\*P[1]-(x-x[1])\*P[0])/(x[1]-x[0]); P[0,1]:=apply(P[0,1],x): P[0,1]:=unapply(P[0,1],x);**

$$P_{0,1} := x \mapsto \frac{(x-x_0) \cdot P_1 - (x-x_1) \cdot P_0}{x_1 - x_0}$$

$$P_{0,1} := x \mapsto 2 \cdot x + 1 \quad (12)$$

**> P[1,2]:=x->((x-x[1])\*P[2]-(x-x[2])\*P[1])/(x[2]-x[1]); P[1,2]:=apply(P[1,2],x): P[1,2]:=unapply(P[1,2],x);**

$$P_{1,2} := x \mapsto \frac{(x-x_1) \cdot P_2 - (x-x_2) \cdot P_1}{x_2 - x_1}$$

$$P_{1,2} := x \mapsto 3 \quad (13)$$

**> P[2,3]:=x->((x-x[2])\*P[3]-(x-x[3])\*P[2])/(x[3]-x[2]); P[2,3]:=apply(P[2,3],x): P[2,3]:=unapply(P[2,3],x);**

$$P_{2,3} := x \mapsto \frac{(x-x_2) \cdot P_3 - (x-x_3) \cdot P_2}{x_3 - x_2}$$

$$P_{2,3} := x \mapsto (x-2) \cdot P_3 - 3 \cdot x + 9 \quad (14)$$

**> expand(P[2,3](x));**

$$P_3 x - 2 P_3 - 3 x + 9 \quad (15)$$

> P[0,1,2]:=x->((x-x[0])\*P[1,2](x)-(x-x[2])\*P[0,1](x))/(x[2]-x[0]); P[0,1,2]:=apply(P[0,1,2], x): P[0,1,2]:=unapply(P[0,1,2],x);

$$P_{0,1,2} := x \mapsto \frac{(x-x_0) \cdot P_{1,2}(x) - (x-x_2) \cdot P_{0,1}(x)}{x_2 - x_0}$$

$$P_{0,1,2} := x \mapsto \frac{3 \cdot x}{2} - \frac{(x-2) \cdot (2 \cdot x + 1)}{2} \quad (16)$$

> expand(P[0,1,2](x));

$$-x^2 + 3x + 1 \quad (17)$$

> P[1,2,3]:=x->((x-x[1])\*P[2,3](x)-(x-x[3])\*P[1,2](x))/(x[3]-x[1]); P[1,2,3]:=apply(P[1,2,3], x): P[1,2,3]:=unapply(P[1,2,3],x);

$$P_{1,2,3} := x \mapsto \frac{(x-x_1) \cdot P_{2,3}(x) - (x-x_3) \cdot P_{1,2}(x)}{x_3 - x_1}$$

$$P_{1,2,3} := x \mapsto \frac{(x-1) \cdot ((x-2) \cdot P_3 - 3 \cdot x + 9)}{2} - \frac{3 \cdot x}{2} + \frac{9}{2} \quad (18)$$

> expand(P[1,2,3](x));

$$\frac{1}{2} P_3 x^2 - \frac{3}{2} P_3 x - \frac{3}{2} x^2 + P_3 + \frac{9}{2} x \quad (19)$$

> P[1,2,3](2.5);

$$0.3750000000 P_3 + 1.875000000 \quad (20)$$

> solve(0.3750000000\*P\_3 + 1.875000000=3,P\_3);

$$3. \quad (21)$$

> P[0]:=1; P[1]:=3; P[2]:=3; P[3]:=3;

$$P_0 := 1$$

$$P_1 := 3$$

$$P_2 := 3$$

$$P_3 := 3 \quad (22)$$

> P[2,3]:=x->((x-x[2])\*P[3]-(x-x[3])\*P[2])/x[3]-x[2]; P[2,3]:=apply(P[2,3],x): P[2,3]:=unapply(P[2,3],x);

$$P_{2,3} := x \mapsto \frac{(x-x_2) \cdot P_3 - (x-x_3) \cdot P_2}{x_3 - x_2}$$

$$P_{2,3} := x \mapsto 3 \quad (23)$$

> P[1,2,3]:=x->((x-x[1])\*P[2,3](x)-(x-x[3])\*P[1,2](x))/(x[3]-x[1]); P[1,2,3]:=apply(P[1,2,3], x): P[1,2,3]:=unapply(P[1,2,3],x);

$$P_{1,2,3} := x \mapsto \frac{(x-x_1) \cdot P_{2,3}(x) - (x-x_3) \cdot P_{1,2}(x)}{x_3 - x_1} \quad (24)$$

$$P_{1,2,3} := x \mapsto 3 \quad (24)$$

`> P[0,1,2,3]:=x->((x-x[0])*P[1,2,3](x)-(x-x[3])*P[0,1,2](x))/(x[3]-x[0]); P[0,1,2,3]:=apply(P[0,1,2,3],x); P[0,1,2,3]:=unapply(P[0,1,2,3],x);`

$$P_{0,1,2,3} := x \mapsto \frac{(x-x_0) \cdot P_{1,2,3}(x) - (x-x_3) \cdot P_{0,1,2}(x)}{x_3 - x_0}$$

$$P_{0,1,2,3} := x \mapsto x - \frac{(x-3) \cdot \left( \frac{3 \cdot x}{2} - \frac{(x-2) \cdot (2 \cdot x + 1)}{2} \right)}{3} \quad (25)$$

`> expand(P[0,1,2,3](x));`

$$\frac{11}{3} x + \frac{1}{3} x^3 - 2x^2 + 1 \quad (26)$$

`> eval(P[0,1,2,3](x),x=5/2);`

$$\frac{23}{8} \quad (27)$$

`> evalf(%);`

$$2.875000000 \quad (28)$$