Problem 6 of Section 3.2, page 121 in 10th edition

Neville's is used to approximate f(0.5), giving the following table:
x_0=0, P_0= 0
x_1=0.4, P_1=2.8
x_2=0.7, P_2,   P_12,   P_012=27/7

Determine P_2=f(0.7).

```
> restart;
> x[0]:=0; x[1]:=2/5; x[2]:=7/10;
```

$$x_0 := 0$$
$$x_1 := \frac{2}{5}$$
$$x_2 := \frac{7}{10} \quad (1)$$

```
> P[0]:=0; P[1]:=14/5; P[2]:=P_2;
```

$$P_0 := 0$$
$$P_1 := \frac{14}{5}$$
$$P_2 := P\_2 \quad (2)$$

```
> P[0,1]:=x->((x-x[0])*P[1]-(x-x[1])*P[0])/(x[1]-x[0]); P[0,1]:=
  apply(P[0,1],x): P[0,1]:=unapply(P[0,1],x);
```

$$P_{0,1} := x \mapsto \frac{(x-x_0) \cdot P_1 - (x-x_1) \cdot P_0}{x_1 - x_0}$$
$$P_{0,1} := x \mapsto 7 \cdot x \quad (3)$$

```
> P[0,1](0.5);
```

$$3.5 \quad (4)$$

```
> P[1,2]:=x->((x-x[1])*P[2]-(x-x[2])*P[1])/(x[2]-x[1]); P[1,2]:=
  apply(P[1,2],x): P[1,2]:=unapply(P[1,2],x);
```

$$P_{1,2} := x \mapsto \frac{(x-x_1) \cdot P_2 - (x-x_2) \cdot P_1}{x_2 - x_1}$$
$$P_{1,2} := x \mapsto \frac{10 \cdot \left(x - \frac{2}{5}\right) \cdot P\_2}{3} - \frac{28 \cdot x}{3} + \frac{98}{15} \quad (5)$$

```
> expand(P[1,2](x));
```

(6)

$$\frac{10}{3} P\_2\, x - \frac{4}{3} P\_2 - \frac{28}{3} x + \frac{98}{15} \quad (6)$$

```
> P[0,1,2]:=x->((x-x[0])*P[1,2](x)-(x-x[2])*P[0,1](x))/(x[2]-x[0]);
  P[0,1,2]:=apply(P[0,1,2],x): P[0,1,2]:=unapply(P[0,1,2],x);
```

$$P_{0,1,2} := x \mapsto \frac{(x - x_0) \cdot P_{1,2}(x) - (x - x_2) \cdot P_{0,1}(x)}{x_2 - x_0}$$

$$P_{0,1,2} := x \mapsto \frac{10 \cdot x \cdot \left(\frac{10 \cdot \left(x - \frac{2}{5}\right) \cdot P\_2}{3} - \frac{28 \cdot x}{3} + \frac{98}{15} \right) - 10 \cdot \left(x - \frac{7}{10}\right) \cdot x}{7} \quad (7)$$

```
> expand(P[0,1,2](x));
```

$$\frac{100}{21} P\_2\, x^2 - \frac{40}{21} P\_2\, x - \frac{70}{3} x^2 + \frac{49}{3} x \quad (8)$$

For simplicity, we call this last Langrange interpolating polynomial by f(x):

```
> f:=x->((100/21)*P_2-(70/3))*x^2+((49/3)-(40/21)*P_2)*x;
```

$$f := x \mapsto \left(\frac{100 \cdot P\_2}{21} - \frac{70}{3} \right) \cdot x^2 + \left(\frac{49}{3} - \frac{40 \cdot P\_2}{21} \right) \cdot x \quad (9)$$

The value of P_0,1,2(x) at x=0.5 is 27/7, thus

```
> f(1/2);
```

$$\frac{5\, P\_2}{21} + \frac{7}{3} \quad (10)$$

and after solving for P_2, we obtain

```
> solve((5/21)*P_2+7/3=27/7,P_2);
```

$$\frac{32}{5} \quad (11)$$

```
> P_2:=32/5;
```