

Problem 5, Section 3.2

> restart;

> x[0]:=0.0; x[1]:=0.25; x[2]:=0.5; x[3]:=0.75;

$$x_0 := 0.$$

$$x_1 := 0.25$$

$$x_2 := 0.5$$

$$x_3 := 0.75$$

(1)

> P[0]:=1.0; P[1]:=2.0; P[2]:=P_2; P[3]:=8.0;

$$P_0 := 1.0$$

$$P_1 := 2.0$$

$$P_2 := P_2$$

$$P_3 := 8.0$$

(2)

> P[0,1]:=x->((x-x[0])*P[1]-(x-x[1])*P[0])/(x[1]-x[0]); P[0,1]:=apply(P[0,1],x): P[0,1]:=unapply(P[0,1],x); expand(P[0,1](x));

$$P_{0,1} := x \mapsto \frac{(x-x_0) \cdot P_1 - (x-x_1) \cdot P_0}{x_1 - x_0}$$

$$P_{0,1} := x \mapsto 4.000000000 \cdot x + 1.000000000$$

$$4.000000000 x + 1.000000000$$

(3)

> P[0,1](0.4);

$$2.600000000$$

(4)

> P[1,2]:=x->((x-x[1])*P[2]-(x-x[2])*P[1])/(x[2]-x[1]); P[1,2]:=apply(P[1,2],x): P[1,2]:=unapply(P[1,2],x); expand(P[1,2](x));

$$P_{1,2} := x \mapsto \frac{(x-x_1) \cdot P_2 - (x-x_2) \cdot P_1}{x_2 - x_1}$$

$$P_{1,2} := x \mapsto 4.000000000 \cdot (x - 0.25) \cdot P_2 - 8.000000000 \cdot x + 4.000000000$$

$$4.000000000 P_2 x - 1.000000000 P_2 - 8.000000000 x + 4.000000000$$

(5)

> P[2,3]:=x->((x-x[2])*P[3]-(x-x[3])*P[2])/(x[3]-x[2]); P[2,3]:=apply(P[2,3],x): P[2,3]:=unapply(P[2,3],x); expand(P[2,3](x));

$$P_{2,3} := x \mapsto \frac{(x-x_2) \cdot P_3 - (x-x_3) \cdot P_2}{x_3 - x_2}$$

$$P_{2,3} := x \mapsto 32.000000000 \cdot x - 16.000000000 - 4.000000000 \cdot (x - 0.75) \cdot P_2$$

$$32.000000000 x - 16.000000000 - 4.000000000 P_2 x + 3.000000000 P_2$$

(6)

> P[2,3](0.4);

$$-3.200000000 + 1.400000000 P_2$$

(7)

> solve(-3.200000000 + 1.400000000*P_2=2.4, P_2);

$$4.$$

(8)

> P[0,1,2]:=x->((x-x[0])*P[1,2](x)-(x-x[2])*P[0,1](x))/(x[2]-x[0]); P[0,1,2]:=apply(P[0,1,2],x): P[0,1,2]:=unapply(P[0,1,2],x); expand(P[0,1,2](x));

$$P_{0,1,2} := x \mapsto \frac{(x-x_0) \cdot P_{1,2}(x) - (x-x_2) \cdot P_{0,1}(x)}{x_2 - x_0}$$

$$P_{0,1,2} := x \mapsto 2.000000000 \cdot x \cdot (4.000000000 \cdot (x-0.25) \cdot P_2 - 8.000000000 \cdot x + 4.000000000) - 2.000000000 \cdot (x-0.5) \cdot (4.000000000 \cdot x + 1.000000000) \\ 8.000000000 P_2 x^2 - 2.000000000 P_2 x - 24.000000000 x^2 + 10.000000000 x + 1.000000000 \quad (9)$$

> P[1,2,3]:=x->((x-x[1])*P[2,3](x)-(x-x[3])*P[1,2](x))/(x[3]-x[1]);
P[1,2,3]:=apply(P[1,2,3],x): P[1,2,3]:=unapply(P[1,2,3],x);expand
(P[1,2,3](x));

$$P_{1,2,3} := x \mapsto \frac{(x-x_1) \cdot P_{2,3}(x) - (x-x_3) \cdot P_{1,2}(x)}{x_3 - x_1}$$

$$P_{1,2,3} := x \mapsto 2.000000000 \cdot (x-0.25) \cdot (32.000000000 \cdot x - 16.000000000 - 4.000000000 \cdot (x-0.75) \cdot P_2) - 2.000000000 \cdot (x-0.75) \cdot (4.000000000 \cdot (x-0.25) \cdot P_2 - 8.000000000 \cdot x + 4.000000000) \\ 80.000000000 x^2 - 68.000000000 x - 16.000000000 P_2 x^2 + 16.000000000 P_2 x + 14.000000000 - 3.000000000 P_2 \quad (10)$$

> P[1,2,3](0.4);

$$-0.4000000000 + 0.8400000000 P_2 \quad (11)$$

> solve(-0.4000000000 + 0.8400000000*P_2=2.96,P_2);

$$4. \quad (12)$$

> P[0,1,2,3]:=x->((x-x[0])*P[1,2,3](x)-(x-x[3])*P[0,1,2](x))/(x[3]-x[0]); P[0,1,2,3]:=apply(P[0,1,2,3],x): P[0,1,2,3]:=unapply(P[0,1,2,3],x);expand(P[0,1,2,3](x));

$$P_{0,1,2,3} := x \mapsto \frac{(x-x_0) \cdot P_{1,2,3}(x) - (x-x_3) \cdot P_{0,1,2}(x)}{x_3 - x_0}$$

$$P_{0,1,2,3} := x \mapsto 1.333333333 \cdot x \cdot (2.000000000 \cdot (x-0.25) \cdot (32.000000000 \cdot x - 16.000000000 - 4.000000000 \cdot (x-0.75) \cdot P_2) - 2.000000000 \cdot (x-0.75) \cdot (4.000000000 \cdot (x-0.25) \cdot P_2 - 8.000000000 \cdot x + 4.000000000)) - 1.333333333 \cdot (x-0.75) \cdot (2.000000000 \cdot x \cdot (4.000000000 \cdot (x-0.25) \cdot P_2 - 8.000000000 \cdot x + 4.000000000) - 2.000000000 \cdot (x-0.5) \cdot (4.000000000 \cdot x + 1.000000000)) \\ 138.66666666 x^3 - 128.00000000 x^2 - 31.99999999 P_2 x^3 + 31.99999999 P_2 x^2 + 27.33333332 x - 5.999999999 P_2 x + 0.9999999998 \quad (13)$$

> P[0,1,2,3](0.4);

$$0.3279999999 + 0.6719999998 P_2 \quad (14)$$

> solve(0.3279999999 + 0.6719999998*P_2=3.016,P_2);

$$4.000000001 \quad (15)$$