Problem 4 of Section 3.2, page 121 (10th edition)

Let P_3(x) be the interpolating polynomial for the data (0,0), (0.5,y), (1,3), (2,2). Find y if the coefficient of x^3 in P_3(x) is 6.

```
> restart;
```

```
> x[0]:=0; x[1]:=1/2; x[2]:=1; x[3]:=2;
```

$$x_0 := 0$$
$$x_1 := \frac{1}{2}$$
$$x_2 := 1$$
$$x_3 := 2 \qquad (1)$$

```
> P[0]:=0; P[1]:=y; P[2]:=3; P[3]:=2;
```

$$P_0 := 0$$
$$P_1 := y$$
$$P_2 := 3$$
$$P_3 := 2 \qquad (2)$$

```
> P[0,1]:=x->((x-x[0])*P[1]-(x-x[1])*P[0])/(x[1]-x[0]); P[0,1]:=
  apply(P[0,1],x): P[0,1]:=unapply(P[0,1],x);
```

$$P_{0,1} := x \mapsto \frac{(x - x_0)\cdot P_1 - (x - x_1)\cdot P_0}{x_1 - x_0}$$
$$P_{0,1} := x \mapsto 2\cdot x\cdot y \qquad (3)$$

```
> P[1,2]:=x->((x-x[1])*P[2]-(x-x[2])*P[1])/(x[2]-x[1]); P[1,2]:=
  apply(P[1,2],x): P[1,2]:=unapply(P[1,2],x);
```

$$P_{1,2} := x \mapsto \frac{(x - x_1)\cdot P_2 - (x - x_2)\cdot P_1}{x_2 - x_1}$$
$$P_{1,2} := x \mapsto 6\cdot x - 3 - 2\cdot(x - 1)\cdot y \qquad (4)$$

```
> expand(P[1,2](x));
```

$$-2\,x\,y + 6\,x + 2\,y - 3 \qquad (5)$$

```
> P[2,3]:=x->((x-x[2])*P[3]-(x-x[3])*P[2])/(x[3]-x[2]); P[2,3]:=
  apply(P[2,3],x): P[2,3]:=unapply(P[2,3],x);
```

$$P_{2,3} := x \mapsto \frac{(x - x_2)\cdot P_3 - (x - x_3)\cdot P_2}{x_3 - x_2}$$
$$P_{2,3} := x \mapsto -x + 4 \qquad (6)$$

```
> expand(P[2,3](x));
```

$$-x + 4 \qquad (7)$$

```
> P[0,1,2]:=x->((x-x[0])*P[1,2](x)-(x-x[2])*P[0,1](x))/(x[2]-x[0]);
  P[0,1,2]:=apply(P[0,1,2],x): P[0,1,2]:=unapply(P[0,1,2],x);
```

$$P_{0,1,2} := x \mapsto \frac{(x - x_0)\cdot P_{1,2}(x) - (x - x_2)\cdot P_{0,1}(x)}{x_2 - x_0}$$

$$P_{0,1,2} := x \mapsto x\cdot(6\cdot x - 3 - 2\cdot(x-1)\cdot y) - 2\cdot(x-1)\cdot x\cdot y \quad (8)$$

```
> expand(P[0,1,2](x));
```

$$-4x^2y + 6x^2 + 4xy - 3x \quad (9)$$

```
> P[1,2,3]:=x->((x-x[1])*P[2,3](x)-(x-x[3])*P[1,2](x))/(x[3]-x[1]);
  P[1,2,3]:=apply(P[1,2,3],x): P[1,2,3]:=unapply(P[1,2,3],x);
```

$$P_{1,2,3} := x \mapsto \frac{(x - x_1)\cdot P_{2,3}(x) - (x - x_3)\cdot P_{1,2}(x)}{x_3 - x_1}$$

$$P_{1,2,3} := x \mapsto \frac{2\cdot\left(x - \frac{1}{2}\right)\cdot(-x+4)}{3} - \frac{2\cdot(x-2)\cdot(6\cdot x - 3 - 2\cdot(x-1)\cdot y)}{3} \quad (10)$$

```
> expand(P[1,2,3](x));
```

$$-\frac{14}{3}x^2 + 13x - \frac{16}{3} + \frac{4}{3}x^2y - 4xy + \frac{8}{3}y \quad (11)$$

```
> P[0,1,2,3]:=x->((x-x[0])*P[1,2,3](x)-(x-x[3])*P[0,1,2](x))/(x[3]-
  x[0]); P[0,1,2,3]:=apply(P[0,1,2,3],x): P[0,1,2,3]:=unapply(P[0,
  1,2,3],x);
```

$$P_{0,1,2,3} := x \mapsto \frac{(x - x_0)\cdot P_{1,2,3}(x) - (x - x_3)\cdot P_{0,1,2}(x)}{x_3 - x_0}$$

$$P_{0,1,2,3} := x \mapsto \frac{x\cdot\left(\frac{2\cdot\left(x - \frac{1}{2}\right)\cdot(-x+4)}{3} - \frac{2\cdot(x-2)\cdot(6\cdot x - 3 - 2\cdot(x-1)\cdot y)}{3}\right)}{2} - \frac{(x-2)\cdot(x\cdot(6\cdot x - 3 - 2\cdot(x-1)\cdot y) - 2\cdot(x-1)\cdot x\cdot y)}{2} \quad (12)$$

```
> expand(P[0,1,2,3](x));
```

$$-\frac{16}{3}x^3 + 14x^2 - \frac{17}{3}x + \frac{8}{3}x^3y - 8x^2y + \frac{16}{3}xy \quad (13)$$

```
> p:=interp([0,1/2,1,2],[0,y,3,2],x);
```

$$p := \left(-\frac{16}{3} + \frac{8y}{3}\right)x^3 + (-8y + 14)x^2 + \left(\frac{16y}{3} - \frac{17}{3}\right)x \quad (14)$$

```
> solve(-16/3+(8/3)*y=6,y);
```

$$\frac{17}{4} \quad (15)$$

```
> y:=evalf(%);
```

$$y := 4.250000000 \quad (16)$$