Neville's Method -- Problem 2(a), Section 3.2

Use Neville's method to obtain the approximation for Lagrange interpolating polyniomial of dgrees one, two,

and three to approximate f(0.43), if f(0)=1, f(0.25)= 1.64872, f(0.5)=2.71828, and f(0.75)=4.48169.

```
> restart;
> x[0]:=0.0; x[1]:=0.25; x[2]:=0.5; x[3]:=0.75;
```

$$x_0 := 0.$$
$$x_1 := 0.25$$
$$x_2 := 0.5$$
$$x_3 := 0.75 \quad (1)$$

```
> P[0]:=1.0; P[1]:=1.64872; P[2]:=2.71828; P[3]:=4.48169;
```

$$P_0 := 1.0$$
$$P_1 := 1.64872$$
$$P_2 := 2.71828$$
$$P_3 := 4.48169 \quad (2)$$

```
> P[0,1]:=x->((x-x[0])*P[1]-(x-x[1])*P[0])/(x[1]-x[0]); P[0,1]:=
  apply(P[0,1],x): P[0,1]:=unapply(P[0,1],x);
```

$$P_{0,1} := x \mapsto \frac{(x - x_0)\cdot P_1 - (x - x_1)\cdot P_0}{x_1 - x_0}$$
$$P_{0,1} := x \mapsto 2.594880000\cdot x + 1.000000000 \quad (3)$$

```
> P[1,2]:=x->((x-x[1])*P[2]-(x-x[2])*P[1])/(x[2]-x[1]); P[1,2]:=
  apply(P[1,2],x): P[1,2]:=unapply(P[1,2],x);
```

$$P_{1,2} := x \mapsto \frac{(x - x_1)\cdot P_2 - (x - x_2)\cdot P_1}{x_2 - x_1}$$
$$P_{1,2} := x \mapsto 4.278240000\cdot x + 0.5791600000 \quad (4)$$

```
> P[2,3]:=x->((x-x[2])*P[3]-(x-x[3])*P[2])/(x[3]-x[2]); P[2,3]:=
  apply(P[2,3],x): P[2,3]:=unapply(P[2,3],x);
```

$$P_{2,3} := x \mapsto \frac{(x - x_2)\cdot P_3 - (x - x_3)\cdot P_2}{x_3 - x_2}$$
$$P_{2,3} := x \mapsto 7.053640000\cdot x - 0.8085400000 \quad (5)$$

```
> P[0,1,2]:=x->((x-x[0])*P[1,2](x)-(x-x[2])*P[0,1](x))/(x[2]-x[0]);
  P[0,1,2]:=apply(P[0,1,2],x): P[0,1,2]:=unapply(P[0,1,2],x);
```

$$P_{0,1,2} := x \mapsto \frac{(x - x_0)\cdot P_{1,2}(x) - (x - x_2)\cdot P_{0,1}(x)}{x_2 - x_0}$$

$$P_{0,1,2} := x \mapsto 2.000000000\cdot x\cdot(4.278240000\cdot x + 0.5791600000) - 2.000000000\cdot(x - 0.5)\cdot(2.594880000\cdot x + 1.000000000) \quad (6)$$

```
> P[1,2,3]:=x->((x-x[1])*P[2,3](x)-(x-x[3])*P[1,2](x))/(x[3]-x[1]);
  P[1,2,3]:=apply(P[1,2,3],x): P[1,2,3]:=unapply(P[1,2,3],x);
```

$$P_{1,2,3} := x \mapsto \frac{(x - x_1)\cdot P_{2,3}(x) - (x - x_3)\cdot P_{1,2}(x)}{x_3 - x_1}$$

$$P_{1,2,3} := x \mapsto 2.000000000\cdot(x - 0.25)\cdot(7.053640000\cdot x - 0.8085400000) - 2.000000000\cdot(x - 0.75)\cdot(4.278240000\cdot x + 0.5791600000) \quad (7)$$

```
> P[0,1,2,3]:=x->((x-x[0])*P[1,2,3](x)-(x-x[3])*P[0,1,2](x))/(x[3]-
  x[0]); P[0,1,2,3]:=apply(P[0,1,2,3],x): P[0,1,2,3]:=unapply(P[0,
  1,2,3],x);
```

$$P_{0,1,2,3} := x \mapsto \frac{(x - x_0)\cdot P_{1,2,3}(x) - (x - x_3)\cdot P_{0,1,2}(x)}{x_3 - x_0}$$

$$P_{0,1,2,3} := x \mapsto 1.333333333\cdot x\cdot(2.000000000\cdot(x - 0.25)\cdot(7.053640000\cdot x - 0.8085400000) - 2.000000000\cdot(x - 0.75)\cdot(4.278240000\cdot x + 0.5791600000)) - 1.333333333\cdot(x - 0.75)\cdot(2.000000000\cdot x\cdot(4.278240000\cdot x + 0.5791600000) - 2.000000000\cdot(x - 0.5)\cdot(2.594880000\cdot x + 1.000000000)) \quad (8)$$

```
> P[1,2](0.43), P[1,2,3](0.43), P[0,1,2,3](0.43);
```

$$2.418803200,\ 2.348863120,\ 2.360604734 \quad (9)$$

```
> expand(1.333333333*x*((2.000000000*(x-.25))*(7.053640000*
  x-.8085400000)-(2.000000000*(x-.75))*(4.278240000*x+.5791600000))
  -(1.333333333*(x-.75))*(2.000000000*x*(4.278240000*x+.5791600000)
  -(2.000000000*(x-.5))*(2.594880000*x+1.000000000)));
```

$$2.912106666\,x^3 + 1.182640000\,x^2 + 2.117213333\,x + 0.9999999998 \quad (10)$$