Fast converging fixed-point iteration approximations to $\sqrt{3}$ - Function g1=(1/2)*(x+3/x)

And not so fast converging fixed-point iteration schemes:

to $\sqrt{3}$ - Function g2=x-(x^2-3)/6.

The derivative g2'($\sqrt{3}$) $= 0.4226497307$.

to $-\sqrt{3}$ - Function g3=x+(x^2-3)/6.

The derivative g3'($-\sqrt{3}$) $= 0.4226497307$, while

the derivative g3'($\sqrt{3}$) $= 1.577350269$.

```
g1:=x->(1/2)*(x+3/x);
```

$$g1 := x \rightarrow \frac{1}{2}\, x + \frac{3}{2\, x} \tag{1}$$

```
> p0:=1.5;
```

$$p0 := 1.5 \tag{2}$$

```
> for n from 1 to 10 do
  p:=g1(p0);
  err:=abs(p-p0);
  if err>=10^(-10) then
  p0:=p;
  else
  break
  end if;
  end do;
```

$$p := 1.750000000$$
$$err := 0.250000000$$
$$p := 1.732142857$$
$$err := 0.017857143$$
$$p := 1.732050810$$
$$err := 0.000092047$$
$$p := 1.732050808$$
$$err := 2.\,10^{-9}$$
$$p := 1.732050808$$
$$err := 0. \tag{3}$$

```
> evalf((3)^(1/2));
```

$$1.732050808 \tag{4}$$

```
> g2:=x->x-(x^2-3)/6;
```

$$g2 := x \rightarrow x - \frac{1}{6}\, x^2 + \frac{1}{2} \tag{5}$$

```
> p0:=1.0;
```

$$p0 := 1.0 \tag{6}$$

```
> for n from 1 to 20 do
p:=g2(p0);
err:=abs(p-p0);
if err>=10^(-8) then
p0:=p;
else
break
end if;
end do;
```

$$p := 1.333333333$$
$$err := 0.333333333$$
$$p := 1.537037037$$
$$err := 0.203703704$$
$$p := 1.643289895$$
$$err := 0.106252858$$
$$p := 1.693222948$$
$$err := 0.049933053$$
$$p := 1.715388956$$
$$err := 0.022166008$$
$$p := 1.724962411$$
$$err := 0.009573455$$
$$p := 1.729046524$$
$$err := 0.004084113$$
$$p := 1.730779544$$
$$err := 0.001733020$$
$$p := 1.731513239$$
$$err := 0.000733695$$
$$p := 1.731823556$$
$$err := 0.000310317$$
$$p := 1.731954751$$
$$err := 0.000131195$$
$$p := 1.732010208$$
$$err := 0.000055457$$
$$p := 1.732033648$$
$$err := 0.000023440$$
$$p := 1.732043555$$
$$err := 0.000009907$$
$$p := 1.732047742$$
$$err := 0.000004187$$
$$p := 1.732049512$$
$$err := 0.000001770$$
$$p := 1.732050260$$
$$err := 7.48\ 10^{-7}$$
$$p := 1.732050576$$
$$err := 3.16\ 10^{-7}$$
$$p := 1.732050710$$
$$err := 1.34\ 10^{-7}$$
$$p := 1.732050766$$

$$err := 5.6\ 10^{-8} \quad (7)$$

```
> gp2:=D(g2);
```

$$gp2 := x \rightarrow 1 - \frac{1}{3}x \quad (8)$$

```
> evalf(gp2((3)^(1/2)));
```

0.4226497307 (9)

```
> g3:=x->x+(x^2-3)/6;
```

$$g3 := x \rightarrow x + \frac{1}{6}x^2 - \frac{1}{2} \quad (10)$$

```
> p0:=1.0;
```

$$p0 := 1.0 \quad (11)$$

```
> for n from 1 to 20 do
p:=g3(p0);
err:=abs(p-p0);
if err>=10^(-8) then
p0:=p;
else
break
end if;
end do;
```

$p := 0.6666666670$
$err := 0.3333333330$
$p := 0.2407407412$
$err := 0.4259259258$
$p := -0.2495999081$
$err := 0.4903406493$
$p := -0.7392165558$
$err := 0.4896166477$
$p := -1.148143036$
$err := 0.4089264802$
$p := -1.428437631$
$err := 0.280294595$
$p := -1.588365287$
$err := 0.159927656$
$p := -1.667881240$
$err := 0.079515953$
$p := -1.704243268$
$err := 0.036362028$
$p := -1.720169082$
$err := 0.015925814$
$p := -1.727005470$
$err := 0.006836388$
$p := -1.729914154$
$err := 0.002908684$
$p := -1.731146991$
$err := 0.001232837$
$p := -1.731668674$

$$err := 0.000521683$$
$$p := -1.731889274$$
$$err := 0.000220600$$
$$p := -1.731982531$$
$$err := 0.000093257$$
$$p := -1.732021950$$
$$err := 0.000039419$$
$$p := -1.732038611$$
$$err := 0.000016661$$
$$p := -1.732045653$$
$$err := 0.000007042$$
$$p := -1.732048629$$
$$err := 0.000002976 \qquad (12)$$

```
> gp3:=D(g3);
```

$$gp3 := x \rightarrow 1 + \frac{1}{3}x \qquad (13)$$

```
> evalf(gp3((3)^(1/2)));
```

1.577350269 **(14)**

```
> evalf(gp3(-(3)^(1/2)));
```

0.4226497307 **(15)**