2.4 Infinite Limits

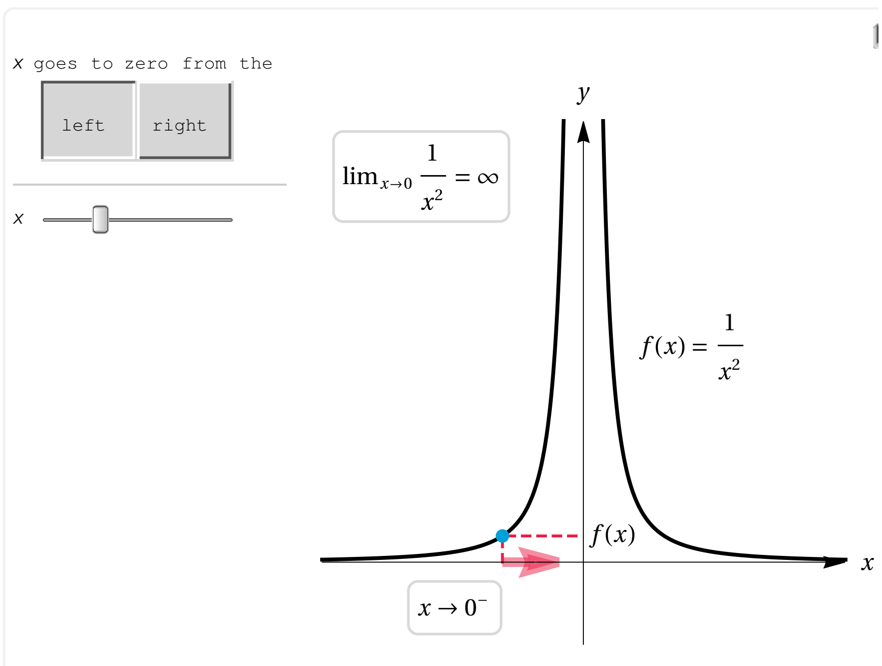
Two more limit scenarios are frequently encountered in calculus and are discussed in this and the following sections. An *infinite limit* occurs when function values increase or decrease without bound near a point. The other type of limit, known as a *limit at infinity*, occurs when the independent variable x increases or decreases without bound. The ideas behind infinite limits and limits at infinity are quite different. Therefore, it is important to distinguish these limits and the methods used to calculate them.

An Overview »

To illustrate the differences between limits at infinity and infinite limits, consider the values of $f(x) = \frac{1}{x^2}$ in Table 2.5. As x approaches 0 from either side, $f(x)$ grows larger and larger. Because $f(x)$ does not approach a finite number as x approaches 0, $\lim_{x\to 0} f(x)$ does not exist. Nevertheless, we use limit notation and write $\lim_{x\to 0} f(x) = \infty$. The infinity symbol indicates that $f(x)$ grows arbitrarily large as x approaches 0. This is an example of an *infinite limit*; in general, the *dependent variable* becomes arbitrarily large in magnitude as the *independent variable* approaches a finite number.

Table 2.5

x	$f(x) = \frac{1}{x^2}$
± 0.1	100
± 0.01	10,000
± 0.001	1,000,000
↓	↓
0	∞



With *limits at infinity*, the opposite occurs: The *dependent variable* approaches a finite number as the *independent variable* becomes arbitrarily large in magnitude. In Table 2.6 we see that $f(x) = \frac{1}{x^2}$ approaches 0 as x increases. In this case, we write $\lim_{x\to\infty} f(x) = 0$.

Table 2.6 dc2s4f23b

x	$f(x) = \frac{1}{x^2}$
10	0.01
100	0.0001
1000	0.000001
↓	↓
∞	0

A general picture of these two limit scenarios—occurring with the same function—is shown in **Figure 2.22**.

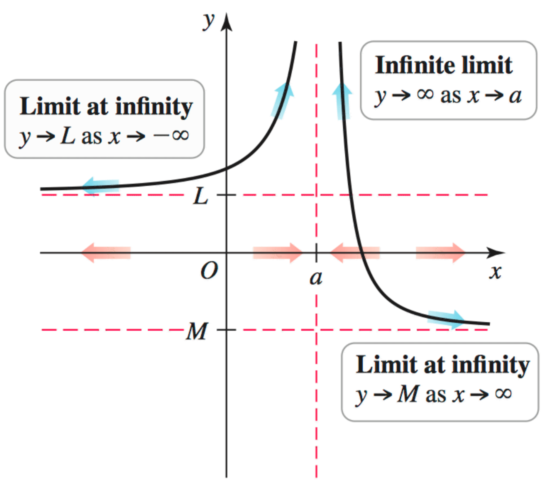


Figure 2.22

Infinite Limits »

The following definition of infinite limits is informal, but it is adequate for most functions encountered in this book. A precise definition is given in Section 2.7.

DEFINITION **Infinite Limits**

Suppose f is defined for all x near a. If $f(x)$ grows arbitrarily large for all x sufficiently close (but not equal) to a (**Figure 2.23a**), we write

$$\lim_{x \to a} f(x) = \infty$$

and say the limit of $f(x)$ as x approaches a is infinity.

If $f(x)$ is negative and grows arbitrarily large in magnitude for all x sufficiently close (but not equal) to a (**Figure 2.23b**), we write

$$\lim_{x \to a} f(x) = -\infty$$

and say the limit of $f(x)$ as x approaches a is negative infinity. *In both cases, the limit does not exist.*

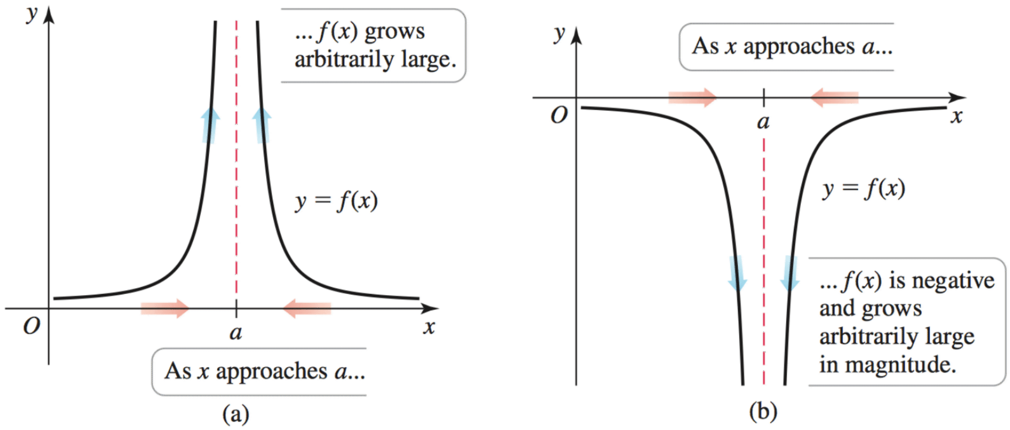


Figure 2.23

EXAMPLE 1 Infinite limits

Analyze $\lim_{x\to 1} \dfrac{x}{(x^2-1)^2}$ and $\lim_{x\to -1} \dfrac{x}{(x^2-1)^2}$ using the graph of the function.

SOLUTION »

The graph of $f(x) = \dfrac{x}{(x^2-1)^2}$ (**Figure 2.24**) shows that as x approaches 1 (from either side), the values of f grow arbitrarily large. Therefore, the limit does not exist and we write

$$\lim_{x\to 1} \frac{x}{(x^2-1)^2} = \infty.$$

As x approaches -1, the values of f are negative and grow arbitrarily large in magnitude; therefore,

$$\lim_{x\to -1} \frac{x}{(x^2-1)^2} = -\infty.$$

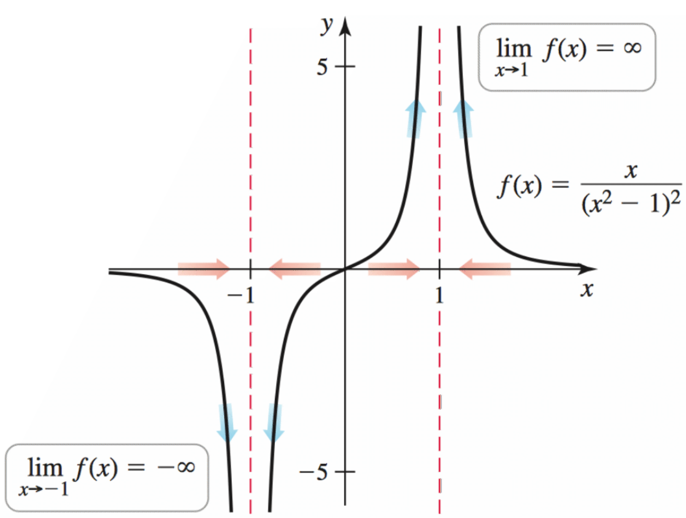


Figure 2.24

Related Exercise 6 ◆

Example 1 illustrates *two-sided* infinite limits. As with finite limits, we also need to work with right-sided and left-sided infinite limits.

DEFINITION One-Sided Infinite Limits

Suppose f is defined for all x near a with $x > a$. If $f(x)$ becomes arbitrarily large for all x sufficiently close to a with $x > a$, we write $\lim_{x \to a^+} f(x) = \infty$ (**Figure 2.25a**). The one-sided infinite limits $\lim_{x \to a^+} f(x) = -\infty$ (**Figure 2.25b**), $\lim_{x \to a^-} f(x) = \infty$ (**Figure 2.25c**), and $\lim_{x \to a^-} f(x) = -\infty$ (**Figure 2.25d**) are defined analogously.

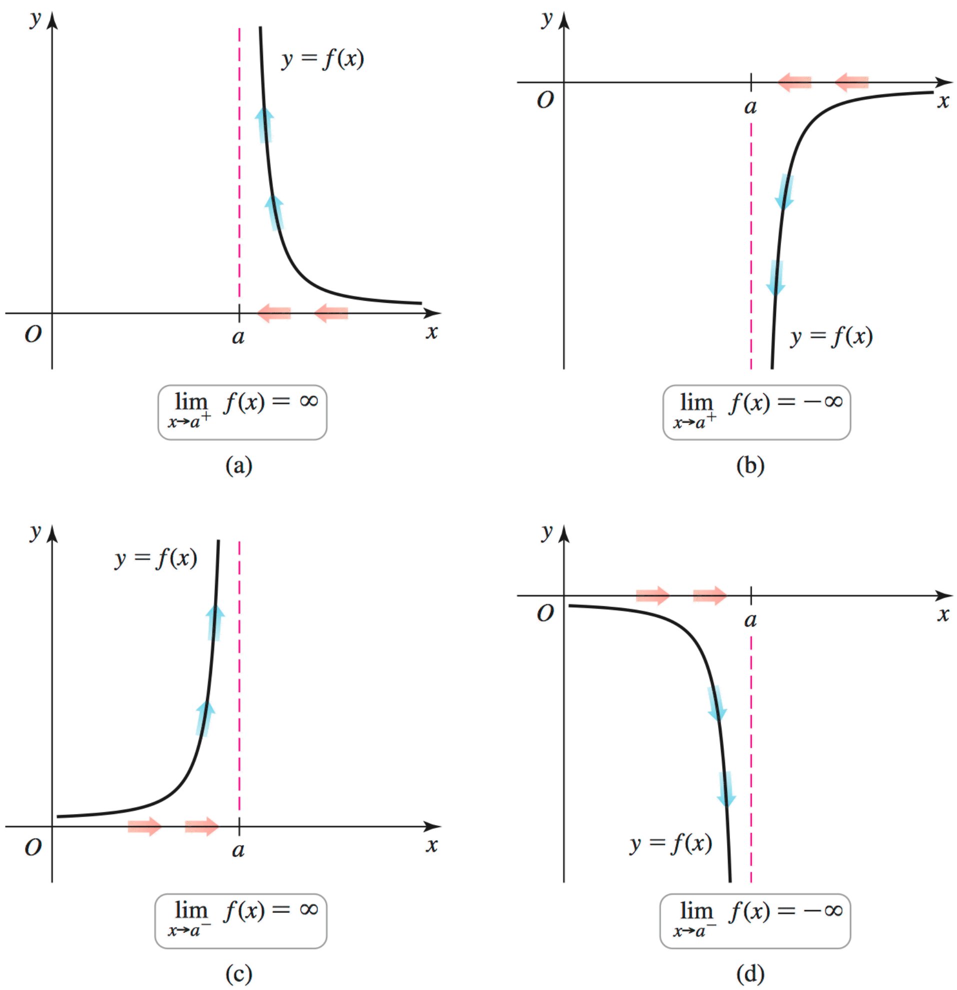


Figure 2.25

In all the infinite limits illustrated in Figure 2.25, the line $x = a$ is called a *vertical asymptote*; it is a vertical line that is approached by the graph of f as x approaches a.

DEFINITION **Vertical Asymptote**

If $\lim_{x\to a} f(x) = \pm\infty$, $\lim_{x\to a^+} f(x) = \pm\infty$, or $\lim_{x\to a^-} f(x) = \pm\infty$, the line $x = a$ is called a **vertical asymptote** of f.

Quick Check 1 Sketch the graph of a function and its vertical asymptote that satisfies the conditions $\lim_{x\to 2^+} f(x) = -\infty$ and $\lim_{x\to 2^-} f(x) = \infty$. ◆

Answer »

Answers will vary, but all graphs should have a vertical asymptote at $x = 2$.

EXAMPLE 2 **Determining limits graphically**

The vertical lines $x = 1$ and $x = 3$ are vertical asymptotes of the function $g(x) = \dfrac{x-2}{(x-1)^2\,(x-3)}$. Use **Figure 2.26** to analyze the following limits.

a. $\lim_{x\to 1} g(x)$

b. $\lim_{x\to 3^-} g(x)$

c. $\lim_{x\to 3} g(x)$

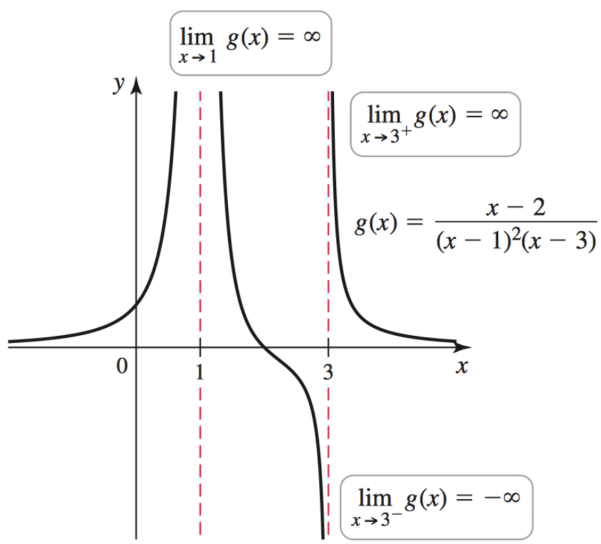


Figure 2.26

SOLUTION »

a. The values of g grow arbitrarily large as x approaches 1 from either side. Therefore, $\lim_{x\to 1} g(x) = \infty$.

b. The values of g are negative and grow arbitrarily large in magnitude as x approaches 3 from the left, so $\lim_{x\to 3^-} g(x) = -\infty$.

c. Note that $\lim_{x\to 3^+} g(x) = \infty$ and $\lim_{x\to 3^-} g(x) = -\infty$. Because g behaves differently as $x \to 3^-$ and as $x \to 3^+$, we do not write $\lim_{x\to 3} g(x) = \infty$ nor do we write $\lim_{x\to 3} g(x) = -\infty$. We simply say that the limit does not exist.

Related Exercises 7–8 ◆

Finding Infinite Limits Analytically »

Many infinite limits are analyzed using a simple arithmetic property: The fraction a/b grows arbitrarily large in magnitude if b approaches 0 while a remains nonzero and relatively constant. For example, consider the fraction $(5 + x)/x$ for values of x approaching 0 from the right (Table 2.7).

Table 2.7

x	$\frac{5+x}{x}$
0.01	$\frac{5.01}{0.01} = 501$
0.001	$\frac{5.001}{0.001} = 5001$
0.0001	$\frac{5.0001}{0.0001} = 50{,}001$
↓	↓
0^+	∞

We see that $\frac{5+x}{x} \to \infty$ as $x \to 0^+$ because the numerator $5 + x$ approaches 5 while the denominator is positive and approaches 0. Therefore, we write $\lim_{x\to 0^+} \frac{5+x}{x} = \infty$. Similarly, $\lim_{x\to 0^-} \frac{5+x}{x} = -\infty$ because the numerator approaches 5 while the denominator approaches 0 through negative values.

EXAMPLE 3 Determining limits analytically

Analyze the following limits.

a. $\lim_{x\to 3^+} \frac{2-5x}{x-3}$

b. $\lim_{x\to 3^-} \frac{2-5x}{x-3}$

SOLUTION »

a. As $x \to 3^+$, the numerator $2 - 5x$ approaches $2 - 5(3) = -13$ while the denominator $x - 3$ is positive and approaches 0. Therefore,

$$\lim_{x\to 3^+} \frac{\overbrace{2-5x}^{\text{approaches } -13}}{\underbrace{x-3}_{\substack{\text{positive and}\\ \text{approaches } 0}}} = -\infty.$$

b. As $x \to 3^-$, $2 - 5x$ approaches $2 - 5(3) = -13$ while $x - 3$ is negative and approaches 0. Therefore,

$$\lim_{x \to 3^-} \frac{\overbrace{2 - 5x}^{\text{approaches } -13}}{\underbrace{x - 3}_{\substack{\text{negative and} \\ \text{approaches } 0}}} = \infty.$$

These limits imply that the given function has a vertical asymptote at $x = 3$; they also imply that the two-sided limit $\lim_{x \to 3} \frac{2x - 5}{x - 3}$ does not exist.

Related Exercises 21–22 ◆

Quick Check 2 Analyze $\lim_{x \to 0^+} \frac{x - 5}{x}$ and $\lim_{x \to 0^-} \frac{x - 5}{x}$ by determining the sign of the numerator and denominator. ◆

Answer »

$-\infty$; ∞

EXAMPLE 4 Determining limits analytically

Analyze $\lim_{x \to -4^+} \frac{-x^3 + 5x^2 - 6x}{-x^3 - 4x^2}$.

SOLUTION »

First we factor and simplify, assuming $x \neq 0$:

$$\frac{-x^3 + 5x^2 - 6x}{-x^3 - 4x^2} = \frac{-x(x - 2)(x - 3)}{-x^2(x + 4)} = \frac{(x - 2)(x - 3)}{x(x + 4)}.$$

Note »

As $x \to -4^+$, we find that

$$\lim_{x \to -4^+} \frac{-x^3 + 5x^2 - 6x}{-x^3 - 4x^2} = \lim_{x \to -4^+} \frac{\overbrace{(x - 2)(x - 3)}^{\text{approaches } 42}}{\underbrace{x(x + 4)}_{\substack{\text{negative and} \\ \text{approaches } 0}}} = -\infty.$$

This limit implies that the given function has a vertical asymptote at $x = -4$.

Related Exercises 28, 31 ◆

Quick Check 3 Verify that $x(x + 4) \to 0$ through negative values as $x \to -4^+$. ◆

Answer »

As $x \to -4^+$, $x < 0$ and $(x + 4) > 0$, so $x(x + 4) \to 0$ through negative values.

EXAMPLE 5 Location of vertical asymptotes

Let $f(x)=\dfrac{x^2-4x+3}{x^2-1}$. Determine the following limits and find the vertical asymptotes of f. Verify your work with a graphing utility.

a. $\lim\limits_{x\to 1} f(x)$

b. $\lim\limits_{x\to -1^-} f(x)$

c. $\lim\limits_{x\to -1^+} f(x)$

Note »

> Example 5 illustrates that $f(x)/g(x)$ might not grow arbitrarily large in magnitude if *both* $f(x)$ and $g(x)$ approach 0. Such limits are called *indeterminate forms* and are examined in detail in Section 4.7.

SOLUTION »

a. Notice that as $x\to 1$, both the numerator and denominator of f approach 0, and the function is undefined at $x=1$. To compute $\lim\limits_{x\to 1} f(x)$, we first factor:

$$\begin{aligned}\lim_{x\to 1} f(x) &= \lim_{x\to 1}\frac{x^2-4x+3}{x^2-1}\\ &= \lim_{x\to 1}\frac{(x-1)(x-3)}{(x-1)(x+1)} && \text{Factor.}\\ &= \lim_{x\to 1}\frac{(x-3)}{(x+1)} && \text{Cancel like factors, } x\neq 1.\\ &= \frac{1-3}{1+1}=-1. && \text{Substitute } x=1.\end{aligned}$$

Note »

> It is permissible to cancel the $x-1$ factors in $\lim\limits_{x\to 1}\dfrac{(x-1)(x-3)}{(x-1)(x+1)}$ because x approaches 1, but is not equal to 1. Therefore $x-1\neq 0$.

Therefore, $\lim\limits_{x\to 1} f(x)=-1$ (even though $f(1)$ is undefined). The line $x=1$ is *not* a vertical asymptote of f.

b. In part (a) we showed that

$$f(x)=\frac{x^2-4x+3}{x^2-1}=\frac{x-3}{x+1},\ \text{provided } x\neq 1.$$

We use this fact again. As x approaches -1 from the left, the one-sided limit is

$$\lim_{x\to -1^-} f(x)=\lim_{x\to -1^-}\frac{\overbrace{x-3}^{\text{approaches } -4}}{\underbrace{x+1}_{\substack{\text{negative and}\\ \text{approaches } 0}}}=\infty.$$

c. As x approaches -1 from the right, the one-sided limit is

$$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} \frac{\overbrace{x-3}^{\text{approaches } -4}}{\underbrace{x+1}_{\text{positive and approaches } 0}} = -\infty.$$

The infinite limits $\lim_{x\to -1^+} f(x) = -\infty$ and $\lim_{x\to -1^-} f(x) = \infty$ each imply that the line $x = -1$ is a vertical asymptote of f. The graph of f generated by a graphing utility *may* appear as shown in **Figure 2.27**. If so, two corrections must be made. A hole should appear in the graph at $(1, -1)$ because $\lim_{x\to 1} f(x) = -1$, but $f(1)$ is undefined. It is also a good idea to replace the solid vertical line with a dashed line to emphasize that the vertical asymptote is not a part of the graph of f.

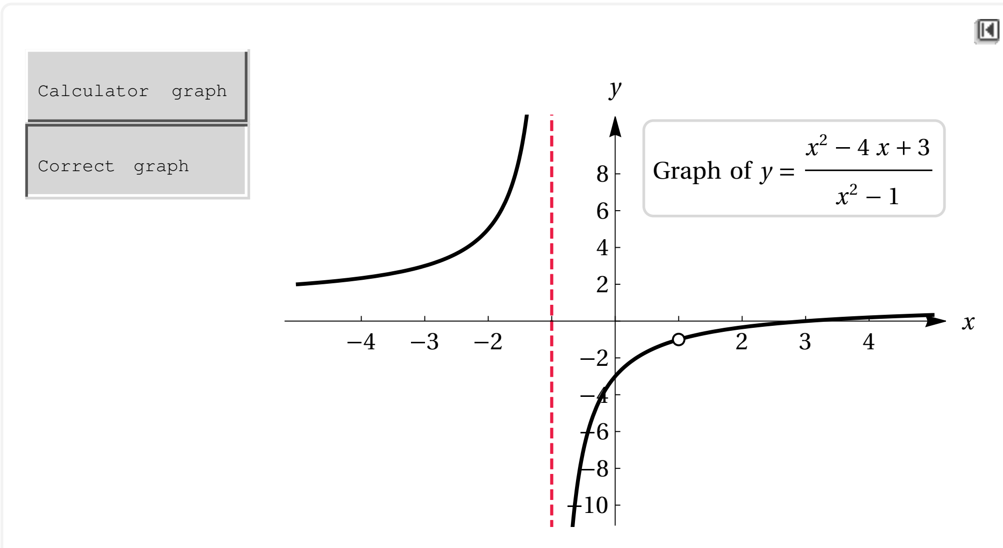


Figure 2.27

Note »

> Graphing utilities vary in how they display vertical asymptotes. The errors shown in Figure 2.27 do *not* occur on all graphing utilities.

Related Exercises 45–46 ◆

Quick Check 4 The line $x = 2$ is not a vertical asymptote of $y = \frac{(x-1)(x-2)}{x-2}$. Why not? ◆

Answer »

> $\lim_{x\to 2} \frac{(x-1)(x-2)}{x-2} = \lim_{x\to 2} (x-1) = 1$, which is not an infinite limit, so $x = 2$ is not a vertical asymptote.

EXAMPLE 6 Limits of trigonometric functions

Evaluate the following limits.

a. $\lim_{\theta \to 0^+} \cot \theta$

b. $\lim_{\theta \to 0^-} \cot \theta$

SOLUTION »

a. Recall that $\cot \theta = \cos \theta / \sin \theta$. Furthermore, it was shown in Section 2.3 that $\lim_{\theta \to 0^+} \cos \theta = 1$ and $\sin \theta$ is positive and approaches 0 as $\theta \to 0^+$. Therefore, as $\theta \to 0^+$, $\cot \theta$ becomes arbitrarily large and positive, which means $\lim_{\theta \to 0^+} \cot \theta = \infty$. This limit is consistent with the graph of $\cot \theta$ (**Figure 2.28**), which has a vertical asymptote at $\theta = 0$.

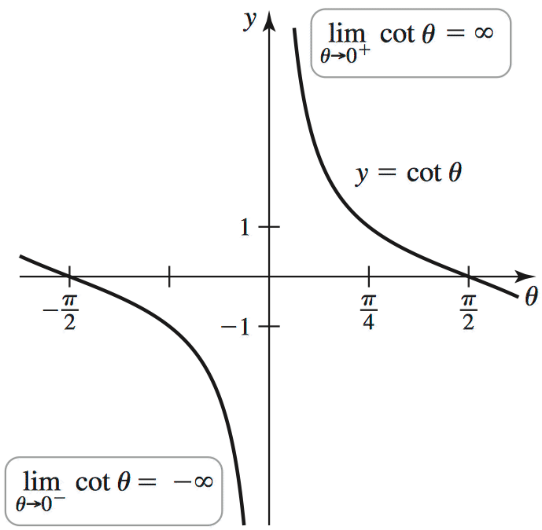


Figure 2.28

b. In this case, $\lim_{\theta \to 0^-} \cos \theta = 1$ and as $\theta \to 0^-$, $\sin \theta \to 0$ with $\sin \theta < 0$. Therefore, as $\theta \to 0^-$, $\cot \theta$ is negative and becomes arbitrarily large in magnitude. It follows that $\lim_{\theta \to 0^-} \cot \theta = -\infty$, as confirmed by the graph of $\cot \theta$.

Related Exercises 39–40 ◆

Exercises »

Getting Started »

Practice Exercises »

21–44. Determining limits analytically *Determine the following limits.*

21.

a. $\lim_{x \to 2^+} \frac{1}{x-2}$

b. $\lim_{x \to 2^-} \frac{1}{x-2}$

c. $\lim_{x \to 2} \frac{1}{x-2}$

22.

a. $\lim_{x\to 3^+} \frac{2}{(x-3)^3}$

b. $\lim_{x\to 3^-} \frac{2}{(x-3)^3}$

c. $\lim_{x\to 3} \frac{2}{(x-3)^3}$

23.

a. $\lim_{x\to 4^+} \frac{x-5}{(x-4)^2}$

b. $\lim_{x\to 4^-} \frac{x-5}{(x-4)^2}$

c. $\lim_{x\to 4} \frac{x-5}{(x-4)^2}$

24.

a. $\lim_{x\to 1^+} \frac{x}{|x-1|}$

b. $\lim_{x\to 1^-} \frac{x}{|x-1|}$

c. $\lim_{x\to 1} \frac{x}{|x-1|}$

25.

a. $\lim_{z\to 3^+} \frac{(z-1)(z-2)}{(z-3)}$

b. $\lim_{z\to 3^-} \frac{(z-1)(z-2)}{(z-3)}$

c. $\lim_{z\to 3} \frac{(z-1)(z-2)}{(z-3)}$

26.

a. $\lim_{x\to -2^+} \frac{(x-4)}{x(x+2)}$

b. $\lim_{x\to -2^-} \frac{(x-4)}{x(x+2)}$

c. $\lim_{x\to -2} \frac{(x-4)}{x(x+2)}$

27.

a. $\lim_{x\to 2^+} \frac{x^2-4x+3}{(x-2)^2}$

b. $\lim_{x \to 2^-} \frac{x^2 - 4\,x + 3}{(x-2)^2}$

c. $\lim_{x \to 2} \frac{x^2 - 4\,x + 3}{(x-2)^2}$

28.

a. $\lim_{t \to -2^+} \frac{t^3 - 5\,t^2 + 6\,t}{t^4 - 4\,t^2}$

b. $\lim_{t \to -2^-} \frac{t^3 - 5\,t^2 + 6\,t}{t^4 - 4\,t^2}$

c. $\lim_{t \to -2} \frac{t^3 - 5\,t^2 + 6\,t}{t^4 - 4\,t^2}$

d. $\lim_{t \to 2} \frac{t^3 - 5\,t^2 + 6\,t}{t^4 - 4\,t^2}$

29.

a. $\lim_{x \to 2^+} \frac{1}{\sqrt{x(x-2)}}$

b. $\lim_{x \to 2^-} \frac{1}{\sqrt{x(x-2)}}$

c. $\lim_{x \to 2} \frac{1}{\sqrt{x(x-2)}}$

30.

a. $\lim_{x \to 1^+} \frac{x-3}{\sqrt{x^2 - 5\,x + 4}}$

b. $\lim_{x \to 1^-} \frac{x-3}{\sqrt{x^2 - 5\,x + 4}}$

c. $\lim_{x \to 1} \frac{x-3}{\sqrt{x^2 - 5\,x + 4}}$

31.

a. $\lim_{x \to 0} \frac{x-3}{x^4 - 9\,x^2}$

b. $\lim_{x \to 3} \frac{x-3}{x^4 - 9\,x^2}$

c. $\lim_{x \to -3} \frac{x-3}{x^4 - 9\,x^2}$

32.

a. $\lim_{x \to 0} \frac{x-2}{x^5 - 4\,x^3}$

b. $\lim_{x\to 2} \dfrac{x-2}{x^5-4x^3}$

c. $\lim_{x\to -2} \dfrac{x-2}{x^5-4x^3}$

33. $\lim_{x\to 0} \dfrac{x^3-5x^2}{x^2}$

34. $\lim_{t\to 5} \dfrac{4t^2-100}{t-5}$

35. $\lim_{x\to 1^+} \dfrac{x^2-5x+6}{x-1}$

36. $\lim_{z\to 4} \dfrac{z-5}{\left(z^2-10z+24\right)^2}$

37. $\lim_{x\to 6^+} \dfrac{x-7}{\sqrt{x-6}}$

38. $\lim_{x\to 2^-} \dfrac{x-1}{\sqrt{(x-3)(x-2)}}$

39. $\lim_{\theta\to 0^+} \csc\theta$

40. $\lim_{x\to 0^-} \csc x$

41. $\lim_{x\to 0^+} (-10\cot x)$

42. $\lim_{\theta\to \pi/2^+} \dfrac{1}{3}\tan\theta$

43. $\lim_{\theta\to 0} \dfrac{2+\sin\theta}{1-\cos^2\theta}$

44. $\lim_{\theta\to 0^-} \dfrac{\sin\theta}{\cos^2\theta-1}$

45. **Location of vertical asymptotes** Analyze the following limits and find the vertical asymptotes of $f(x)=\dfrac{x-5}{x^2-25}$.

a. $\lim_{x\to 5} f(x)$

b. $\lim_{x\to -5^-} f(x)$

c. $\lim_{x\to -5^+} f(x)$

46. Location of vertical asymptotes Analyze the following limits and find the vertical asymptotes of $f(x)=\dfrac{x+7}{x^4-49x^2}$.

a. $\lim_{x\to 7^-} f(x)$

b. $\lim_{x\to 7^+} f(x)$

c. $\lim_{x\to -7} f(x)$

d. $\lim_{x\to 0} f(x)$

47–50. Finding vertical asymptotes *Find all vertical asymptotes $x=a$ of the following functions. For each value of a, determine* $\lim_{x\to a^+} f(x)$, $\lim_{x\to a^-} f(x)$, *and* $\lim_{x\to a} f(x)$.

47. $f(x)=\dfrac{x^2-9x+14}{x^2-5x+6}$

48. $f(x)=\dfrac{\cos x}{x^2+2x}$

49. $f(x)=\dfrac{x+1}{x^3-4x^2+4x}$

50. $f(x)=\dfrac{x^3-10x^2+16x}{x^2-8x}$

51. Checking your work graphically Analyze the following limits. Then sketch a graph of $y=\tan x$ with the window $[-\pi,\pi]\times[-10,10]$ and use your graph to check your work.

a. $\lim_{x\to \pi/2^+} \tan x$

b. $\lim_{x\to \pi/2^-} \tan x$

c. $\lim_{x\to -\pi/2^+} \tan x$

d. $\lim_{x\to -\pi/2^-} \tan x$

T **52. Checking your work graphically** Analyze the following limits. Then sketch a graph of $y=\sec x\tan x$ with the window $[-\pi,\pi]\times[-10,10]$ and use your graph to check your work.

a. $\lim_{x\to \pi/2^+} \sec x\tan x$

b. $\lim_{x\to \pi/2^-} \sec x\tan x$

c. $\lim_{x\to -\pi/2^+} \sec x\tan x$

d. $\lim_{x\to -\pi/2^-} \sec x\tan x$

53. Explain why or why not Determine whether the following statements are true and give an explanation or counterexample.

a. The line $x=1$ is a vertical asymptote of the function $f(x)=\dfrac{x^2-7x+6}{x^2-1}$.

b. The line $x = -1$ is a vertical asymptote of the function $f(x) = \dfrac{x^2 - 7x + 6}{x^2 - 1}$.

c. If g has a vertical asymptote at $x = 1$ and $\lim_{x \to 1^+} g(x) = \infty$, then $\lim_{x \to 1^-} g(x) = \infty$.

54. Matching Match functions a-f with graphs A-F in the figure without using a graphing utility.

a. $f(x) = \dfrac{x}{x^2 + 1}$

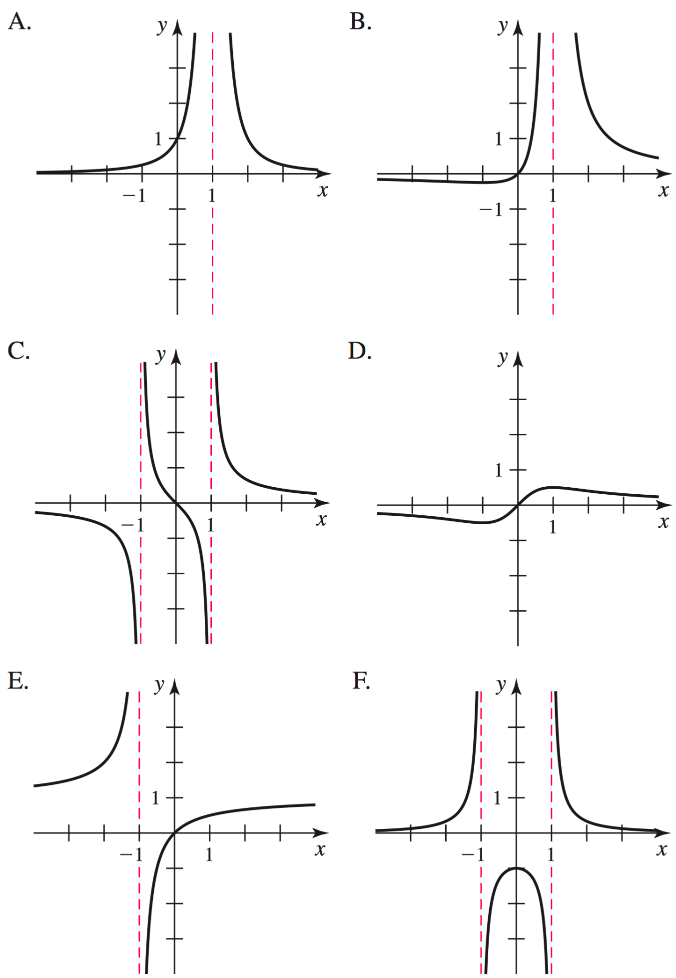
b. $f(x) = \dfrac{x}{x^2 - 1}$

c. $f(x) = \dfrac{1}{x^2 - 1}$

d. $f(x) = \dfrac{x}{(x - 1)^2}$

e. $f(x) = \dfrac{1}{(x - 1)^2}$

f. $f(x) = \dfrac{x}{x + 1}$



Explorations and Challenges »

55. Finding a rational function Find a rational function $r(x)$ such that $r(1)$ is undefined, $\lim_{x \to 1} r(x) = 0$, and $\lim_{x \to 2} r(x) = \infty$.

56. Finding a function with vertical asymptotes Find polynomials p and q such that $f = p/q$ is undefined at 1 and 2, but f has a vertical asymptote only at 2. Sketch a graph of your function.

57. Finding a function with infinite limits Give a formula for a function f that satisfies $\lim_{x \to 6^+} f(x) = \infty$ and $\lim_{x \to 6^-} f(x) = -\infty$.

T 58–66. Asymptotes *Use analytical methods and/or a graphing utility to identify the vertical asymptotes (if any) of the following functions.*

58. $f(x) = \dfrac{x^2 - 1}{x^4 - 1}$

59. $f(x) = \dfrac{x^2 - 3x + 2}{x^{10} - x^9}$

60. $g(x) = \cot\left(x - \dfrac{\pi}{2}\right)$, for $|x| \le \pi$

61. $h(x) = \dfrac{\cos x}{(x+1)^3}$

62. $p(x) = \sec \dfrac{\pi x}{2}$, for $|x| < 2$

63. $g(\theta) = \tan \dfrac{\pi \theta}{10}$

64. $q(s) = \dfrac{\pi}{s - \sin s}$

65. $f(x) = \dfrac{1}{\sqrt{x} \sec x}$

66. $g(x) = \dfrac{1}{\sqrt{x\left(x^2 - 1\right)}}$

67. Limits with a parameter Let $f(x) = \dfrac{x^2 - 7x + 12}{x - a}$.

a. For what values of a, if any, does $\lim_{x \to a^+} f(x)$ equal a finite number?

b. For what values of a, if any, does $\lim_{x \to a^+} f(x) = \infty$?

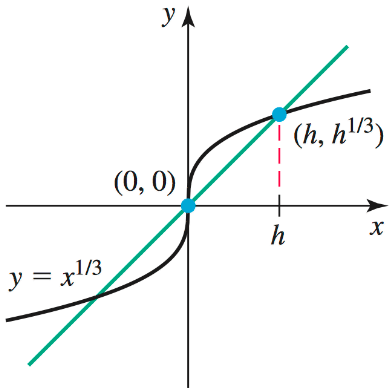
c. For what values of a, if any, does $\lim_{x \to a^+} f(x) = -\infty$?

68–69. Steep secant lines

a. *Given the graph of f in the following figures, find the slope of the secant line that passes through* $(0,\ 0)$ *and* $(h,\ f(h))$ *in terms of h for* $h > 0$ *and* $h < 0$.

b. *Analyze the limit of the slope of the secant line found in part (a) as* $h \to 0^+$ *and* $h \to 0^-$. *What does this tell you about the line tangent to the curve at* $(0,\ 0)$*?*

68. $f(x) = x^{1/3}$



69. $f(x) = x^{2/3}$