Various practice problems II

Problem 1.
Use the linear approximation to estimate $(15)^{1/4}$

Problem 2.
Approximate $f(x) = (1 + 2x)^{-n}$ at $a = 0$ by the linear approximation. (Here, n is a positive integer.)

Problem 3.
The measurement of x is accurate within 2%. Use the linear approximation to determine the error Δf in the calculation of f and the percentage error $100\frac{\Delta f}{f}$ when

$$f(x) = \frac{1}{1+x} \quad \text{and} \quad x = 4.$$

Problem 4.
A child is flying a kite. If the kite is 90 feet above child's hand level and the wind is blowing it on a horizontal course at 5 feet per second, how fast is the child paying out cord when 150 feet of cord is out? (Assume that the child is not moving and the cord remains straight from hand to kite, an unrealistic assumption.)

Problem 5.
A ladder 20 feet long leans against a building. If a bottom of the ladder slides away from the building horizontally at the rate of 2 ft/sec, how fast is the top of the ladder sliding down the building when the top of ladder is 12 feet above the ground.

Problem 6.
Use the linear approximation to estimate $\sqrt[4]{81.6}$.

Problem 7.
Assuming that the equator is a circle whose radius is approximately 4000 miles, how much longer than the equator would a concentric, coplanar circle be if each point on it were 2 feet above equator?

Hint: *Use the linear approximation.*

Problem 8.
Use the linear approximation to estimate $f(x)$ at a.

$$f(x) = (1 - x)^{-n} \quad \text{at} \quad a = 0. \qquad \text{(Assume that n is a positive integer.)}$$

Problem 9.
The speed v of blood flowing along the central axis of an artery of radius R is given by Poiseuille's law

$$v(R) = cR^2$$

where c is a constant. If you can determine the radius of the artery within an accuracy of 5%, how accurate is your calculation of the speed?

Problem 10.
(*Tilman's resource model*) Suppose that the rate of growth of a plant in a certain habitat depends on a single resource: for instance, nitrogen. Assume that the growth rate $f(R)$ depends on the resource level R as

$$f(R) = a\frac{R}{k+R}$$

where a and k are constants. Express the percentage error of the growth rate, $100\frac{\Delta f}{f}$, as a function of the percentage error of resource level, $100\frac{\Delta R}{R}$.

Problem 11.
Determine (using derivatives) where $f(x) = \dfrac{x}{x^2+1}$ is increasing and where is decreasing.

Problem 12.
In the following problems find the critical points and use the test you prefer to decide which give a local maximum value and which give a local minimum. What are these local maximum and local minimum values? Finally, find the inflection points.

(a) $f(x) = x^4 - 2x^2 + 3$ (b) $g(x) = x^4 + 2x^3$ (c) $h(x) = 2x + x^{\frac{2}{3}}$ (d) $F(x) = \dfrac{x^2}{\sqrt{x^2+1}}$

Problem 13.
Let f be a continuous function defined for all real numbers whose first derivative is
$f'(x) = \dfrac{x-1}{3x^{2/3}}$ and whose second derivative is $f''(x) = \dfrac{x+2}{9x^{5/3}}$.

(a) Find the critical points of f;

(b) On what intervals is f increasing and on what intervals is f decreasing?

(c) Find local extrema of f;

(d) On what intervals is f concave up and on what intervals is f concave down?

(e) Find points of inflection for f;

(f) In addition, we know that $f(0) = f(4) = 0$. Use the information found in parts (a), (b), (c), (d), and (e) to sketch the graph of $y = f(x)$.

Problem 14.
For the function given by $f(x) = (x^2 - 9)/(x^2 - 4)$ find

(a) The critical points of f,

(b) The local maxima and the local minima of f,

(c) The concavity of f and its points of inflections,

(d) All horizontal and vertical asymptotes,

(e) Finally, sketch the graph of the function.

Problem 15.
A man 6 feet tall is walking away from a street lights 18 feet high at a speed of 6ft/sec. How fast is the tip of his shadow moving along the ground?

Problem 16.
Determine where the graph of the given function is increasing, decreasing, concave up, and concave down. Find its local extrema. Then sketch its graph.

(a) $f(x) = x^6 - 3x^4$.

(b) $g(x) = 3x^5 - 5x^3 + 1$.

(c) $h(x) = x^{\frac{2}{3}}(1-x)$.

Problem 17.
If $f'(x) = 2(x+2)(x+1)^2(x-2)^4(x-3)^3$, what values of x make $f(x)$ a local maximum? A local minimum?

Problem 18.
On the interval $[0, 6]$, sketch a *possible* graph of a continuous function that satisfies all of the stated conditions.

$f(0) = 3$; $f(3) = 0$; $f(6) = 4$;

$f'(x) < 0$ on $(0, 3)$; $f'(x) > 0$ on $(3, 6)$;

$f''(x) > 0$ on $(0, 5)$; $f''(x) < 0$ on $(5, 6)$.

Note: *There are infinitely many graphs that satisfy all the conditions.*

Problem 19.
Suppose a rectangle has its lower base on the x-axis and upper vertices on the graph of the function $y = 8 - x^2$. Find the area of the largest such rectangle.

Problem 20.
A car rental agency has 24 identical cars. The owner of the agency finds that at a price of \$10 per day, all the cars are rented. However, for each \$1 increase in rental price, one of the cars is not rented. How much should be charged to maximize the income to the agency?

Hint: *Let x denote the increase in rental above* \$10. *Find the income for car rental agency in terms of x.*

Problem 21.
Sketch the graphs of the following functions. Label critical points, local extrema, points of inflection, and all asymptotes (vertical and horizontal).

(a) $f(x) = \dfrac{8x}{x^2+4}$
(b) $f(x) = x^{2/3}$
(c) $f(x) = (x-2)^{1/3}$
(d) $f(x) = \dfrac{1}{x^5+1}$

Problem 22.
Decide whether the Mean Value Theorem applies to the given function on the given interval. If it does find all possible values of c; if not, state the reason.

(a) $f(x) = x^{\frac{5}{3}}; \quad [-1,1]$ (b) $g(x) = |x|; \quad [-2,2]$

Problem 23.
Use the Mean Value Theorem to prove that

$$\lim_{x\to\infty} \left(\sqrt{x+2} - \sqrt{x}\right) = 0.$$

Problem 24.
Find the antiderivatives (i.e., indefinite integrals) of the given functions.

(a) $\int \left(x - \dfrac{1}{x}\right)^2 dx$

(b) $\int \dfrac{1}{\sqrt{3x+2}}\, dx$

(c) $\int x(1-x^2)^{1/4}\, dx$

(d) $\int \dfrac{(1+\tan(x))^{1/3}}{\cos^2(x)}\, dx$

Problem 25.

Evaluate the following sums:

(a) $\sum_{i=1}^{5} [(3i+4)^{10} - (3i+1)^{10}]$,

(b) $\sum_{k=1}^{n} (3k^2 - 2k + 1)$.

Problem 26.
Find the area under the curve $y = x^3$ over the interval $[0,2]$. To do this, divide the interval into n equal subintervals, calculate the area of the corresponding circumscribed polygon, and then let $n \to \infty$.

Problem 27.
Use **only** geometry to calculate $\int_{-1}^{2} (2+4x)\, dx$.

Problem 28.
Evaluate $\int_2^6 (x^2+6)\, dx$ using the definition of the definite integral. In other words, first evaluate the corresponding Riemann sum and then take a suitable limit.