Math. 150a, Spring 2024

Various practice problems I

Problem 1.
Solve the following quadratic equations.

$$\text{(a)} \quad 8x^2 + 2x - 3 = 0 \qquad\qquad \text{(b)} \quad x^2 - 2x - 5 = 0$$

Problem 2.
Solve the following inequalities.

$$\text{(a)} \quad x^2 + 2x - 15 \geq 0 \qquad \text{(b)} \quad x^2 + x + 1 \leq 0 \qquad \text{(c)} \quad \frac{2x+1}{2-x} \leq 1$$

Problem 3.
Find the composite functions $f \circ g$ and $g \circ f$ where

$$f(x) = \sqrt{x+1} \qquad \text{and} \qquad g(x) = \frac{1}{x-1}.$$

Simplify your answers as much as you can !

Problem 4.
If $f(x) = 1/x$, find and simplify

$$\frac{f(-1+h) - f(-1)}{h}, \qquad\qquad \text{where } h \neq 0 \text{ and } h \neq 1.$$

Problem 5.
Sketch the graph of the following functions.

(a) $f(x) = \sqrt{x+1} - 1$ (b) $f(x) = |x-1| + 1$ (c) $f(x) = \begin{cases} -2x+4 & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ x^2 + 2 & \text{if } x > 1 \end{cases}$ (d) $f(x) = \begin{cases} |x-2| & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$

Problem 6.
Find an equation of the line that passes through the point $(-1, 0)$ and is perpendicular to the line with the equation $4x + 5y + 16 = 0$.

Problem 7.
For $f(x) = \dfrac{2x}{x+5}$ and $g(x) = \dfrac{x}{3x-8}$, find $(f \circ g)(x)$ and $(g \circ f)(x)$. **Simplify your results !**

Problem 8.
Find the following limits.

$$\text{(a)} \qquad \lim_{x \to -1} \frac{3x^2 + 4x + 1}{x+1} \qquad\qquad \text{(b)} \qquad \lim_{x \to -\infty} \frac{-2x^4 + 3x^3 - 7x - 10}{3x^4 + 6x^2 - x + 100}$$

Problem 9.
Find the following limits

$$\text{(a)} \lim_{x \to -2} \frac{x^2 - 4}{x+2} \quad \text{(b)} \quad \lim_{x \to 4} \frac{x-4}{\sqrt{x} - 2} \quad \text{(c)} \quad \lim_{x \to 1} \frac{\sqrt{x+3} - 2}{x-1} \quad \text{(d)} \quad \lim_{x \to 0} \frac{1 - \cos(2x)}{3x^2}$$

Problem 10.
Determine the values of x, if any, at which the given function is discontinuous. At each point of discontinuity, state the condition(s) for continuity that are violated.

$$f(x) = \begin{cases} \frac{x^2-1}{x+1}, & \text{if } x \neq -1 \\ 1, & \text{if } x = -1. \end{cases}$$

Problem 11.
Explain in details why $f(x) = \begin{cases} 2x - 3, & \text{if } x \leq -1 \\ x^2 - 4, & \text{if } x > -1 \end{cases}$ is **NOT** continuous at $x = -1$.

Problem 12.
Let $y = -x^2$.
(a) Find the average rate of change of y with respect to x on the interval $[2, 3]$.

(b) Find the (instantaneous) rate of change at $x = 3$.

Problem 13.
Find the derivatives of the following functions

$$\text{(a)}\quad g(s)=2s^2-\frac{4}{s}+\frac{2}{\sqrt{s}} \qquad \text{(b)}\quad h(x)=\left(x+\frac{1}{x}+\frac{1}{x^2}\right)^5 \qquad \text{(c)}\quad F(x)=\sqrt{\frac{x^2+1}{x^4+2}+10}$$

Problem 14.
For the function

$$g(t)=\sqrt{2t^2+3}$$

find g'' and g'''.

Problem 15.
Differentiate the following functions with respect to the indicated variable.

$$\text{(a)}\quad h(t)=\frac{t^2-3t+1}{t+1} \qquad\qquad \text{(b)}\quad f(x)=\sqrt{c^2x^2+2}\quad (c \text{ is a constant})$$

Problem 16.
For each of the following find the limit or show that it does not exist:

(a) $\lim_{x\to 0} x^3\sin(\frac{1}{x})$

(b) $\lim_{x\to 1}\dfrac{\sqrt{x+3}-2}{x-1}$

(c) $\lim_{w\to -2}\dfrac{(w+2)(w^2-w-6)}{w^2+4w+4}$

Problem 17.
For $f(x)=3x^2-5$ find

$$\text{(a)}\quad \frac{[f(x)-f(2)]}{(x-2)}; \qquad \text{(b)}\quad \lim_{x\to 2}\frac{[f(x)-f(2)]}{(x-2)}.$$

Problem 18.
Give an ϵ, δ proof of

$$\lim_{x\to 5}\sqrt{x-1}=2.$$

Problem 19.
Give an $\epsilon-\delta$ proof of the following limit:

$$\lim_{x\to -1}(x^2-2x-1)=2.$$

Problem 20. (4 points)
Show that the equation $x^5+4x^3-7x+14=0$ has at least one real solution.
Hint: Use the Intermediate Value Theorem.

Problem 21.
Find the equation of the tangent line to $y=\dfrac{2}{x-2}$ at $(0,-1)$.

Problem 22.
Use $f'(x)=\lim_{h\to 0}\dfrac{[f(x+h)-f(x)]}{h}$ to find the derivative at $x>0$ of $f(x)=\sqrt{10x}$.

Problem 23.
Develop a rule for $D_x[f(x)g(x)h(x)]$.

Problem 24.
Find the equation of the tangent line to $y=(x^2+1)^4(x^4+1)^3$ at $(1,128)$.

Problem 25.
Find

$$\text{(a)}\qquad D_x\left[\cos^4\left(\frac{x^2+1}{x+1}\right)\right], \qquad\qquad \text{(b)}\qquad D_t\{\cos^2[\cos(\cos t)]\}.$$

Problem 26.
Let $n!=n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$. Thus, $4!=4\cdot 3\cdot 2\cdot 1=24$ and $5!=5\cdot 4\cdot 3\cdot 2\cdot 1$, and so on.
We give $n!$ the name **n factorial**. Find

$$\text{(a)}\qquad D_x^n(x^n), \qquad\qquad \text{(b)}\qquad D_x^n\left(\frac{1}{x}\right).$$

Note: express your results in terms of $n!$.