

Matador Math Society: Integration Contest

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Problem 1 of 10

Find an expression that is equivalent to:

$$\int_0^{\infty} x^n e^{-cx} dx$$

for any $n \in (\mathbb{N} \cup \{0\})$ and $c \in \mathbb{R}$

Your expression must be written WITHOUT an integral sign.
Your expression will, obviously, be in terms of n and c .

Problem 1 of 10 (Answer)

Find an expression that is equivalent to:

$$\int_0^{\infty} x^n e^{-cx} dx$$

for any $n \in (\mathbb{N} \cup \{0\})$ and $c \in \mathbb{R}$

$$\frac{n!}{c^{n+1}}$$

Problem 2 of 10

Evaluate

$$\int (\cos 2x)(e^{-x}) dx$$

Problem 2 of 10 (Answer)

$$\int (\cos 2x)(e^{-x}) dx$$

$$\begin{aligned} u &= e^{-x} & dv &= \cos 2x \\ du &= -e^{-x} & v &= \frac{1}{2} \sin 2x \end{aligned}$$

$$\int (\cos 2x)(e^{-x}) dx = \frac{1}{2} e^{-x} \sin 2x + \frac{1}{2} \int (\sin 2x)(e^{-x}) dx \quad (\text{Eqn. 1})$$

$$\int (\sin 2x)(e^{-x}) dx$$

$$\begin{aligned} u &= e^{-x} & dv &= \sin 2x \\ du &= -e^{-x} & v &= \frac{-1}{2} \cos 2x \end{aligned}$$

$$\int (\sin 2x)(e^{-x}) dx = -\frac{1}{2} e^{-x} \cos 2x - \frac{1}{2} \int (\cos 2x)(e^{-x}) dx \quad (\text{Eqn. 2})$$

We plug Eqn. 2 into Eqn. 1, giving:

$$\int (\cos 2x)(e^{-x}) dx = \frac{1}{2} e^{-x} \sin 2x + \left[\frac{1}{2} \right] \left[-\frac{1}{2} e^{-x} \cos 2x - \frac{1}{2} \int (\cos 2x)(e^{-x}) dx \right]$$

$$\int (\cos 2x)(e^{-x}) dx = \frac{1}{2} (\sin 2x)(e^{-x}) - \frac{1}{4} (\cos 2x)(e^{-x}) - \frac{1}{4} \int (\cos 2x)(e^{-x}) dx$$

$$\frac{5}{4} \int (\cos 2x)(e^{-x}) dx = \frac{1}{2} (\sin 2x)(e^{-x}) - \frac{1}{4} (\cos 2x)(e^{-x})$$

$$\int (\cos 2x)(e^{-x}) dx = \frac{2}{5} (\sin 2x)(e^{-x}) - \frac{1}{5} (\cos 2x)(e^{-x})$$

Problem 3 of 10

Simplify:

$$\sqrt{4 + 2\sqrt{3}} - \sqrt{28 + 10\sqrt{3}}$$

Hint: It simplifies to just one integer.

Problem 3 of 10 (Answer)

$$\sqrt{4 + 2\sqrt{3}} - \sqrt{28 + 10\sqrt{3}}$$

Note: $4 + 2\sqrt{3} = (1 + \sqrt{3})^2$

Note: $28 + 10\sqrt{3} = (5 + \sqrt{3})^2$

So the expression is:

$$(1 + \sqrt{3}) - (5 + \sqrt{3})$$

or just:

-4

Problem 4 of 10

Evaluate:

$$\int (x^2)(\sin(5x))dx$$

During one scene of *Stand and Deliver* (1988), this integral appears on the chalkboard.

Problem 4 of 10 (Answer)

$$\int (x^2)(\sin(5x))dx$$

$$\left(\frac{2}{125}\right) \cos(5x) - \left(\frac{x^2}{5}\right) \cos(5x) + \left(\frac{2}{25}\right) (x)(\sin(5x))$$

Problem 5 of 10

Evaluate:

$$\int (x^{1/4})(\ln(x))dx$$

Problem 5 of 10 (Answer)

$$\int (x^{1/4})(\ln(x))dx$$

$$\left(\frac{4}{25}\right) (x^{5/4}) (5\ln(x) - 4)$$

Problem 6 of 10

Evaluate:

$$\int \frac{x}{(2-x)^3} dx$$

Problem 6 of 10 (Answer)

$$\int \frac{x}{(2-x)^3} dx$$

$$\frac{x-1}{(x-2)^2}$$

Problem 7 of 10

Evaluate:

$$\int (x) \sqrt{\frac{1-x^2}{1+x^2}} dx$$

Problem 7 of 10 (Answer)

$$\int (x) \sqrt{\frac{1-x^2}{1+x^2}} dx$$

$$\frac{\sqrt{\frac{1-x^2}{x^2+1}} \left(\sqrt{1-x^2}(x^2+1) + 2\sqrt{x^2+1} \arcsin \frac{\sqrt{x^2+1}}{\sqrt{2}} \right)}{2\sqrt{1-x^2}}$$

Problem 8 of 10

Evaluate:

$$\int_{-1}^3 e^{|x|} dx$$

Problem 8 of 10 (Answer)

$$\int_{-1}^3 e^{|x|} dx$$

$$-2 + e + e^3 \approx 20.804$$

Problem 9 of 10

Evaluate:

$$\int e^{\sqrt[3]{x}} dx$$

Problem 9 of 10 (Answer)

$$\int e^{\sqrt[4]{x}} dx$$

$$(4) \left(e^{\sqrt[4]{x}} \right) \left(x^{\frac{3}{4}} - 3\sqrt{x} + 6\sqrt[4]{x} - 6 \right)$$

Problem 10 of 10

Evaluate:

$$\int_0^1 \ln(x) dx$$

Problem 10 of 10 (Answer)

$$\int_0^1 \ln(x) dx$$

Note: $\int \ln(x) dx = x \ln(x) - x$

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