Matador Math Society: Integration Contest

compiled by Samuel Lurie

April 15, 2016

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Problem 1 of 10

Find an expression that is equivalent to:

$$\int_0^\infty x^n e^{-cx} dx$$

for any n $\in (\mathbb{N} \cup \{0\})$ and $c \in \mathbb{R}$

Your expression must be written WITHOUT an integral sign.
Your expression will, obviously, be in terms of n and c.

Problem 1 of 10 (Answer)

Find an expression that is equivalent to:

$$\int_0^\infty x^n e^{-cx} dx$$

for any n $\in (\mathbb{N} \cup \{0\})$ and $c \in \mathbb{R}$

$$\frac{n!}{c^{n+1}}$$

Problem 2 of 10

Evaluate

$$\int (cos2x)(e^{-x})dx$$

Problem 2 of 10 (Answer)

$$\int (cos2x)(e^{-x})dx$$

$$u = e^{-x} \qquad dv = cos2x$$
$$du = -e^{-x} \qquad v = \frac{1}{2}sin2x$$
$$\int (cos2x)(e^{-x})dx = \frac{1}{2}e^{-x}sin2x + \frac{1}{2}\int (sin2x)(e^{-x})dx \quad (Eqn.\ 1)$$

$$\int (sin2x)(e^{-x})dx$$

$$u = e^{-x} \qquad dv = sin2x$$
$$du = -e^{-x} \qquad v = \frac{-1}{2}cos2x$$
$$\int (sin2x)(e^{-x})dx = -\frac{1}{2}e^{-x}cos2x - \frac{1}{2}\int (cos2x)(e^{-x})dx \quad (Eqn.\ 2)$$

We plug Eqn. 2 into Eqn. 1, giving:

$$\int (cos2x)(e^{-x})dx = \frac{1}{2}e^{-x}sin2x + \left[\frac{1}{2}\right]\left[-\frac{1}{2}e^{-x}cos2x - \frac{1}{2}\int (cos2x)(e^{-x})dx\right]$$

$$\int (cos2x)(e^{-x})dx = \frac{1}{2}(sin2x)(e^{-x}) - \frac{1}{4}(cos2x)(e^{-x}) - \frac{1}{4}\int (cos2x)(e^{-x})dx$$

$$\frac{5}{4}\int (cos2x)(e^{-x})dx = \frac{1}{2}(sin2x)(e^{-x}) - \frac{1}{4}(cos2x)(e^{-x})$$

$$\int (cos2x)(e^{-x})dx = \frac{2}{5}(sin2x)(e^{-x}) - \frac{1}{5}(cos2x)(e^{-x})$$

Problem 3 of 10

Simplify:

$$\sqrt{4+2\sqrt{3}}-\sqrt{28+10\sqrt{3}}$$

Hint: It simplifies to just one integer.

Problem 3 of 10 (Answer)

$$\sqrt{4+2\sqrt{3}}-\sqrt{28+10\sqrt{3}}$$

Note: $4+2\sqrt{3}=(1+\sqrt{3})^2$

Note: $28+10\sqrt{3}=(5+\sqrt{3})^2$

So the expression is:

$$(1+\sqrt{3})-(5+\sqrt{3})$$

or just:

-4

Problem 4 of 10

Evaluate:

$$\int (x^2)(sin(5x))dx$$

During one scene of *Stand and Deliver* (1988), this integral appears on the chalkboard.

Problem 4 of 10 (Answer)

$$\int (x^2)(sin(5x))dx$$

$$\left(\frac{2}{125}\right) cos(5x) - \left(\frac{x^2}{5}\right) cos(5x) + \left(\frac{2}{25}\right) (x)(sin(5x))$$

Problem 5 of 10

Evaluate:

$$\int (x^{1/4})(ln(x))dx$$

Problem 5 of 10 (Answer)

$$\int (x^{1/4})(ln(x))dx$$

$$\left(\frac{4}{25}\right)\left(x^{5/4}\right)\left(5ln(x)-4\right)$$

Problem 6 of 10

Evaluate:

$$\int \frac{x}{(2-x)^3} dx$$

Problem 6 of 10 (Answer)

$$\int \frac{x}{(2-x)^3} dx$$

$$\frac{x-1}{(x-2)^2}$$

Problem 7 of 10

Evaluate:

$$\int (x)\sqrt{\frac{1-x^2}{1+x^2}}dx$$

Problem 7 of 10 (Answer)

$$\int (x)\sqrt{\frac{1-x^2}{1+x^2}}dx$$

$$\frac{\sqrt{\frac{1-x^2}{x^2+1}}\left(\sqrt{1-x^2}(x^2+1)+2\sqrt{x^2+1}arcsin\frac{\sqrt{x^2+1}}{\sqrt{2}}\right)}{2\sqrt{1-x^2}}$$

Problem 8 of 10

Evaluate:

$$\int_{-1}^{3} e^{|x|} dx$$

Problem 8 of 10 (Answer)

$$\int_{-1}^{3} e^{|x|} dx$$

$$-2 + e + e^3 \approx 20.804$$

Problem 9 of 10

Evaluate:

$$\int e^{\sqrt[4]{x}} dx$$

Problem 9 of 10 (Answer)

$$\int e^{\sqrt[4]{x}} dx$$

$$(4)\left(e^{\sqrt[4]{x}}\right)\left(x^{\frac{3}{4}} - 3\sqrt{x} + 6\sqrt[4]{x} - 6\right)$$

Problem 10 of 10

Evaluate:

$$\int_0^1 ln(x)dx$$

Problem 10 of 10 (Answer)

$$\int_0^1 ln(x)dx$$

Note: $\int ln(x)dx = xln(x) - x$

-1