2017-2018 Annual Program Assessment Report

Please submit report to your department chair or program coordinator, the Associate Dean of your College, and to james.solomon@csun.edu, Director of the Office of Academic Assessment and Program Review, by September 28, 2018. You may, but are not required to, submit a separate report for each program, including graduate degree programs, which conducted assessment activities, or you may combine programs in a single report. Please identify your department/program in the file name for your report.

College: Science and Mathematics

Department: Mathematics

Program: B.A. and B.S. and M.S., in addition to assessment of General Education Basic Skills SLOs in Mathematics.

Assessment liaison: Daniel Katz

1. Please check off whichever is applicable:
   A. ___X__ Measured student work within program major/options.
   B. ___X__ Analyzed results of measurement within program major/options.
   C. ______ Applied results of analysis to program review/curriculum/review/revision major/options.
   D. ______ Focused exclusively on the direct assessment measurement of General Education Natural Sciences learning outcomes

2. Overview of Annual Assessment Project(s). On a separate sheet, provide a brief overview of this year’s assessment activities, including:
   • an explanation for why your department chose the assessment activities (measurement, analysis, application, or GE assessment) that it enacted
   • if your department implemented assessment option A, identify which program SLOs were assessed (please identify the SLOs in full), in which classes and/or contexts, what assessment instruments were used and the methodology employed, the resulting scores, and the relation between this year’s measure of student work and that of past years: (include as an appendix any and all relevant materials that you wish to include)
   • if your department implemented assessment option B, identify what conclusions were drawn from the analysis of measured results, what changes to the program were planned in response, and the relation between this year’s analyses and past and future assessment activities
   • if your department implemented option C, identify the program modifications that were adopted, and the relation between program modifications and past and future assessment activities
   • if your program implemented option D, exclusively or simultaneously with options A, B, and/or C, identify the basic skill(s) assessed and the precise learning outcomes assessed, the assessment instruments and methodology employed, and the resulting scores
   • in what way(s) your assessment activities may reflect the university’s commitment to diversity in all its dimensions but especially with respect to underrepresented groups
   • any other assessment-related information you wish to include, including SLO revision (especially to ensure continuing alignment between program course offerings and both program and university student learning outcomes), and/or the creation and modification of new assessment instruments

Overview of Assessment Projects
2017–2018 Academic Year
Report to the Office of Academic Assessment

Prepared by the Assessment Committee
(Alberto Candel, Daniel Katz, and Jason Lo)
in Collaboration with the Graduate Committee
Department of Mathematics
California State University, Northridge

14 September 2018
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Chapter 1

Introduction

1.1 Synopsis of Activities Done for Academic Year 2017–2018

This report describes all the assessment activities performed by the Department of Mathematics at California State University, Northridge for the 2017–2018 academic year:

- For General Education Basic Skills SLO #1 in Mathematics (Represent, understand and explain mathematical information symbolically, graphically, numerically and verbally), we assessed Math 150A (Calculus I) because it has a high DFU rate (38% for Fall 2011–Spring 2017). Our measurements indicate satisfactory achievement of the SLO.

- Since Math 150A is also a required course for the Mathematics Bachelor Program, we also assessed SLO #2 for this Program (Rigorously establish fundamental analytic properties and results such as limits, continuity, differentiability and integrability). Our measurements indicate satisfactory achievement of the SLO.

- We also assessed SLO #4 for the Mathematics Bachelors Program (Demonstrate facility with the terminology, use of symbols, and concepts of probability) in Math 340 (Introductory Probability) because it is a required course for many options within the program and has a high DFU rate (35% for Fall 2011–Spring 2017), so could serve as a bottleneck for graduation. We also assessed Math 341 (Applied Statistics I), since all math majors who are not required to take Math 340 are required to take Math 341 to learn probability.
  - For Math 340, our measurement in Fall 2017 had a flawed signature assignment, and so was inconclusive.
  - For Math 340, our measurement in Spring 2018 indicate satisfactory achievement of the SLO.
  - For Math 341, our measurements indicate satisfactory achievement of the SLO.

- We assessed SLO #4 for the Master’s Program in Mathematics, which is at the same time SLO #4 for the Master’s Program in Applied Mathematics (Communicate abstract mathematical ideas clearly and cogently) in the Math 501 class (Topology), a course required of all master’s students, and one in which the content of SLO is introduced. Our measurements indicate satisfactory achievement of the SLO.
1.2 Preview of Planned Assessment Activities for Academic Year 2018–2019

In the 2015–2016 academic year, the Department of Mathematics revised the undergraduate program SLOs and created graduate program SLOs. We have been assessing these new undergraduate and graduate SLOs to obtain a first measure of our programs so as to build a baseline for future assessment and a reference point to assist in curriculum development. So far we have assessed the undergraduate program SLOs #1, #2, #3, and #4 (out of five total), SLOs #3 and #4 for the M.S. Program in Mathematics (out of five total), and SLOs #2 and #4 for the M.S. Program in Applied Mathematics (out of five total). We plan to continue to assess these new SLOs over the next few years until we have covered the full set of SLOs for each program. So we plan to assess at least one SLO in the undergraduate program and to assess at least one SLO in the graduate program in 2018–2019.

In 2016–2017, we also assessed all four of the General Education Basic Skills SLOs in Mathematics, and we have added an additional measurement to that assessment this year. During our assessment of the General Education Basic Skills SLOs in 2016–2017, we mentioned to the Coordinator for Program Review and to the Director of the Office of Academic Assessment and Program Review that the General Education Basic Skills SLOs for Mathematics would benefit from revision to state clearer, more assessable goals. If the Office of Academic Assessment and Program Review is ready for us to begin this task, we may also be able to work on this revision in 2018–2019.
Chapter 2

Fall 2017: Measurement of Math 150A Class and Analysis for General Education

2.1 SLO Assessed

The following SLO for General Education Basic Skills in Mathematics was assessed:

General Education SLO #1 for Basic Skills: Mathematics: Represent, understand and explain mathematical information symbolically, graphically, numerically and verbally.

2.2 Role in the University

Math 150A is a required class for many majors in natural science, mathematics, and engineering.

2.3 Sections

Twelve sections of this course were offered in Fall 2017: Section 1 (36 students), Section 2 (35 students), Section 3 (38 students), Section 4 (36 students), Section 5 (36 students), Section 6 (33 students), Section 7 (36 students), Section 9 (32 students), Section 10 (32 students), Section 11 (28 students), Section 12 (30 students), and Section 13 (32 students), for a total of 404 students. There was a total of nine total instructors, of whom three taught two sections each and the other six taught only one section each. These sections were randomly labeled Sections A through L for the purposes of this report. The course was coordinated with a common final exam for all sections, which represented 30% of a student’s course grade. For uniformity in grading across sections, each problem on the full set of final exams was the responsibility of a single instructor who followed a uniform grading procedure under the direction of the course coordinator. The students were prepared for this exam via homework and midterm exams designed individually by the instructors but aligned to a common syllabus based on the textbook, which was Calculus Early Transcendentals (second edition, 2015) by Briggs, Cochran, and Gillett.
2.4 Signature Assignment

A five-part question on the common final exam was used as the signature assignment.

2.5 Results, Analysis, and Recommendations

The problem was divided into five sub-problems, each of which was scored from 0 to 2 using the rubrics that are included in the Appendix A.1, for a total score from 0 to 10. The scoring for formal assessment, based on these scoring rubrics, was performed by two full-time faculty of the Department of Mathematics on the 350 exams that were turned in (out of 404 total students in Sections A through L still on the class rosters at the end of the term). The two scorings were combined to give a list of 700 scores (two per exam).

The mean score is listed for each of the twelve sections as well as for the class as a whole in the table below, and the distribution of scores for the class is presented in Figure 2.1.

<table>
<thead>
<tr>
<th>Section</th>
<th>Mean Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>3.62</td>
</tr>
<tr>
<td>I</td>
<td>4.74</td>
</tr>
<tr>
<td>E</td>
<td>5.00</td>
</tr>
<tr>
<td>C</td>
<td>5.03</td>
</tr>
<tr>
<td>A</td>
<td>5.12</td>
</tr>
<tr>
<td>J</td>
<td>5.44</td>
</tr>
<tr>
<td>entire class</td>
<td>5.59</td>
</tr>
<tr>
<td>F</td>
<td>5.65</td>
</tr>
<tr>
<td>B</td>
<td>5.66</td>
</tr>
<tr>
<td>L</td>
<td>6.19</td>
</tr>
<tr>
<td>G</td>
<td>6.30</td>
</tr>
<tr>
<td>H</td>
<td>6.68</td>
</tr>
<tr>
<td>D</td>
<td>7.10</td>
</tr>
</tbody>
</table>

We consider these results to be evidence that the SLO is being achieved in the Math 150A class. The source of the variability in mean scores among sections is unclear. This variability is unlikely to be explained solely by method and style of teaching because it is apparent that it occurs among sections taught by the same instructor.

The average score (out of 2) for each sub-problem mostly declined as the problem went on, as can be seen in the following table.

<table>
<thead>
<tr>
<th>Sub-Problem</th>
<th>Mean Score for Entire Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1.30</td>
</tr>
<tr>
<td>II</td>
<td>1.32</td>
</tr>
<tr>
<td>III</td>
<td>1.09</td>
</tr>
<tr>
<td>IV</td>
<td>1.08</td>
</tr>
<tr>
<td>V</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Less than one percent of the students made no attempt at part (a) of the problem, which covers Sub-problems I and II, while students left unmarked the later parts (b)–(e), which cover Sub-problems III–V, at a rate of ten percent. Sub-problems I and II require computing the first and second
derivatives of $f(x) = x^4 - 6x^2$, respectively, and determining where they are positive and negative. Students are quite proficient with computing derivatives of polynomials. But determining positivity or negativity for the degree 3 polynomial is slightly harder than for the degree 2 polynomial here, which could explain the slightly higher mean score of sub-problem I over sub-problem II. Sub-problems III and IV have a slightly higher degree of difficulty than I and II, and correctly solving Sub-problems I and II facilitates correctly solving Sub-problems III and IV. Sub-problem V requires more than the correct solution to all previous sub-problems: students must graphically interpret the information collected on those parts. We suggest that instructors put a stronger emphasis on the connection between textual (including symbolic and numerical) information and graphical display thereof.

This course was selected for assessment because of its high DFU rate, reported at 38% from Fall 2011 to Spring 2017 by the CSU Student Success Dashboard. It is noted that of the 404 students enrolled at the end of the term, only 350 turned in exams, which is a non-completion rate of 13.4%. We recommend that the Department of Mathematics continues its efforts to identify the causes of the high DFU rate, and take appropriate steps. In particular, to lower the non-completion rate, the Department could continue its effort in identifying and warning students who are at high risk of failure before the drop deadline. Such risk factors include not attending classes from the outset, not completing homework and quizzes, and responding neither to offers of help from instructors nor warnings before the drop date that their lack of work so far will make it very difficult for them to pass. It must be recognized, however, that some students may be fully aware of their risk of failing the course, and yet choose to stay enrolled for a variety of reasons, perhaps including satisfaction of minimum unit requirements for financial aid. This presents an
issue that may not be resolved by the Department of Mathematics alone. We recommend that the Department of Mathematics and its instructors establish communications with the academic advisors and graduation and retention advisors of students who are at risk of failure. Since most of the students in Math 150A are not mathematics majors, this will require coordination with advisors from other colleges and departments. The Department of Mathematics should also continue to look for ways to reduce the DFU rate for students who do complete the course work, including continued efforts to provide the best possible preparation in the prerequisite courses (Math 102, 104, and 105) and mentoring in the laboratory associated to Math 150A, perhaps by expanding mandatory laboratory participation.
Chapter 3

Fall 2017: Measurement of Math 150A Class and Analysis for Bachelor’s Program

3.1 SLO Assessed

The following SLO for the Mathematics Bachelor’s Program was assessed:

SLO #2: Rigorously establish fundamental analytic properties and results such as limits, continuity, differentiability and integrability.

3.2 Role in the Program

Math 150A is a required class for all math majors.

3.3 Sections

See section 2.3 above. The identical random section labels were used in this part of the report to facilitate comparison.

3.4 Signature Assignment

A question on the common final exam was used as the signature assignment.

3.5 Results, Analysis, and Recommendations

The signature assignment was scored from 0 to 3 using a rubric that is included in the Appendix A.2. The scoring for formal assessment, based on this scoring rubric, was performed by two full-time faculty of the Department of Mathematics on the 350 exams that were turned in (out of 404 total students in Sections A through L still on the class rosters at the end of the term). The two scorings were combined to give a list of 700 scores (two per exam).
The mean score is listed for each of the twelve sections as well as for the class as a whole in the table below, and the distribution of scores for the class is presented in Figure 3.1.

<table>
<thead>
<tr>
<th>Section</th>
<th>Mean Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>1.42</td>
</tr>
<tr>
<td>C</td>
<td>1.48</td>
</tr>
<tr>
<td>K</td>
<td>1.58</td>
</tr>
<tr>
<td>L</td>
<td>1.60</td>
</tr>
<tr>
<td>F</td>
<td>1.61</td>
</tr>
<tr>
<td>A</td>
<td>1.69</td>
</tr>
<tr>
<td>entire class</td>
<td>1.71</td>
</tr>
<tr>
<td>E</td>
<td>1.72</td>
</tr>
<tr>
<td>G</td>
<td>1.75</td>
</tr>
<tr>
<td>I</td>
<td>1.79</td>
</tr>
<tr>
<td>J</td>
<td>1.81</td>
</tr>
<tr>
<td>H</td>
<td>1.92</td>
</tr>
<tr>
<td>D</td>
<td>2.21</td>
</tr>
</tbody>
</table>

Figure 3.1: Score distribution for Math 150A bachelor’s program assessment

The scores show that on average, students are closest to score category 2, that is, substantial knowledge with nontrivial incompleteness or flaws. This is evidence that the SLO is being achieved at an appropriate level in the Math 150A class, where the concepts of rigorous calculus are first introduced. We note that there is a considerable variation in mean scores among the sections, but
the source of this variation is not clear. Among the three instructors who taught two sections, two of them had sections with average scores both below the mean of the class and above the mean. Thus it is unlikely that the variation between sections can be explained entirely by method and style of teaching. We note that if we take the mean scores of the various sections for this assessment measurement and compare them with their mean scores for the general education assessment measurement (see Section 2.5), the Pearson correlation is moderate (0.56), which suggests that students who work hard on the class will tend to do better across the board.

The comments in the last paragraph of the analysis of this same class for general education (see Section 2.5) about high DFU rate and students signing up for the class and not completing it hold equally well here.
Chapter 4

Fall 2017: Attempted Measurement of Math 340 Class

4.1 SLO We Attempted to Assess

We attempted to assess the following SLO for the Mathematics Bachelor’s Program:

SLO #4: Demonstrate facility with the terminology, use of symbols, and concepts of probability.

We do not believe that our measurement gave useful results for assessment, due to a flaw in the Signature Assignment (see Section 4.4 below).

4.2 Role in the Program

Math 340 is a required class for all bachelor’s programs in mathematics except the Secondary Teaching, Four-Year Integrated (FYI) Mathematics Subject Matter Program for the Single Subject Credential, and Junior-Year Integrated (JYI) Mathematics Subject Matter Program for the Single Subject Credential options of the Bachelor of Arts. These programs require Math 341 rather than Math 340.

4.3 Sections

Three sections of this course were offered in Fall 2017. Section 1 (27 students), Section 2 (17 students), Section 3 (31 students). These were randomly labeled Sections A, B, and C for the purposes of this report. Sections A and B used Introduction to Mathematical Statistics (seventh edition, 2012) by Hogg, McKean, and Craig. Section C used A First Course in Probability (ninth edition, 2014) by Ross.
4.4 Signature Assignment

One problem was chosen from each of the final exams of the three sections. The instructors were asked to agree on a problem or problems that all three final exams would have, with the possibility of varying minor details between exams. It soon became clear that the different versions of the course placed different emphases on discrete versus continuous probability problems, so it would be difficult to find a lot in common between the exams. Because of the smallness of overlap between the approaches, it would have been difficult to have a large set of common problems. So it was finally decided to have just one single common problem (with variants between the different exams).

Given the nature of formal assessment, uniformity or at least near-uniformity among all versions of the signature assignment is highly desirable, for otherwise it is difficult to meaningfully integrate the results of the different sections into a common judgement about how the whole body of students is doing. However, in this case, the content of different sections of the course were so divergent that the selection of this single problem (in its three variants) turned out to be fatally flawed. In order to help the assessment process, the instructor of Section C agreed to use a variant of one of the problems assigned by the instructor of Sections A and B: this was the genesis of the three variants used. But Section C’s course work had much less coverage of problems like the signature assignment. After the assessment, we discovered that the variants of the signature assignment used for Sections A and B were problems in the textbook used by Sections A and B and had been assigned as homework problems to those sections during the term.

Because of this difference in the students’ exposure, we consider the signature assignment to be too flawed to provide a useful assessment as to whether students are achieving SLO #4 in the Math 340 course. We continue presenting our failed attempt at assessment because it does reveal that there have been very divergent approaches to the Math 340 course (which are nonetheless both consistent with the Math 340 catalog description). We believe that this divergence makes assessment of Math 340 difficult, but that it is possible, and future attempts will greatly benefit from a description and analysis of the difficulties that we encountered here.

4.5 Results, Analysis, and Recommendations

As stated above in Section 4.4, the signature assignment was too flawed to provide data useful for determining whether students are achieving SLO #4 in Math 340. Sections A and B had their problems in the textbook and assigned homework exercises during the term, while Section C’s course work had significantly less coverage of problems like this. The only conclusion we can draw is that the sections covered significantly different material. In light of the divergence we discovered between the sections and their previous exposure to these problems, this outcome in unsurprising, and does not really tell us much about whether individual students are achieving the SLO#4 within the widely different contexts that they experienced in the various sections.

The catalog description for Math 340 lists topics to be covered, but does not go so far as to prescribe details of emphasis, depth, and difficulty level. The difference between the sections appears to be in the choice of emphasis: for example, the amounts of time spent on discrete versus continuous probability problems were considerably different. The textbooks also have significant difference in emphasis and difficulty level for the topics in the catalog description. Since the catalog entry prescribes only the topics to be covered and does not indicate how much time is to be spent on each, we cannot say that one approach or the other is inconsistent with the expectations codified
in the catalog. The fact that such different versions of the class can be consistent with the catalog description may be of interest to the department, especially as Math 340 is a prerequisite for other mathematics courses (440A, 442A-Z, and 483).

Both approaches to the class can provide many ways for students to demonstrate SLO #4, and in the future we shall attempt to design signature assignments that can handle such divergent approaches. This will probably require multiple problems that differ from section to section, but with every problem designed to uncover the students' assimilation of the content described in SLO #4. But we should emphasize that formal assessment protocols, which require judgements that ought to have some stable meaning across different sections, make this difficult, given the inevitable differences between signature assignments tailored to different sections. For example, as a problem becomes more difficult, even quite capable students are less likely to demonstrate their learning by attempting that problem and more likely to shift their efforts to an easier problem that might not be part of the signature assignment. And as we have seen, the amount of prior exposure to a particular form of problem can influence how much the students are willing to attempt it, and thus demonstrate what they have learned in a way that can be observed and measured.
Chapter 5

Spring 2018: Measurement of Math 340 Class

5.1 SLO Assessed

The following SLO for the Mathematics Bachelor’s Program was assessed:

SLO #4: Demonstrate facility with the terminology, use of symbols, and concepts of probability.

5.2 Role in the Program

Math 340 is a required class for all bachelor’s programs in mathematics except the Secondary Teaching, Four-Year Integrated (FYI) Mathematics Subject Matter Program for the Single Subject Credential, and Junior-Year Integrated (JYI) Mathematics Subject Matter Program for the Single Subject Credential options of the Bachelor of Arts. These programs require Math 341 rather than Math 340.

5.3 Sections

Three sections of this course were offered in Spring 2018. Section 1 (30 students), Section 2 (20 students), Section 4 (31 students). These were randomly labeled Sections A, B, and C for the purposes of this report. For their textbook, all three sections used Introduction to Mathematical Statistics (seventh edition, 2012) by Hogg, McKean, and Craig.

5.4 Signature Assignment

Two problems on each final exam were used as the signature assignment. Problem 1 consisted of four multiple choice questions that tested the students’ grasp of terminology, use of symbols, and concepts of probability. Problem 2 asked the students to calculate the probability of an event that is defined by both continuous and discrete criteria; this requires them to understand the terminology
and concepts of probability. Two different versions of each problem were used, as the final exams for the three exams were administered at two different times.

5.5 Results, Analysis, and Recommendations

Problem 1 was scored from 0 to 4, where the score indicates the number of correct answers among the four multiple choice questions. Problem 2 was scored from 0 to 3 using using a rubric that is included in the Appendix A.4. The scoring for formal assessment, based on these scoring rubrics, was performed by two full-time faculty of the Department of Mathematics on the 74 exams that were turned in (out of 81 total students in Sections A, B, and C still on the class rosters towards the end of the term). The two scorings were combined to give a list of 148 scores (two per exam). The scores for the multiple choice Problem 1 agreed exactly between the scorers.

The mean and median scores for the three topics are given in the below table, and the distribution of scores is presented in Figures 5.1 and 5.2 below. Recall that Problem 1 is out of 4 points possible, and Problem 2 is out of 3 points possible.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Mean Score</th>
<th>Median Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.36</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1.54</td>
<td>2</td>
</tr>
</tbody>
</table>

Figure 5.1: Score distribution for Math 340 Problem 1

Recalling the text of the SLO under consideration, “Demonstrate facility with the terminology, use of symbols, and concepts of probability,” we feel that the scores show that the students have demonstrated a level of facility that is reasonable given that this is the first course in probability. We should emphasize that facility does not necessarily imply a deep penetration into or ability to communicate well the underlying concepts, and although these other goals go beyond the scope of the SLO, we invite the Department of Mathematics to consider what the goal for this course is within their program. Since this course serves mathematics majors who need general background in the underlying theory of probability and also mathematics majors specializing in statistics who will build upon the probability concepts taught here, as well as computer science majors who need
a practical grasp of probability, we recognize that is difficult to balance the many needs that this course serves.

As a concrete example of the tension between practical facility and theoretical depth, consider Problem 2, where students would typically express three ideas to fully justify their calculation: (i) independence and identical distribution of the original continuous random variables leads to independence and identical distribution of the boolean random variables derived from them, allowing (ii) invocation of the binomial probability mass function, which requires (iii) the calculation of the success or failure of one of these boolean random variables (by integration of the density function of the original variable). As students were given a formula sheet that contained the formula for the probability of a given outcome of a binomial random variable, and as the students were not asked to explain all their logical steps in this problem, it is sometimes difficult to discern the thought process behind an answer. Given that this question is on a timed test with many other problems, the most knowledgeable students might be those who said very little to justify their work, knowing that they would get the points with the correct final answer and just enough work to show that they had used the right formula and computed the correct integral. But there is also a possibility that some students who got the right answer and showed enough work were just applying a given formula to a well-known type of problem. Could those students have solved the following problem?

**Problem.** Let the independent random variables $X_1, X_2, X_3$ have pdf $f_n(x) = (n+1)x^n$, $0 < x < 1$ and zero elsewhere, $n = 1, 2, 3$, respectively. Find the probability that exactly two of these three variables exceeds 0.5.

This problem depends on the same concepts but would not be reducible to application of a well-known formula.

On the other hand, we should also be clear that the problem actually given, where the distributions are identical, is an instance of a commonly-occurring question that must be answered in many areas of pure and applied mathematics, statistics, computer science, engineering, and natural science. And understanding that one can use the binomial probability formula to get the answer quickly is valuable to know, and it is important to have facility in such calculations, per the SLO. We are aware that time is finite, and that there may be tradeoffs involved in the teaching of probability. Do we move quickly from the foundations so as to expose the students to many
topics and show them how to do many commonly-occurring probability calculations that will arise in applications? Or do we dwell on the foundations more, but perhaps cover fewer topics that will appear in later applications? This is not a question for the Assessment Committee to answer, but one which the Department of Mathematics faculty might want to consider.
Chapter 6

Fall 2017: Measurement of Math 341
Class and Analysis

6.1 SLO Assessed

The following SLO for the Mathematics Bachelor’s Program was assessed:

SLO #4: Demonstrate facility with the terminology, use of symbols, and concepts of probability.

6.2 Role in the Program

Math 341 is a required class for the Secondary Teaching, Four-Year Integrated (FYI) Mathematics Subject Matter Program for the Single Subject Credential, and Junior-Year Integrated (JYI) Mathematics Subject Matter Program for the Single Subject Credential options of the Bachelor of Arts in Mathematics.

6.3 Sections

Three sections of this course were offered in Fall 2017, Section 2 (19 students), Section 3 (33 students), and Section 5 (25 students). Section 3 was not assessed because the instructor did not test probability in the final exam. Sections 2 and 5 were randomly labeled Sections A and B for the purposes of this report. Both these sections used Probability and Statistics for Engineering and the Sciences (ninth edition, 2015) by Devore.

6.4 Signature Assignment

The instructors of Sections A and B agreed on a few common multi-part problems on probability to appear on their final exams that would serve as the signature assignment. These problems require the students to understand the terminology of probability, for example, words and their abbreviations like “event,” “pdf” (probability density function), “pmf” (probability mass function) “cdf” (cumulative distribution function), “mean,” “standard deviation,” and “median.” They
also require students to understand the common notation “\( P(E) \),” for the probability of event \( E \) described using either set-theoretic notation or a specified range for the random variable. And they require the student to understand probabilistic concepts like conditional probability (phrased as “the probability of \( X \) given \( Y \)”), the fact that a probability density or mass function must be properly normalized, and the relation between a cumulative distribution function and a probability density function.

### 6.5 Results, Analysis, and Recommendations

The various parts of the signature assessment were grouped into three topics: Topic I (Facility with Probability Spaces), Topic II (Facility with Density, Mass, and Cumulative Distribution Functions), and Topic III (Facility with Moments and Medians), with each topic containing multiple parts, each of which was scored using the point system detailed in the rubrics included in the Appendix A.5. This produced three percentage scores, one for each topic (where the percentage indicates points earned out of total possible). The scoring for formal assessment, based on these scoring rubrics, was performed by two full-time faculty of the Department of Mathematics on the 38 exams that were turned in (out of 44 total students in Sections A and B still on the class rosters towards the end of the term). The two scorings were combined to give a list of 76 scores (two per exam).

The mean and median scores for the three topics are given in the below table, and the distribution of scores is presented in Figures 6.1–6.3 below.

<table>
<thead>
<tr>
<th>Topic</th>
<th>Mean Score (%)</th>
<th>Median Score (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>59.2</td>
<td>50.0</td>
</tr>
<tr>
<td>II</td>
<td>61.3</td>
<td>62.5</td>
</tr>
<tr>
<td>III</td>
<td>54.4</td>
<td>66.7</td>
</tr>
</tbody>
</table>

![Figure 6.1: Score distribution for Math 341 Topic I](image)

Based on our guidelines for interpreting the percentage scores, we feel that the students overall have demonstrated achievement of SLO #4. Topic III seems to be the most challenging, with the mean score indicating significant imperfection in the students’ overall level of knowledge. To see
which parts of problems caused the most difficulty, we tabulate the mean scores for each. We tabulate them in percentage out of total possible, which is either 1 point for the parts of Topic I, or 2 points for the parts of Topics II and III.
We see that part (C) for Topic III on the median of a continuous random variable was the most difficult for the students. As part (C) of Topic III accounts for one-third of the score for Topic III, this explains why students had most difficulty with this topic. In addition to the median of a continuous random variable, other topics that caused difficulty were calculation of conditional probability (part (B) of Topic I), normalization of the distribution function for a discrete random variable (part (B) of Topic II), and calculation of a cumulative distribution function from a density function by integration (part (C) of Topic II). We recommend that additional care be given to these topics in the future.
Chapter 7

Fall 2017: Measurement of Math 501
Class and Analysis

7.1 SLO Assessed

The following SLOs for the Mathematics Master’s Program were assessed:

SLO #4 for M.S. in Mathematics, and at the same time SLO #4 for M.S in Applied Mathematics: Communicate abstract mathematical ideas clearly and cogently.

7.2 Role in the Program

Math 501 is a required class for all options of the Master’s program and is the prerequisite for other courses, including Math 552 (required for all master’s degrees) and Math 655 (required for the M.S. in Mathematics option).

7.3 Section

There course had a single section of 17 students. The course was taught using the instructor’s course notes rather than a textbook.

7.4 Signature Assignment

A three-part problem in the final exam was used as the signature assignment.

7.5 Results, Analysis, and Recommendations

Each of the three parts was scored from 0 to 3 using a rubric that is included in the Appendix A.6. The scoring for formal assessment, based on this scoring rubric, was performed by two full-time faculty of the Department of Mathematics on the 14 exams that were turned in (out of 17 total students). The two scorings were combined to give a list of 28 scores (two per exam).
The distribution of scores for the three parts is presented in Figure 7.1 below. The following are the average scores for each part.

<table>
<thead>
<tr>
<th>Part</th>
<th>Average Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>2.00</td>
</tr>
<tr>
<td>(b)</td>
<td>0.75</td>
</tr>
<tr>
<td>(c)</td>
<td>1.04</td>
</tr>
</tbody>
</table>

Figure 7.1: Score distribution for Math 501

In retrospect, the scoring criteria we used for SLO# 4, “Communicate abstract mathematical ideas clearly and cogently,” would benefit from revisions. Criteria more clearly aligned to SLO# 4 would allow for higher scores for clear communication of abstract ideas even if those clearly expressed ideas do not lead to complete and correct proofs. Most students know topology, compactness and connectedness by definition. The relatively high mean score for part (a) can likely be attributed to the unambiguous steps that must be undertaken in order to prove that a family of sets is a topology. The greater difficulty with parts (b) and (c) may be due in part to their greater abstractness and, in the case of part (b), to the fact that the real line is compact in the topology assumed in the problem, but not compact in the usual topology of the real line. While we would like to see future improvement, all in all, we found that first year graduate student performance relative to SLO# 4 this year to be satisfactory.
Appendix A

Rubrics

A.1 Rubric for Math 150A (General Education Assessment)

Students shall be scored based on five sub-problems, each of which is scored 0, 1, or 2. The total score is the sum of the scores on the five sub-problems, so the total score is from 0 to 10. The five sub-problems are:

- Sub-Problem I: Student shall determine where the function is increasing and where it is decreasing.
- Sub-Problem II: Student shall determine where the function is concave up and where it is concave down.
- Sub-Problem III: Student shall find both coordinates of every local minimum and every local maximum of the function.
- Sub-Problem IV: Student shall find both the coordinates of every inflection point of the function.
- Sub-Problem V: Student shall draw a graph of the function $y = f(x)$ for $x$ in $[-2, 2]$.

The 0-1-2 scoring rubric for all of these parts is:

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Student has no substantial idea</td>
</tr>
<tr>
<td>1</td>
<td>Student has a substantial idea, but also a substantial incompleteness or flaw</td>
</tr>
<tr>
<td>2</td>
<td>Solution is perfect, or has only trivial flaws</td>
</tr>
</tbody>
</table>

In applying this rubric to the specific sub-problems, the following guidelines should be considered:

- In solving Sub-Problem III, the student may use derivatives, which are likely calculated in other parts of the problem. These can count as a substantial idea only if there is clear evidence that the student actually applied them to Sub-Problem III.

- A similar principle holds for second derivatives calculated elsewhere on the exam, but applicable to solving Sub-Problem IV.
• In Sub-Problem V, a 2-point score (near perfect to perfect) is only obtained if the graph shows increase and decrease on the correct intervals, some decent attempt to get concavity right (so straight line segments between extrema are not sufficient). If at least the increasing and decreasing behavior is correct (implying correct local extrema), then the student should not get 0. If the graph is so poor as to be no better than a random guess regarding increase and decrease and overall shape, then 0 points should be given.

Once the sub-problem scores are summed to give an overall score from 0 to 10, the overall performance is evaluated:

<table>
<thead>
<tr>
<th>Score</th>
<th>Student’s ability to represent, understand, and explain mathematical information</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–2</td>
<td>little or no ability</td>
</tr>
<tr>
<td>3–5</td>
<td>some ability, but imperfect</td>
</tr>
<tr>
<td>6–8</td>
<td>substantial ability</td>
</tr>
<tr>
<td>9–10</td>
<td>excellent ability</td>
</tr>
</tbody>
</table>

### A.2 Rubric for Math 150 (Bachelor’s Program Assessment)

Student shall calculate the derivative of a rational function using the limit definition of the derivative.

Students shall be given a score from 0 to 3 based on the following rubric:

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Student expresses no substantial idea about how to use the definition of derivative to find ( f'(x) )</td>
</tr>
<tr>
<td>1</td>
<td>Student demonstrates a substantial idea, but falls far short of what is needed to use the definition of derivative to find ( f'(x) )</td>
</tr>
<tr>
<td>2</td>
<td>Student makes substantial progress toward a solution, but with nontrivial incompleteness and/or flaws</td>
</tr>
<tr>
<td>3</td>
<td>Student gives a valid solution with no flaws or only trivial flaws</td>
</tr>
</tbody>
</table>

### A.3 Assessment Rubric for Math 340 (Fall 2017)

Student shall calculate the probability density function of a random variable \( Y \) from that of a related random variable \( X \) and determine where the density function of \( Y \) is positive.

Students shall be given a score from 0 to 3 based on the following rubric:
<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Student expresses no substantial ideas relevant to a solution</td>
</tr>
<tr>
<td>1</td>
<td>Student demonstrates a substantial idea, but falls far short of what is needed for a full solution</td>
</tr>
<tr>
<td>2</td>
<td>Student makes substantial progress toward a solution, but with nontrivial incompleteness and/or flaws</td>
</tr>
<tr>
<td>3</td>
<td>Student gives a valid solution with no flaws or only trivial flaws</td>
</tr>
</tbody>
</table>

### A.4 Assessment Rubrics for Math 340 (Spring 2018)

**Problem 1:** Students shall answer four multiple-choice questions that require them to demonstrate knowledge of terminology, use of symbols, and concepts of probability.

A student receives 1 point for each correct answer and 0 points for each wrong answer, giving a total score from 0 to 4.

**Problem 2:** Students shall calculate the probability of an event that is defined by both continuous and discrete criteria.

Students shall be given a score from 0 to 3 based on the following rubric:

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Student expresses no substantial ideas relevant to a solution (where these ideas include understanding or use of terminology or notation that constitutes a meaningful step toward a solution)</td>
</tr>
<tr>
<td>1</td>
<td>Student demonstrates some substantial idea relevant to a solution, but falls far short of what is needed for a full solution</td>
</tr>
<tr>
<td>2</td>
<td>Student makes substantial progress toward a solution, but with nontrivial incompleteness and/or flaws, including nontrivial misunderstanding or misuse of terminology or notation</td>
</tr>
<tr>
<td>3</td>
<td>Student gives a valid solution with no flaws or only trivial flaws¹</td>
</tr>
</tbody>
</table>

¹ It is acceptable for the student to express the answer as a product of binomial coefficients and numbers raised to powers, provided that the entries of the binomial coefficient, the numbers, and the exponents are all fully worked out (no variables or unworked integrals should be present).

### A.5 Assessment Rubrics for Math 341

We group parts of the above signature assignment to obtain three topics.

**Topic I: Facility with Probability Spaces**

Student shall calculate probability of four events in a probability space.
These are the four parts of Problem 1, and each shall be scored 0 or 1.

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Student gives no solution or a solution with</td>
</tr>
<tr>
<td></td>
<td>nontrivial flaws and/or incompleteness</td>
</tr>
<tr>
<td>1</td>
<td>Student gives a valid solution with no flaws or</td>
</tr>
<tr>
<td></td>
<td>only trivial flaws</td>
</tr>
</tbody>
</table>

We sum the scores for the four parts, to give an overall score from 0 to 4.

**Topic II: Facility with Density, Mass, and Cumulative Distribution Functions**

Student shall perform four calculations involving density, mass, and cumulative distribution functions.

These are Problems 2(a), 2(b), 3(a), and 3(b), and students shall be given a score from 0 to 2 for each part, based on the following rubric:

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Student has no substantial idea</td>
</tr>
<tr>
<td>1</td>
<td>Student has a substantial idea, but also</td>
</tr>
<tr>
<td></td>
<td>a substantial incompleteness or flaw</td>
</tr>
<tr>
<td>2</td>
<td>Solution is perfect, or has only trivial flaws</td>
</tr>
</tbody>
</table>

We sum the scores for the four parts, to give an overall score from 0 to 8.

**Topic III: Facility with Moments and Medians**

Student shall perform four calculations involving the mean, standard deviation, and median of a probability distribution.

These are Problems 3(c), (d), and (e), and students shall be given a score from 0 to 2 for each part, based on the following rubric:

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Student has no substantial idea</td>
</tr>
<tr>
<td>1</td>
<td>Student has a substantial idea, but also</td>
</tr>
<tr>
<td></td>
<td>a substantial incompleteness or flaw</td>
</tr>
<tr>
<td>2</td>
<td>Solution is perfect, or has only trivial flaws</td>
</tr>
</tbody>
</table>

We sum the scores for the four parts, to give an overall score from 0 to 6.

Once we have scores for the three measurements, we turn these into percentages out of the total possible (4 for Topic I, 8 for Topic II, and 6 for Topic III). We can evaluate the students according to the following scheme in each of the three areas:
A.6 Assessment Rubric for Math 501

How well the student demonstrated clear and cogent communication of abstract mathematical ideas was evaluated using the following rubric:

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Demonstrates no relevant idea about how to present the proof</td>
</tr>
<tr>
<td>1</td>
<td>Presents some ideas relevant to a proof, but falls short of what is needed to construct the complete argument.</td>
</tr>
<tr>
<td>2</td>
<td>Presents a proof that contains substantial correct ideas, but has some flaws in its logic or in presentation.¹</td>
</tr>
<tr>
<td>3</td>
<td>Presents a valid proof in a clear manner. The proof has no flaws or negligible flaws.</td>
</tr>
</tbody>
</table>

¹For example, the order of statements in the proof is such that the flow of the argument is partially obscured.