2015-2016 Annual Program Assessment Report

Please submit report to your department chair or program coordinator, the Associate Dean of your College, and to james.solomon@csun.edu, Director of the Office of Academic Assessment and Program Review, by September 30, 2016. You may, but are not required to, submit a separate report for each program, including graduate degree programs, which conducted assessment activities, or you may combine programs in a single report. Please identify your department/program in the file name for your report.

College: Science and Mathematics

Department: Mathematics

Program: B.A. and B.S. (assessment and revised SLOs). At the end is also an appendix on the M.S. program. (This contains the new graduate program SLOs that were approved by the department. As background material, it also contains proposed alignment matrices, and examples of potential assessment methods that show the feasibility of measuring these SLOs.)

Assessment liaison: Vladislav Panferov

1. **Please check off whichever is applicable:**
   A. ____X___ Measured student work.
   B. ____X___ Analyzed results of measurement.
   C. ________ Applied results of analysis to program review/curriculum/review/revision.

2. **Overview of Annual Assessment Project(s).** On a separate sheet, provide a brief overview of this year's assessment activities, including:
   - an explanation for why your department chose the assessment activities (measurement, analysis, and/or application) that it enacted
   - if your department implemented assessment **option A**, identify which program SLOs were assessed (please identify the SLOs in full), in which classes and/or contexts, what assessment instruments were used and the methodology employed, the resulting scores, and the relation between this year's measure of student work and that of past years: (include as an appendix any and all relevant materials that you wish to include)
   - if your department implemented assessment **option B**, identify what conclusions were drawn from the analysis of measured results, what changes to the program were planned in response, and the relation between this year's analyses and past and future assessment activities
   - if your department implemented **option C**, identify the program modifications that were adopted, and the relation between program modifications and past and future assessment activities
   - in what way(s) your assessment activities may reflect the university's commitment to diversity in all its dimensions but especially with respect to underrepresented groups
   - any other assessment-related information you wish to include, including SLO revision (especially to ensure continuing alignment between program course offerings and both program and university student learning outcomes), and/or the creation and modification of new assessment instruments

3. **Preview of planned assessment activities for next year.** Include a brief description and explanation of how next year's assessment will contribute to a continuous program of ongoing assessment.

Overview of Annual Assessment Projects

Summary of Assessment Activities Undertaken:

1. In Fall of 2015, we measured undergraduate student work to evaluate undergraduate program **SLO #3: Present clear and rigorous proofs**. We used an embedded instrument in the final exam of the Math 320 class (Foundations of Higher Mathematics), the main class where this objective is introduced and practiced.

2. We revised the undergraduate program SLOs.

3. In Spring of 2016, we measured undergraduate student work to evaluate the undergraduate program **Revised SLO #1: Devise proofs of basic results concerning sets and number systems** and **Revised SLO #2: Rigorously establish fundamental analytic properties and results such as limits, continuity, differentiability and integrability.** Both of these new SLOs cover different aspects that were covered by the old SLO # 3. This time, we used an embedded instrument in the final exam of the Math 350 class (Advanced Calculus I), where these objectives are practiced and demonstrated.

4. We analyzed all of our measurements for this year and made recommendations to the department.

Rationale: The ability to present clear and rigorous proofs is the hinge upon which student success in higher mathematics turns. The mathematics department educates a highly diverse population of students, with widely varying levels of background in formal mathematics. The Math 320 course is designed to make sure that all students, regardless of the background, have the intellectual tools in logic and mathematical presentation needed to succeed in their further upper division classes, so it was natural for us to embed our first assessment there. The Math 350 course is more advanced, and makes intensive use of basic proof techniques, and so we felt that it is a natural place to measure the students' continued development in ability to write proofs. Since Math 350 includes both problems about number systems and analytic concepts, assessment of this course enabled us to cover two SLOs in one assessment exercise.

1. Fall 2015: Measurement of Math 320 Class and Analysis

SLO #3: Present clear and rigorous proofs

Assessment data collected: One specially designed question was included in the final exam for the Fall 2015 course Math 320 (Foundations of Higher Mathematics). Data were collected from two class sections: 16145 and 17258, referred further as Section 1 and Section 2 (these designations were chosen without reference to the actual section numbers to preserve anonymity).

Course specifics: Math 320 is a junior level course which serves as a gateway into the upper division math program at CSUN. It focuses on techniques of formal mathematical reasoning in context of number theory, combinatorics and analysis. The question chosen represents one of the basic techniques (using the induction axiom to establish the validity of a certain logical statement for every natural number).

Question Statement (as it appeared on the exam): We'll define a sequence of numbers as follows: $a_1 = 1$, and for each natural number $n \geq 2$, let $a_n = 3a_{n-1} - 1$. (a) Find a_2 and a_3. (b) Use induction to prove that for all $n \in \mathbb{N}$, $a_n = \frac{1}{2}(3^{n-1} + 1)$.

Mathematical background: The first part of the question refers only to basics of mathematical notation and arithmetic skills. One would expect every student at this level to complete it correctly, which was in fact the case, with very few exceptions, attributable perhaps to such factors as lack of attention, exam environment, etc. The second part was the one that our scoring was focused on. The usual technique of proof in this case consists of checking the truth of the "base case" $n = 1$ (interpreting the notation $n \in \mathbb{N}$ correctly as "for all natural numbers starting with 1"), and performing the induction step: assuming the result be true for a given natural n show, with a few lines of algebraic manipulations, that the result also holds for the next natural number $n + 1$. An informal thinking about the problem would in fact involve the crucial step of "verifying the algebra":

$$3\big(\tfrac{1}{2}(3^{n-1} + 1)\big) - 1 = \tfrac{1}{2}(3^n + 1).$$

Once this fact is established, the rest of the problem amounts to presenting the proof in a proper format, as discussed in class, and practiced in homework assignments and prior exams.

Formal assessment procedure: The formal assessment of the student work was done based on the following **scoring rubric**:

Score	0	1	2	3
Description	Demonstrates no relevant idea about how to present the proof	Presents some ideas relevant to a proof, but falls short of what is needed to construct the complete argument.	Presents a proof that contains substantial correct ideas, but has some flaws in its logic or in presentation.[1]	Presents a valid proof in a clear manner. The proof has no flaws or negligible flaws.

[1]For example, the order of statements in the proof is such that the flow of the argument is partially obscured.

Results and analysis of the formal assessment: The scoring for formal assessment, based on the above scoring rubric, was done independently by two members of the department assessment committee. The results for the scores given to the two sections by the scorers are presented in Figure 1.

The average score for Sections 1 and 2 are 1.95 (65%) and 2.12 (71%), respectively. It is noteworthy that score distribution for Section 2 is clearly bimodal, with peaks at 1 and 3, while the distribution for Section 1 is uniform between the scores 1, 2 and 3.

In both sections consistently about 65% of the students received scores 2 and 3, while about 35% received scores below 2, with very few receiving score 0.

The overall results show that a typical Math 320 student has a substantial, but not perfect, grasp of inductive proof techniques. This is to be expected for students

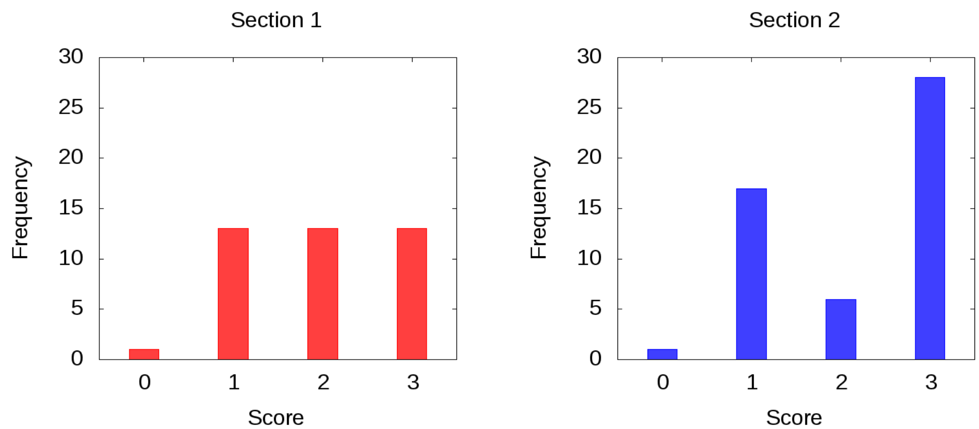

Figure 1: Score distribution for Sections 1 and 2

who are working with these concepts for the first time.

Supplemental appraisal of student performance: To supplement the formal assessment described in the previous sections, additional data were collected based on the following supplemental appraisal scheme with four indicators, I_1 to I_4:

Score	0	1	2
I_1: Outlines the formal structure of the induction principle and applies it properly	no	partially	yes
I_2: Works the algebra correctly to show the induction step	no	partially	yes
I_3: Checks the induction base with $n = 1$	no	yes	
I_4: Demonstrates proper use of mathematical and formal logical notation	no	yes	

The objective of using the supplemental appraisal was to collect data about frequency of occurrence of typical mistakes, without interfering with the overall assessment score.

Analysis of supplemental appraisal: We checked for correlations between the data obtained from the supplemental appraisal and the formal assessment. The results in the table below show the values of correlation coefficients between the different supplementary appraisal indicators I_1 to I_4 listed in the table at the top of this page and the score from the formal assessment scoring rubric at the bottom of page 2. (Higher positive values indicate stronger correlations; negative values indicate negative correlations.)

Indicator	Section 1	Section 2
I_1	0.60	0.61
I_2	0.79	0.92
I_3	0.40	-0.11
I_4	0.13	0.44
Composite: $\frac{1}{2}(I_1 + I_2 + I_3 + I_4)$	0.86	0.95

Particularly noticeable is the correlation between the "algebra" score I_2 and the the overall level of success: one may conclude that students who were able to grasp the key idea behind the problem were overall successful in presenting the proof in a clear manner. (The correlation is further improved by considering the composite score as can be seen from the last row of the table.)

The negative value for the correlation with I_3 in Section 2 is due to the fact that a large number of students in that class (54%) failed to check the base of induction with $n = 1$, perhaps by analogy with some other problem, or due to some unknown factor.

Analysis of instructional approaches:

The instructors used different approaches to present the Induction Principle: the instructor in Section 1 used the language of predicate calculus (the equality of n-th terms of the sequences was denoted $P(n)$), while in Section 2 the main vehicle was the notion of inductive set. Both approaches were met with difficulties: in Section 1 students had difficulty with using the notation $P(n)$ correctly, while in Section 2 the set of indices was often confused with the set of elements of the sequence. Many students chose to forego the formal approach entirely and to present the proof without using either $P(n)$ or inductive set notation: overall it seemed that using the formal language did not serve to improve clarity of the proof. This type of difficulty is perhaps to be expected by the end of a one-semester course providing an introduction to rigorous mathematics. Nevertheless, we hope that further effort by instructors of the course could be directed towards training the students in proper use of mathematical logic and set theory notation to improve clarity in communicating mathematical ideas.

2. Revision of Undergraduate Program SLOs

We revised the undergraduate program SLOs, so that they are now as follows:

Students shall be able to:

1. Devise proofs of basic results concerning sets and number systems.
2. Rigorously establish fundamental analytic properties and results such as limits, continuity, differentiability and integrability.
3. Demonstrate facility with the objects, terminology and concepts of linear algebra.
4. Demonstrate facility with the terminology, use of symbols, and concepts of probability.
5. Write simple computer programs to perform computations arising in the mathematical sciences.

Curriculum Alignment Matrix
for Mathematics courses required in the majority of Options

Course	SLO 1	SLO 2	SLO 3	SLO 4	SLO 5	Options Requiring Course
150A		I				All
150B		I				All
250		I				All
262			I/P			All
320	I/P					All
340				I/P/D		A,C,D,E
350	P/D	P/D				All
351		P	P			A,C,D
360	P/D					A,B,C
382/L					I/P/D	All
462	P/D		P/D			A,C,D

I = introduced P = practiced D=demonstrated

Option Key:
A = B.A., Mathematics Option
B = B.A., Secondary Teaching Option
C = B.S., Mathematics
D = B.S., Statistics
E = B.S., Applied Mathematical Sciences

3. Spring 2016: Measurement of Math 350 Class and Analysis

Revised SLO #1: Devise proofs of basic results concerning sets and number systems.

Revised SLO #2: Rigorously establish fundamental analytic properties and results such as limits, continuity, differentiability and integrability.

Assessment data collected: In Spring 2016 the Assessment Committee decided to continue with the assessment of the SLOs pertaining to presenting proofs. The original single SLO had been revised to emphasize two different aspects:

SLO #1: Devise proofs of basic results concerning sets and number systems;

SLO #2: Rigorously establish fundamental analytic properties and results such as limits, continuity, differentiability and integrability.

We assessed these SLOs in MATH 350, the natural follow-up course to MATH 320, used for the assessment of the Proofs SLO in Fall 2015. We chose two questions on the MATH 350 final exam given by the instructor for the course. We chose questions that we felt were representative of the basic sort of proof problems students encounter in MATH 350.

Course specifics: MATH 350 is a required course for all bachelor's options and has MATH 320 as a prerequisite. In this course students learn the rigorous foundations of calculus. Topics include the real number system, continuous functions, differentiation, and Riemann integration of functions of one real variable. The Mathematics Department regards this course as comprising the core material underpinning much of advanced mathematics.

Question Statements

1. Let u_0 be an upper bound of an open set A and suppose that u_0 is also a boundary point of A. Prove that u_0 is the least upper bound of A.

2. Give an epsilon-delta proof that $\lim\limits_{x \to \frac{1}{4}} \frac{3}{1-x} = 4$.

Mathematical background: The first question invokes the concept of least upper bound, which is a pillar in the rigorous development of calculus. Solving the question requires solid understanding of the concepts of upper bound and boundary point of a set of real numbers, and combining them using formal logic techniques. This question serves primarily as the assessment instrument for SLO #1.

Epsilon-delta proofs are very common tools in mathematical analysis (a broad area which includes calculus and differential equations). Some students may have been introduced to such proofs in their introductory calculus courses, however for most students this topic is first developed in earnest in a rigorous analysis course such as MATH 350. The second question requires the students to apply the formal definition of limit, perform algebraic manipulations and use the technique of manipulating inequalities which is fundamental for other developments in the course. This question serves primarily as the assessment instrument for SLO #2.

Formal assessment procedure: The scoring rubric was identical to that used for the Fall 2015 assessment of MATH 320, with possible scores from 0 to 3. We repeat the rubric here.

Score	0	1	2	3
Description	Demonstrates no relevant idea about how to present the proof	Presents some ideas relevant to a proof, but falls short of what is needed to construct the complete argument.	Presents a proof that contains substantial correct ideas, but has some flaws in its logic or in presentation.[1]	Presents a valid proof in a clear manner. The proof has no flaws or negligible flaws.

[1]For example, the order of statements in the proof is such that the flow of the argument is partially obscured.

Results and analysis of the formal assessment: The scoring for formal assessment, based on the above scoring rubric, was done independently by two members of the department assessment committee. The results for the frequency of scores given by the two scorers for the two questions are presented in Figure 2:

The mean scores were 1.82 for Question 1 and 1.74 for Question 2, respectively. Thus overall student learning was rated nearly the same for SLO 1 as for SLO 2, as for both questions the overall average was a bit below 2. This indicates that the solutions given by typical student in this MATH 350 class were approaching the standard corresponding to a score of 2: "Presents a proof that contains substantial correct ideas, but has some flaws in its logic or in presentation". There

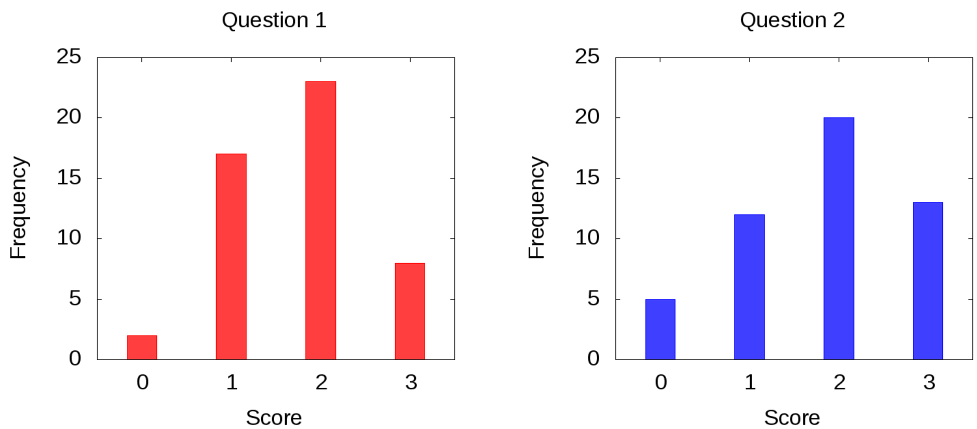

Figure 2: Score distribution for Questions 1 and 2

was substantial variability in the scores from student to student, however nearly all students in the class exhibited at least some positive progress toward a solution. These results are on par with those found in the MATH 320 assessment.

Supplemental appraisal of student performance: To supplement the general assessment described above, and to pinpoint the specific areas of difficulty for the students answering exam questions, additional data were collected based on the following appraisal scheme with three indicators specific to each question:

Table 1: Question 1 Indicator.

Question 2 Indicator	0	1
J_1: States or applies the definition of upper bound correctly	no	yes
J_2: States or applies the definition of boundary point correctly	no	yes
J_3: Combines the two to arrive at the conclusion using definitions and properties of least upper bounds	no	yes

Table 2: Question 2 Indicator.

Score	0	1
I_1: Follows the essential steps to apply the definition of limit (given ε find δ...)	no	yes
I_2: Performs algebraic manipulations correctly	no	yes
I_3: Uses inequalities correctly, restricts δ according to the behavior of the function	no	yes

The data for frequencies of positive ("yes") results (out of a total of 25 students) for the indicators for the two questions are presented in Figure 3.

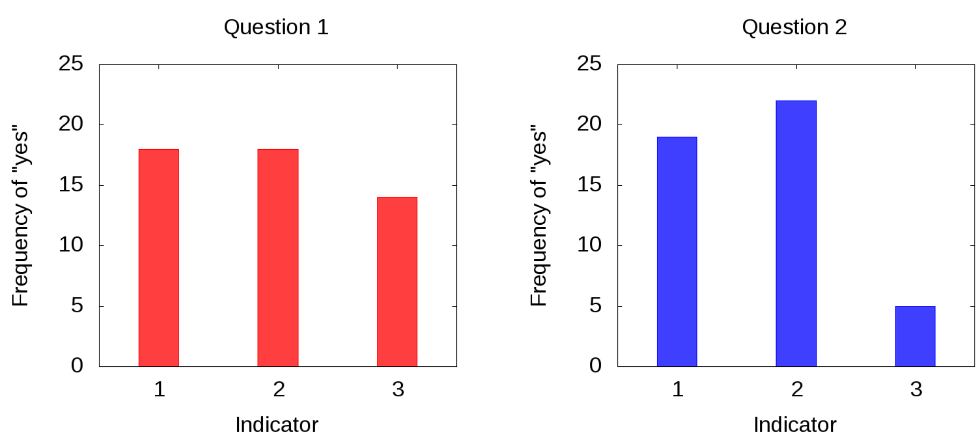

Figure 3: Frequencies of positive results for the specific indicators for Questions 1 and 2 (out of a total of 25 students).

Analysis of supplemental appraisal: The data collected for Question 1 indicate that combining the definitions to arrive at a proof remains the strongest challenge for the students. It is recommended that definitions are illustrated by real-line diagrams (few students included this type of diagrams in their work) and their use is emphasized as a technique for finding a path towards the solution before attempting to present the work in the formal logic form.

The data collected clearly pinpoint a weakness in answers to Question 2: overall only four students noticed the pitfall that the function blows up at $x = 1$; a straightforward manipulation of the inequalities yields incorrect answers. Overall the majority of the students followed correct path for the solution of the problem (Indicator 1) and demonstrated adequate algebraic manipulation skills (Indicator 2). However, given the low overall performance on Indicator 3 it is recommended that this aspect be specifically included in the instruction of the course in the future.

4. Recommendations

We shall recommend the following to our instructors:

1. When teaching mathematical induction, emphasize the importance of establishing a base case.
2. When teaching the completeness property of the real numbers, encourage students to use diagrams to reinforce their intuitions before proceeding with formal proofs.
3. When teaching δ-ϵ proofs, emphasize that one can (and often must) restrict these parameters to values small enough to avoid pathological features of the functions being investigated.

Preview of Planned Assessment Activities for Next Year

Since we revised the SLOs for the bachelor's programs this year (2015–16), the natural course is to assess each one in turn. Having assessed SLOs 1 and 2 this past year, we intend to assess one or two of the remaining three SLOs in 2016-17.

Appendix: New Graduate Program SLOs

Student Learning Outcomes
M.S. in Mathematics and M.S. in Applied Mathematics
—Proposal by the Department Graduate Committee—

Student Learning Outcomes: M.S. in Mathematics (Option I)

1. Demonstrate proficiency in abstract algebra, including group theory and Galois theory.
2. Demonstrate proficiency in real and complex analysis, including measure theory.
3. Demonstrate proficiency in general topology, including metric spaces.
4. Communicate abstract mathematical ideas clearly and cogently.
5. Solve mathematical problems at the graduate level.

Student Learning Outcomes: M.S. in Applied Mathematics (Option II)

1. Demonstrate proficiency in real analysis, including measure theory.
2. Demonstrate proficiency in general topology, including metric spaces.
3. Demonstrate proficiency in numerical methods or statistics.
4. Communicate abstract mathematical ideas clearly and cogently.
5. Solve mathematical problems at the graduate level.

Background Information

1 Student Learning Outcomes: M.S. in Mathematics

Program Requirements:

- Topology, MATH 501 (3)
- Real Analysis, MATH 552 (3)
- Calculus on Manifold, MATH 550 (3)
- Abstract Algebra, MATH 560 (3)
- Complex Analysis, MATH 655 (3)
- Graduate Seminar, MATH 589 (1)
- Electives (15 units)
- Thesis or Comprehensive Exam

Student Learning Outcomes

1. Demonstrate proficiency in abstract algebra, including group theory and Galois theory.
2. Demonstrate proficiency in real and complex analysis, including measure theory.
3. Demonstrate proficiency in general topology, including function spaces and metric spaces.
4. Communicate abstract mathematical ideas clearly and cogently.
5. Solve mathematical problems at the graduate level.

OPTION I	SLO 1	SLO 2	SLO 3	SLO 4	SLO 5
Topology (MATH 501)			IP	I	I
Real Analysis (MATH 552)		IP		I	I
Calculus on Manifolds (MATH 550)	P		P	P	I
Abstract Algebra (MATH 560)	IPD			I	I
Complex Analysis (MATH 655)		PD	PD	P	P
Graduate Seminar (MATH 589)				PD	
Electives	P	P	P	P	P
Thesis or Comprehensive Exam	D	D	D	D	D

Table 1: I=Introduced, P=Practiced, D=Demonstrated

Method of Assessment. The Graduate Committee forms an Assessment Subcommittee of at least two members. In each academic year, the subcommittee assesses two SLOs as outlined below.

1. SLO 1: The assessment instruments are embedded in the course work for MATH 560 (Abstract Algebra). The Assessment Subcommittee coordinates with the instructor of the course, and assesses two components of the course work (an assignment or a test question), independently of grades.

2. SLO 2: The assessment instruments are embedded in the course work for MATH 552 (Real Analysis) or MATH 655 (Complex Analysis). The Assessment Subcommittee coordinates with the instructor of the courses, and assesses two components of the course work (an assignment or a test question), independently of grades.
3. SLO 3: The assessment instruments are embedded in the course work for MATH 501 (Topology). The Assessment Subcommittee coordinates with the instructor of the courses, and assesses two components of the course work (an assignment or a test question), independently of grades.
4. SLO 4: The assessment instruments are embedded in the course work for MATH 589 (Graduate Seminar). The Assessment Subcommittee coordinates with the instructor of the seminar, prepares a *rubric*, and assesses one presentation of every student in the seminar, independently of grades.
5. SLO 5: The assessment instruments are embedded in the Thesis and the Comprehensive Exam. The Assessment Subcommittee prepares a rubric, and checks all master theses defended in the academic year. Simultaneously, the Assessment Subcommittee coordinates with the authors of the Comprehensive Exam, and assesses the answers to one question independently of grades.

2 Student Learning Outcomes: M.S. in Applied Mathematics

Program requirements:

- Topology, MATH 501 (3)
- Real Analysis, MATH 552 (3)
- Topics in Probability/Statistics, MATH 542A-D (3)
  or Numerical Methods for Linear Systems, MATH 581 (3)
- Two courses chosen from the following of courses:
  Regression Analysis, MATH 540 (3)
  Topics in Probability/Statistics, MATH 542A-D (3)
  Advanced Numerical Analysis, MATH 581 (3)
  Topics in Numerical Analysis, MATH 582 AD (3)
  Topics in Applied Mathematics, MATH 592A-D (3)
  Advanced Mathematical Modeling, MATH 625 (3)
  Complex Analysis, MATH 655 (3)
  Applied Functional Analysis I/II, MATH 680A/B (3)
- Graduate Seminar, MATH 589 (1)
- Electives (15 units)
- Thesis or Comprehensive Exam

Student Learning Outcomes

1. Demonstrate proficiency in real analysis, including measure theory.
2. Demonstrate proficiency in general topology, including function spaces and metric spaces.
3. Demonstrate proficiency in numerical methods or statistics.
4. Communicate abstract mathematical ideas clearly and cogently.
5. Solve mathematical problems at the graduate level.

OPTION II	SLO 1	SLO 2	SLO 3	SLO 4	SLO 5
Topology (MATH 501)		IP		IP	I
Real Analysis (MATH 552)	IP		IP	I	I
MATH 542A-D or MATH 581	P	P	I	P	I
Two of 8 courses	P	P	P	P	P
Graduate Seminar (MATH 589)				PD	
Electives	P	P	P	P	P
Thesis or Comprehensive Exam	D	D	D	D	D

Table 2: I=Introduced, P=Practiced, D=Demonstrated

Method of Assessment. The Graduate Committee forms an Assessment Subcommittee of at least two members. In each academic year, the subcommittee assesses two SLOs as outlined below.

1. SLO 1: The assessment instruments are embedded in the course work for MATH 552 (Real Analysis). The Assessment Subcommittee coordinates with the instructor of the courses, and assesses two components of the course work (an assignment or a test question), independently of grades.
2. SLO 2: The assessment instruments are embedded in the course work for MATH 501 (Topology). The Assessment Subcommittee coordinates with the instructor of one of these courses, and assesses two components of the course work (an assignment or a test question), independently of grades.
3. SLO 3: The assessment instruments are embedded in the course work for MATH 542 (Topics in Probability/Statistics) or MATH 581 (Numerical Methods for Linear Systems) The Assessment Subcommittee coordinates with the instructor of the courses, and assesses two components of the course work (an assignment or a test question), independently of grades.
4. SLO 4: The assessment instruments are embedded in the course work for MATH 589 (Graduate Seminar). The Assessment Subcommittee coordinates with the instructor of the seminar, prepares a *rubric*, and assesses one presentation of every student in the seminar, independently of grades.
5. SLO 5: The assessment instruments are embedded in the Thesis and the Comprehensive Exam. The Assessment Subcommittee prepares a rubric, and checks all master theses defended in the academic year. Simultaneously, the Assessment Subcommittee coordinates with the authors of the Comprehensive Exam, and assesses the answers to one question independently of grades.