MECHANICS LAB
AM 317

EXP 5
COLUMN BEHAVIOR
BUCKLING
I. OBJECTIVES

I.1 To determine the effect the slenderness ratio has on the load carrying capacity of columns of varying lengths.

I.2 To observe short, intermediate and long column behavior under the application of a compressive load.

I.3 To compare experimentally observed values of critical stress with the theoretical values.

II. BACKGROUND

A straight slender member subjected to an axial compressive load is called a column. If such a member is relatively short, it will remain straight when loaded, and failure will occur by yielding of the material (in the case of wood crushing of the fibers will occur). However, if the member is relatively long a different type of behavior will be observed. When the compressive load reaches a so called “critical load” a long column will undergo a bending action in which the lateral deflection will become very large with little increase in load. This behavior is called “buckling” and can occur even though the maximum stress in the column is less than the yield stress of the material. The load at which a column will buckle is affected by material properties, column length and cross section, and end conditions.

\[ P_{cr} = \frac{\pi^2 EI}{(KL)^2} \]

where:

\[ E \] = modulus of elasticity
\[ I \] = minimum moment of inertia of cross-sectional area about an axis through the centroid
\[ L \] = length of the bar
\[ K \] = effective length constant (\( KL \) = effective length)
This formula was first obtained by the Swiss mathematician, Leonard Euler (1707-1783) and the load $P_{cr}$ is called the Euler buckling load (see Appendix A for the derivation).

Euler's formula defines a boundary above which elastic instability occurs in a compression member. To make it independent of the size of the member it is frequently written in terms of stress rather than load.

\[
\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2 E}{\left(\frac{K}{r}\right)^2}
\]

where $L/r$ is called the effective-slenderness ratio and $r$ is the radius of gyration of the cross-sectional area. The radius of gyration can be computed from the equation:

\[
r = \sqrt{\frac{I}{A}}
\]

The effective length $L_e = KL$ depends on the support boundary conditions which are summarized in the figure below:

![Figure 1](image)

**Figure 1** Effective length of column for various end conditions

In this experiment, the members tested are made of Douglas Fir and have a cross section of approximately 0.7 x 0.7 inches. The radius of gyration can then be
written in terms of the minimum thickness \( (d_{\text{min}}) \) of the member. It is the minimum cross section dimension \( (d_{\text{min}}) \) that usually defines the axis about which buckling takes place experimentally.

\[
\sigma = \frac{\pi^2 E}{12 \left( \frac{KL}{d_{\text{min}}} \right)^2}
\]

where \( L/d \) is an alternate definition of slenderness ratio for members with a rectangular cross section.

If the cross section is square, the moment of inertia and the radius of gyration are the same about any axis through the centroid and buckling is likely to occur about any axis. During the testing of each specimen try to observe the axis about which bending is taking place. If the member bends about a diagonal then the moment required to reach the proportional limit is a minimum because the distance to the extreme fiber is one half the diagonal rather than one half the thickness.

A second boundary to the safe stress range for a compression member is the yield stress or crushing strength. The experimental data points may therefore be expected to fall below both Euler curve and the yield stress line as shown in the following sample figure.

![Figure 2 An example of column critical stress and slenderness ratio relation](image-url)
The two material properties of interest in this experiment are the crushing strength and modulus of elasticity in bending for Douglas Fir. Mark’s handbook implies, based on data from the U.S. Forest Products Laboratory, that both of these material properties are directly related to the specific gravity of the wood. Average values at 12% moisture content, for Douglas Fir are:

- specific gravity \((SG)\) = 0.51
- modulus of elasticity in bending \((E)\) = 1,950,000 psi
- maximum crushing strength \((\sigma_{ys})\) = 7430 psi

Since the modulus and the crushing strength vary directly with the specific gravity, the critical stress values obtained from the experiment may be adjusted using the values you determine for the specific gravity:

\[
\sigma = \left( \frac{0.51}{SG_{exp}} \right) \sigma_{exp}
\]

III. EQUIPMENT

III.1 Instron test machine

III.2 Assorted small tools

III.3 Computer program QTEST

III.4 Materials

Six specimens of nominal 0.7 in. x 0.7 in. Douglas Fir of varying lengths.

These specimens used in this experiment are selected at random from the wood shop. No attempt has been made to cut each length member from the same piece of wood. Thus the results may be expected to vary considerably; however, by adjusting the data for specific gravity an attempt will be made to reduce the scatter and explain the results.

The weight of each member, in ounces, will be needed to calculate the specific gravity of each piece. First, the weight per cubic inch of each member is obtained by dividing its weight by its actual volume in cubic inches \((\text{in}^3)\) then, dividing by standard weight of water \((0.5778 \text{ oz/in}^3)\), the specific gravity of each member is obtained.
IV. PROCEDURE

IV.1 Measure and weigh the wood samples.

IV.2 Calculate the theoretical Euler buckling load and stress. Calculate the theoretical maximum crushing load (the maximum crushing stress is given).

IV.3 Turn on the Instron machine. Turn on Computer MONITOR.

IV.4 Pull out the emergency stop button (red button) on the right hand side of the Instron machine. Two green arrows will light.

IV.5 Check that the upper and lower bearing plates are aligned vertically then place the longest column in the bearing plates.

IV.6 Place the wire cage around the machine.

IV.7 Computer Procedure

QTEST Program

1. Double click icon “TestWorks QT 2.02” to launch QTEST.

2. Enter QTEST for username and AM317 for password.

3. Click on TEST and from the drop down window on the right, select “COMPRESSION AM317”.

4. Use the HANDSET MENU to move the crosshead of QTEST machine. The crosshead can be moved at a maximum speed of 0.2 IN/MIN to adjust height for positioning a column member.

CAUTION: SLOW CROSSHEAD MOTION PRIOR TO CONTACT WITH SPECIMEN. Use the green arrow keys (next to RED knob) for micro adjustment.

5. In the window “loadcell10” click on “zero” and in the window “Crosshead Position” click on “zero” AFTER column is positioned in the supports.

6. Click RUN

In window enter the SAMPLE ID. (e.g., “24 inch”)  
Enter CROSS SECTION THICKNESS, and then click SAVE/NEXT  
Enter CROSS SECTION WIDTH, and then click SAVE/NEXT  
Enter LENGTH, and then click OK
NOTE: For the test, the crosshead speed should be set to 0.01 IN/MIN. Check it by click INPUT and select TEST. If it needs to be changed, set INITIAL SPEED to 0.01 IN/MIN.

CAUTION: If for any reason you want to stop the test in progress press <SPACE> bar to abort the experiment.

7. A load deflection plot will be created until the column fails. When test is complete, record the peak load and peak stress. Take a picture of the sample at fracture.

8. To run a new sample, click “NEXT” and repeat steps 4 through 7, and procedures IV.5 and IV.6.

IV.8 At the end of the experiment, push in RED knob, and turn off the Instron machine. “EXIT” the program and turn off the MONITOR.

V. REPORT

V.1 Plot the following curves on the same graph using $\sigma_{cr} = P_{cr}/A$ as ordinate and $L/d$ as abscissa:

- Two experimental curves, using $P_{cr}/A$ and $L/d$ as obtained and measured in the laboratory. Plot the “raw” data and the adjusted data.
- The theoretical curve plotted from the Euler column formula (Eq. 5.4). Set an appropriate maximum scale value for the $y$ axis. This is required because the Euler buckling stress will approach infinity as the length approaches zero.
- The horizontal line indicating the maximum crushing stress.

V.2 Indicate directly on the figure (as in the sample figure in Fig. 2) the range of values of $L/d$ corresponding to short columns, intermediate columns, and long columns.

V.3 Discuss the results and draw appropriate conclusions.

VI. SELECTED REFERENCES


## Table I Raw Data of Column Behavior

<table>
<thead>
<tr>
<th>Specimen number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<tbody>
<tr>
<td>Specimen length (in.)</td>
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<tr>
<td>Effective length constant $K$</td>
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<tr>
<td>Least cross section dimension $d_{\text{min}}$ (in.)</td>
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<td>Greatest cross section dimension $d_{\text{max}}$ (in.)</td>
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<td>Cross section area (in$^2$)</td>
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<tr>
<td>Slenderness ratio $(L/d_{\text{min}})$</td>
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<td>Weight (oz.)</td>
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<tr>
<td>Specific gravity = Density/0.5778</td>
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<tr>
<td>Max. crushing strength $\sigma_y$ (psi)</td>
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<tr>
<td>Max. crushing load (lb)</td>
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<tr>
<td>Calculated Euler $P_{\text{cr}}$ (lb)</td>
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<tr>
<td>Euler calculated stress $P_{\text{cr}}/A$ (psi)</td>
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<tr>
<td>Experimental failure load $P_{\text{exp}}$ (lb)</td>
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<td>Experimental failure stress $P_{\text{exp}}/A$ (psi)</td>
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<tr>
<td>Adjusted experimental stress $P_{\text{adj}}/A$ (psi)</td>
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Appendix A Derivation of Euler’s Equation

In 1757, Leonard Euler (pronounced Oiler) developed a relationship for the critical column load which would produce buckling. A very brief derivation of Euler's equation goes as follows:

A loaded pinned-pinned column is shown in the diagram. A top section of the diagram is shown with the bending moment indicated. In terms of the load $P$, and the lateral deflection $y$, we can write an expression for the bending moment $M$:

$$M = -Py(x) \quad 5.A1$$

Note that the lateral deflection $y$ is a function of $x$. We can also state that for beams and columns, the bending moment is proportional to the curvature of the beam, which, for small deflection can be expressed as:

$$M/EI = d^2y/dx^2 \quad 5.A2$$

where $E$ = Young's modulus and $I$ = moment of Inertia. Substituting Eq. 5.A1 into Eq. 5.A2, we obtain the following differential equation:

$$d^2y/dx^2 = -(P/EI)y$$

or

$$d^2y/dx^2 + (P/EI)y = 0 \quad 5.A3$$

This is a second order differential equation which has the general solution of:

$$y = A\sin\left(\sqrt{\frac{P}{EI}} \cdot x\right) + B\cos\left(\sqrt{\frac{P}{EI}} \cdot x\right) \quad 5.A4$$

We next apply boundary conditions: at $x = 0$, $y = 0$ and at $x = L$, $y = 0$. That is, the deflection of the column must be zero at each end since it is pinned. Applying
the first boundary condition, it is noted that $B$ must be zero since $\cos(0) = 1$. The second boundary condition implies that either $A$ must be zero (which leaves us with no equation at all) or that:

$$\sin\sqrt{\frac{P}{EI}} = 0$$

Noting that $\sin(\pi) = 0$, we can solve for $P$:

$$\sqrt{\frac{P}{EI}} L = \pi$$

so that

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

where $P_{cr}$ stands for the critical load at which the column is predicted to buckle.

By replacing $L$ with the effective length, $KL$, we can generalize the formula to predict the critical load for fixed-pinned, fixed-fixed, and fixed-free columns.

It should be noted that buckling is a complicated phenomena, and the buckling in any individual column may be influenced by misalignment in loading, variations in straightness of the member, presence of initial unknown stresses in the column, and defects in the material.