AM 317

MECHANICS LAB

EXP 1

BEAM DEFLECTIONS

I. OBJECTIVES

I.1 To observe, evaluate and report on the load-deflection relationship of a simply supported beam and a cantilever beam.

I.2 To determine the modulus of elasticity of the beam and what the material the beam is made of using beam deflection theory.

I.3 To verify the principle of superposition and Maxwell's Reciprocity Theorem.

II. INTRODUCTION AND BACKGROUND

The deflections of a beam are engineering concerns as they can create an unstable structure if they are large. People don't want to work in a building in which the floor beams deflect an excessive amount, even though it may be in no danger of failing. Consequently, limits are often placed upon the allowable deflections of a beam, as well as upon the stresses.

When loads are applied to a beam their originally straight axes become curved. Displacements from the initial axes are called bending or flexural deflections. The amount of flexural deflection in a beam is related to the beam's cross-sectional area moment of inertia (I), the single applied concentrated load (P), length of the beam (L), the modulus of elasticity (E), and the position of the applied load on the beam. The amount of deflection due to a single concentrated load P, is given by:

$$\delta = \frac{PL^3}{kEI} \qquad \textbf{1.1}$$

where k is a constant based on the position of the load, and on the end conditions of the beam. For deflection of specific loading conditions refer to Table I.

The bending stress at any location of a beam section is determined by the flexure formula:

$$\sigma = -\frac{My}{I} \qquad \textbf{1.2}$$

where:

- M **- internal moment at the section**
- y **- distance from the neutral axis to the point of interest**
- I **- moment of inertia of the cross-sectional area about the neutral axis**

The largest stress at the same section follows from this relation, Eq. (1.2), by taking y at an extreme fiber at distance c from the neutral axis which leads to:

$$\sigma_{\max} = \frac{Mc}{I} \quad \textbf{1.3}$$

III. APPARATUS

III.1 Simply-supported and Cantilever beams

III.2 Weights & Hangers

III.3 Micrometer

III.4 Ruler or Tape measure

III.5 Dial gauges

IV. PROCEDURE

PART A - SIMPLY SUPPORTED BEAM

IV.1A Measure and record the beam dimensions and calculate the cross-sectional area moment of inertia (*I*) using:

$$I = \frac{bh^3}{12} \quad \textbf{1.4}$$

IV.2A Draw the shear force and bending moment diagrams (V.D. and M.D.) for a concentrated load *P* for each of the mid-span and quarter-span loading cases. Determine the maximum bending stress in terms of *P* for each of the loading cases using Eq. (1.3). Calculate the maximum permissible loads, if the allowable stress of the material the beam is made from is 18,000 psi (Beer, et. al., 2019).

IMPORTANT - Check these calculated maximum permissible loads with instructor before proceeding with the experiment.

IV.3A Load the beam at the mid-span in 5-lb increments, until the maximum load limit is reached. Record the deflection at the point of loading at each incremental load.

IV.4A Repeat the above procedure at the quarter-span. Load the beam at the quarter-span in 5-lb increments, until the maximum load limit is reached. Record the deflection at the point of loading at each incremental load.

PART B - CANTILEVER BEAM

IV.1B **Measure and record the beam dimensions and calculate the area moment of inertia (I) using Eq. (1.4).**

IV.2B **Draw the shear force and bending moment diagrams (V.D. and M.D.) for a concentrated load P for each of the mid-span and free-end loading cases. Determine the maximum bending stress in terms of P for each of the loading cases using Eq. (1.3). Calculate the maximum permissible loads, if the allowable stress of the material the beam is made from is 18,000 psi (Beer, et. al., 2019).**

IMPORTANT - Check these calculated maximum permissible loads with instructor before proceeding with the experiment.

IV.3B **Load the beam at the mid-span in 2-lb increments, until the maximum load limit is reached. Record the deflection at the point of loading at each increment.**

IV.4B **Repeat the above procedure at the free end of the beam. Care must be taken so that the displacement does not exceed the maximum travel of the dial gauge.**

PART C - THE PRINCIPLE OF SUPERPOSITION

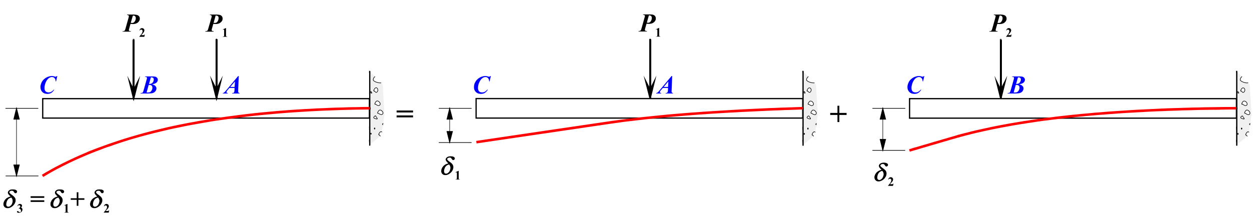


Figure 1 Illustration of the principle of superposition

IV.1C **Choose and record a convenient point on either beam. All deflections will be measured at this point. The example illustrated in Fig. 1 shows the deflections are measured at point C of the cantilever beam.**

IV.2C **Place a single concentrated load P_1 at some point other than the convenient point (record the location of this loaded point) and measure the resulting deflection (δ_1) at the convenient point.**

IV.3C **Remove the first load, and place a second load P_2 at another point on the beam (record the location of this loaded point) and measure the resulting deflection (δ_2) at the convenient point.**

IV.4C **Apply both loads P_1 and P_2 simultaneously, as placed in IV.2C and IV.3C, and measure the resulting deflection (δ_3) at the convenient point. The deflection δ_3 should be close to the sum of the single deflections; i.e.,**

$$\delta_3 = \delta_1 + \delta_2 \qquad \textbf{1.5}$$

PART D - MAXWELL'S RECIPROCITY THEOREM

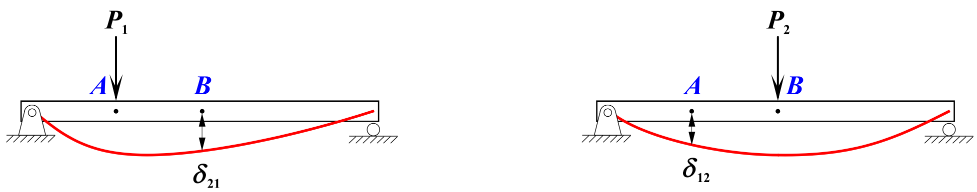


Figure 2 Illustration of the Maxwell's reciprocity theorem

IV.1D **Choose two non-symmetrical points on either beam (record the location of these points, A and B). An example is illustrated in Fig. 2.**

IV.2D **Apply a concentrated load (P_1) at point A and measure the resulting deflection (δ_{21}) at point B.**

IV.3D **Remove the load P_1. Place a different load (P_2) at point B. Measure the resulting deflection (δ_{12}) at point A. The loads and the deflections should satisfy the following relationship:**

$$P_1\delta_{12} = P_2\delta_{21} \qquad \textbf{1.6}$$

V. REPORT

V.1 **Plot the curve of load versus deflection for two loading cases of the simply supported beam. Show loads as ordinates and deflections as abscissas. Include this plot in the Results section with proper legend and axis labels. Include the figure number and caption. Raw data goes in the Appendix.**

V.2 **Plot the curve of load versus deflection for the two loading cases of the cantilever beam. Show loads as ordinates and deflections as abscissas. Include this plot in the Results section with proper legend and axis labels. Draw data goes in the Appendix.**

V.3 **Referring to the configuration in Table I and selecting the appropriate equations, determine the values of modulus of elasticity for each loading**

cases. Create a table of the results you obtained for the modulus of elasticity for the simply supported beam (two load cases) and the cantilever beam (two load cases). Raw data goes in the Appendix.

V.4 Determine what material the beams are made from by comparing the modulus of elasticity you calculated to values referenced in an Engineering Materials or Strength of Materials textbook, or online Open Educational Resources (OER).

V.5 Verify the validity of the Principle of Superposition by performing all necessary calculations. Was δ_3 close to $\delta_1 + \delta_2$? Present the results in the Results section.

V.6 Verify the validity of the Maxwell's Reciprocity Theorem by performing all necessary calculations. Was $P_1\delta_{12} = P_2\delta_{21}$ satisfied? Present these results in the Results section.

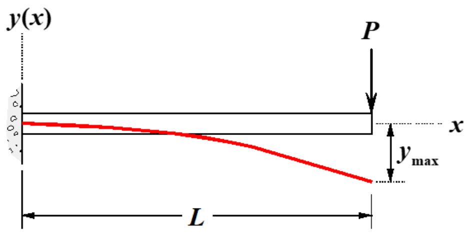
VI. REFERENCES

Hibbeler, R.C., 2022, *Mechanics of Materials*, 11th edition, Pearson, p. 804-806.

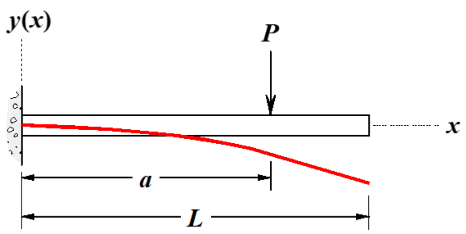
Beer, F., Johnson, E., and DeWolf, J., 2019, *Mechanics of Materials*, 8th edition, McGraw Hill, p. A21-32.

Table I Deflection Equations for Cantilever and Simply-Supported Beams (Hibbeler, 2022)

Cantilever Beam: Deflection y as a function of x



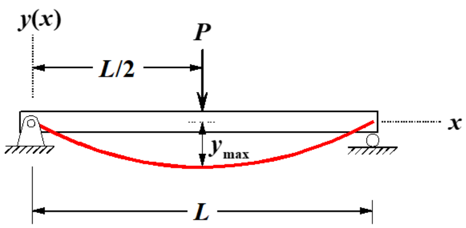
$$x = L \qquad y_{max} = -\frac{PL^3}{3EI}$$



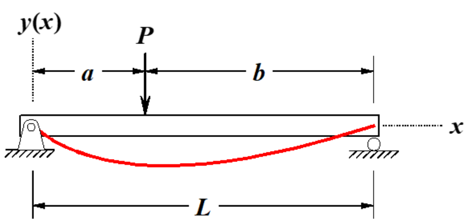
$$0 \le x \le a \qquad y(x) = -\frac{Px^2}{6EI}(3a - x)$$

$$a \le x \le L \qquad y(x) = -\frac{Pa^2}{6EI}(3x - a)$$

Simply-Supported Beam: Deflection y as a function of x



$$x = \frac{L}{2} \qquad y_{max} = -\frac{PL^3}{48EI}$$



$$0 \le x \le a \qquad y(x) = -\frac{Pbx}{6EIL}(L^2 - x^2 - b^2)$$

$$x = a = \frac{L}{4} \qquad y = -\frac{3PL^3}{256EI}$$

Table II Simply Supported Beam Data

Length L (inches)	
Cross-Section Height h (inches)	
Cross-Section Width b (inches)	
Area Moment of Inertia I (in.^{4})	

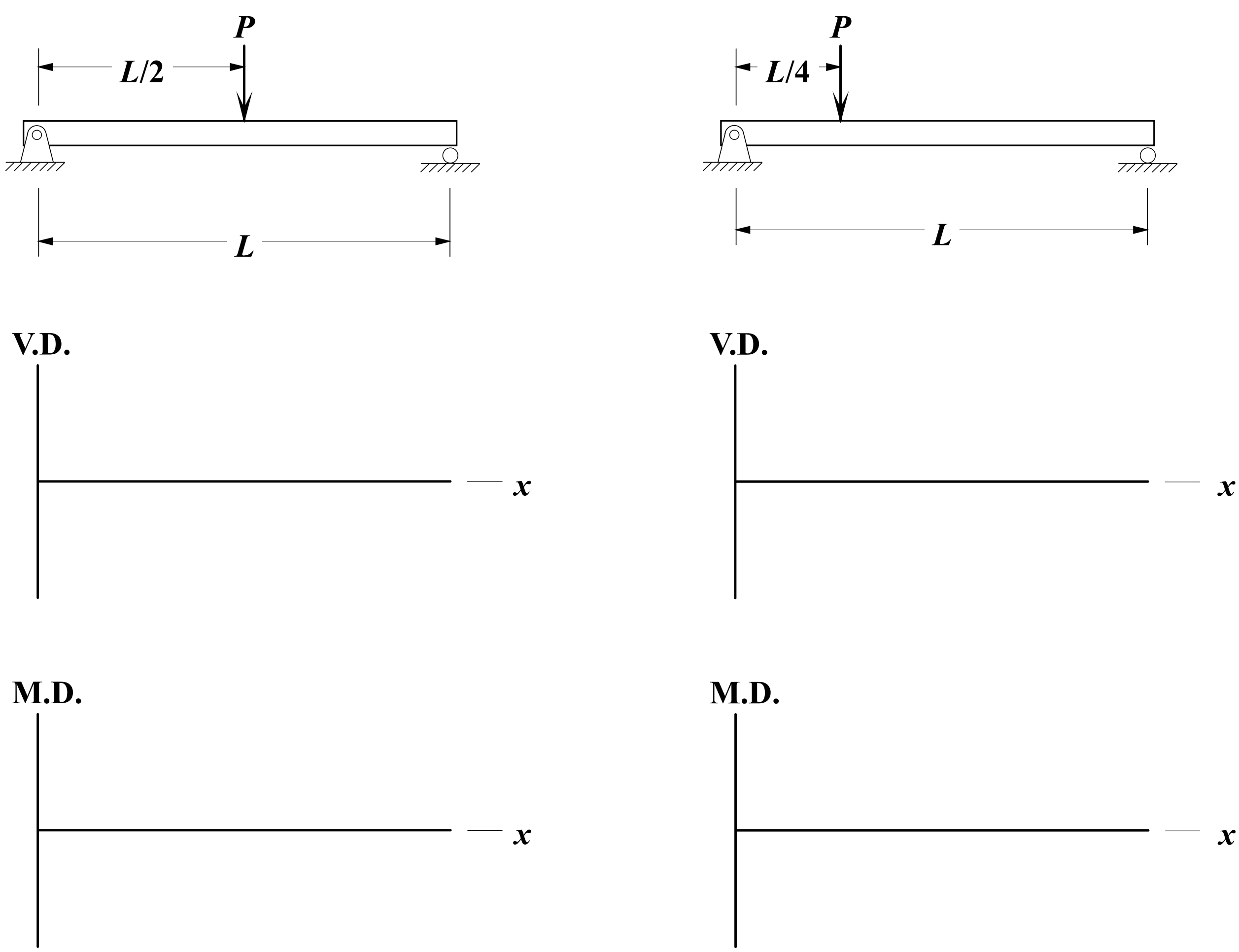


Figure 3 Shear force and moment diagrams for the simply supported beam for each of the loading cases

Max. Permissible Load = ____________ **Max. Permissible Load = ____________**

Table III Cantilever Beam Data

Length L **(inches)**	
Cross-Section Height h **(inches)**	
Cross-Section Width b **(inches)**	
Area Moment of Inertia I **(in.^{4})**	

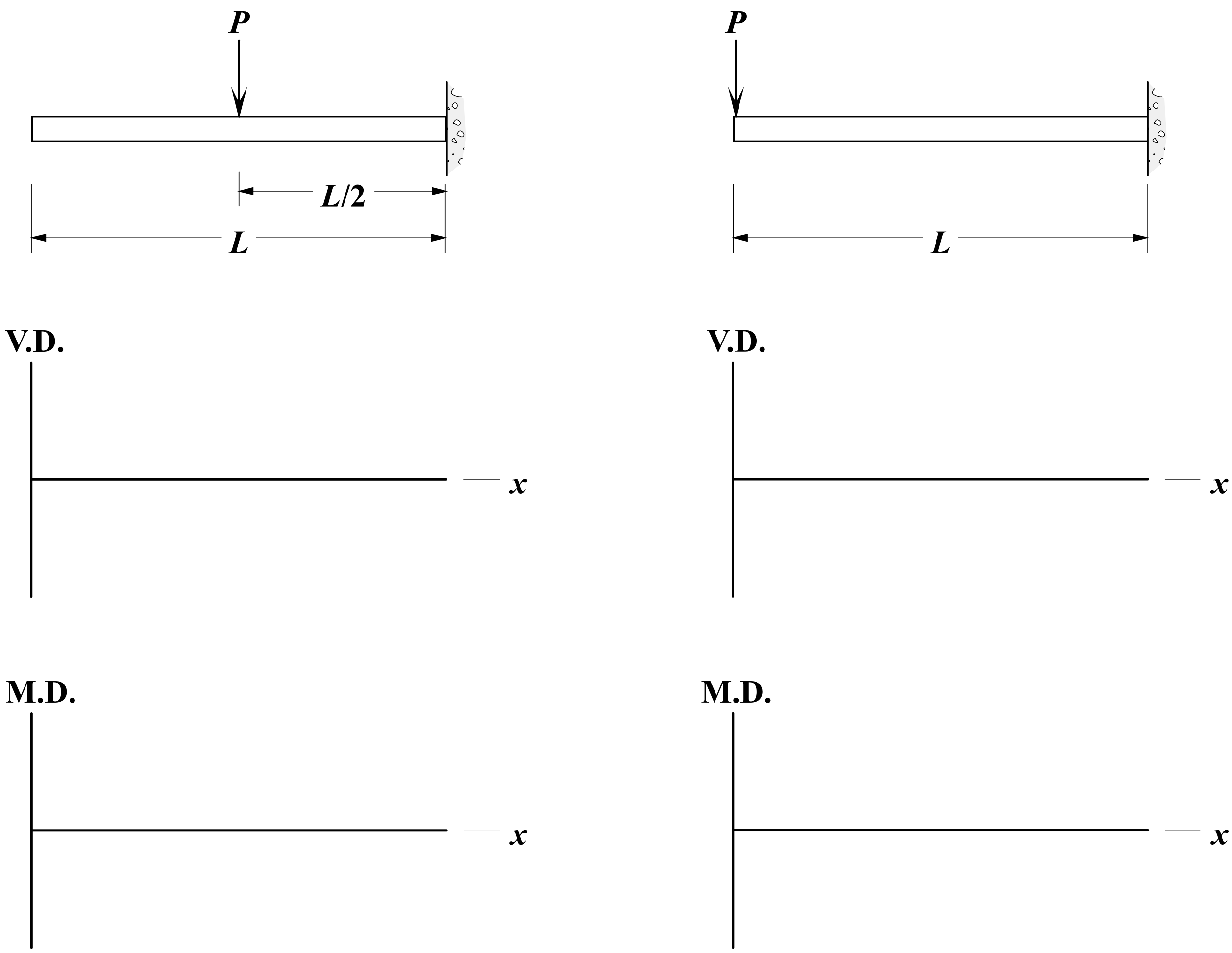


Figure 4 Shear force and moment diagrams for the cantilever beam for each of the loading cases

Max. Permissible Load = ____________   **Max. Permissible Load = ____________**

Table IV Simply Supported Beam Data

Load (lb)	Mid-Span Deflection (in.)	Modulus of Elasticity, E (psi)	¼ Span Deflection (in.)	Modulus of Elasticity, E (psi)
5				
10				
15				
20				
25				
30				
35				
40				
45				
50				
	Average E =		Average E =	

Table V Cantilever Beam Data

Load (lb)	Mid-Span Deflection (in.)	Modulus of Elasticity, E (psi)	Free-End Deflection (in.)	Modulus of Elasticity, E (psi)
2				
4				
6				
8				
10				
12				
14				
16				
18				
20				
22				
24				
26				
28				
	Average E =		Average E =	

Table VI Superposition Data

Beam Used:		
Loading Location	A at (in.)	B at (in.)
Deflection Location	C at (in.)	
Load (lb)	Experimental Deflection (in.)	
P_1 =	δ_1 =	
P_2 =	δ_2 =	
Both P_1 and P_2	δ_3 =	
Theoretical Deflection (in.) $\delta_1 + \delta_2 = \delta_3$ =		
% Error of δ_3 (Ref. to Experimental Value) =		

Table VII Maxwell's Reciprocity Data

Beam Used:		
Loading/Deflection Location	A at (in.)	B at (in.)
	Deflection (in.)	
Load (lb)	Experimental Value	Theoretical Value
P_1 =	δ_{21} =	δ_{21} =
P_2 =	δ_{12} =	δ_{12} =
$P_1\delta_{12}$ =		
$P_2\delta_{21}$ =		