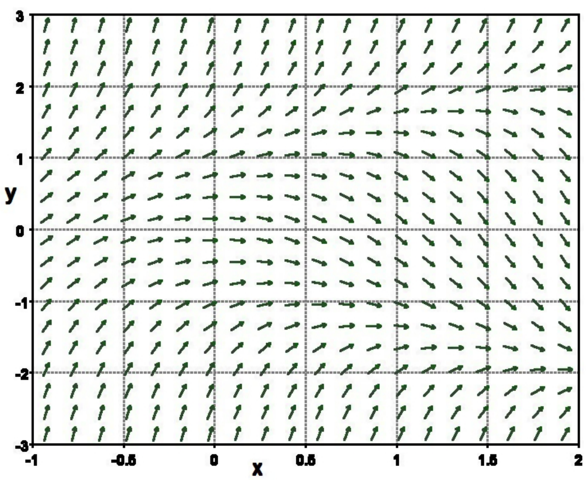
Math 280: Final Study Guide, Fall 2019 Answers

1.2: 3-6, 15-16, 17-24, 25-28, 29-34

1) (a) $x_0 \neq \frac{\pi}{2} + k\pi, y_0 \neq 0$.
   (b) $x \neq \pm 1, y \neq 2$.

2.1: 1-4, 19-20, 21-27, 29-30, 38-40



2) (a)
   (b)
   (c)

3) (a)
   (b)

4) Sketch the solutions to the initial value problems. Do not solve. (Hint: It may help to sketch the general solution first.)
   (a)
   (b)

2.2: 1-11, 15-18, 21-22, 23-26, 29-30, 31-33, 36, 43

5) (a) $y = \tan(\ln(x) - \frac{\pi}{4})$.
   (b) $y + \ln(y) = \frac{1}{2}(\ln(x))^2 + \frac{1}{2}$.

6) (a) $y = \frac{1}{3-x}, \quad (-\infty, 3)$.

(b) $y = \frac{4}{x^2}, \quad (0, \infty)$.

7) (a) $y = \int_1^x \frac{1}{s} e^{2s}\, ds + 3$.

(b) $y = \left(3 \int_2^x e^{s^2}\, ds + 27\right)^{\frac{1}{3}}$

2.3: 1-24, 25-36, 43-48

8) (a) $y = -\frac{1}{2} + Ce^{x^2}$

(b) $y = x^2 + Cx^2e^{-x}$

(c) $y = 1 + \frac{C}{\sqrt{x^2+1}}$

2.2, 2.3: Be able to recognize if an equation is linear or separable.

9) (a) $y = \frac{1}{3-2xe^{-x}-2e^{-x}}$

(b) $y = \sin(x) + 2\cos(x)$

2.5: 1-8, 11-12, 15-19, 21-22

10) (a) $y = x\frac{2\ln(x)-3}{2\ln(x)-1}$

(b) $y = 2x\sqrt{\ln(x)+1}$

3.1: 1-8, 11-12, 13-18, 21-25, 27-28, 29-30, 35-38

11) (a) $T = 325 - 300e^{kt}$

(b) $k = \frac{1}{20}\ln\left(\frac{2}{3}\right)$.

(c)

12) (a) $x(t) = 10 + 10e^{-.03t}$

(b)

13) $x(t) = .1(100 + 2t) + \frac{100}{\sqrt{100+2t}}$

14) (a) $\mathbf{i}(t) = 3e^{-5t} + 10t - 2$.

(b)

15) (a) $k = 98$

(b) $v(t) = 45e^{-10t} + 10$

(c)

4.1: 1-6, 9-10, 15-19, 21, 23-28

16) (a) $W(x) = e^{4x}$. Linearly independent.
(b) $W(x) = -2x^3$. Linearly independent.
(c) $2\sin^2(x) - 1 + \cos(2x) = 0$. Linearly dependent.

17) (a) Need to plug x^3 and $x^3\ln(x)$ into the ODE and get 0. $W(x) = x^5$.
(b) Need to plug e^x and $x^2 - 2x + 2$ into the ODE and get 0. $W(x) = -4e^x + 4xe^x - x^2e^x$.

4.2: 1-4, 6, 9-12, 16

18) (a) $x^2y'' - 7xy' + 15y = 0, \quad y_1 = x^3$. $y = c_1x^3 + c_2x^5$
(b) $y = c_1e^x + c_2(2x-1)e^{-x}$

4.3: 1-11, 15-24, 29-31, 35-36, 49-58

19) (a) $y = c_1 + c_2e^{-2x}\cos(x) + c_3e^{-2x}\sin(x)$.
(b) $y = c_1e^{-x} + c_2xe^{-x} + c_3x^2e^{-x}$
(c) $y = c_1\cos(x) + c_2\sin(x) + c_3x\cos(x) + c_4x\sin(x)$

20) (a) $y'' + 6y' + 10y = 0$.
(b) $y''' - 6y'' + 12y' + 8y = 0$.
(c) $y'''' + 25y'' = 0$.

4.4: 1-14, 19, 21, 27-34; 4.5: 35-50, 55-58

21) (a) $y = -2 + 2e^{2x} - x^2 - x + e^{-x}$.
(b) $y = e^x + 2xe^x + x^2e^x - e^{2x}$.

22) (a) $y = -\frac{\gamma}{\omega(\omega^2-\gamma^2)}\sin(\omega x) + \frac{1}{\omega^2-\gamma^2}\sin(\gamma x)$.
(b) $y = \frac{1}{2\omega^2}\sin(\omega x) - \frac{1}{2\omega}x\cos(\omega x)$.

4.6: 1-6, 9, 11-12, 15, 23-26, 27-28; 4.7: 19-24

23) $y = c_1 e^{-x} + c_2 e^{2x} + \frac{1}{4} e^{3x}$.

24) (a) $y_p = -\frac{1}{2}\sec(x) + \tan(x)\sin(x)$

(b) $y = c_1\cos(x) + c_2\sin(x) - \frac{1}{2}\sec(x) + \tan(x)\sin(x)$

(c) $y = \frac{1}{2}\cos(x) - \sin(x) + \frac{1}{2}\sec(x) + \tan(x)\sin(x)$

25) $y = c_1 x + c_2 x^3 + \frac{1}{8}x^5$.

26) $y = c_1 x + c_2 x\ln(x) + \frac{1}{6}x(\ln(x))^3$

27) $y = \frac{1}{2}e^{-x}\int_0^x \frac{-e^s}{s^4+1}\,ds + \frac{1}{2}e^x\int_0^x \frac{e^{-s}}{s^4+1}\,ds$.

4.4, 4.6: Know when to use undetermined coefficients and when to use variation of parameters.

28) (a) $y = c_1 e^{-x} + c_2 x e^{-x} + \frac{1}{2x}e^{-x}$

(b) $y = c_1 e^{-x}\cos(x) + c_2 e^{-x}\sin(x) + \frac{1}{2}xe^{2x} + \frac{1}{2}e^{2x}$.

4.9: 1-7, 21-22

29) (a) $x = 4e^t - 2te^t + t - 3\,,\ y = -5e^t + 2te^t - 2t + 5$.

(b) $x = c_1 e^t\cos(t) - c_2 e^t\sin(t) - e^{2t}\,,\ y(t) = c_2 e^t\cos(t) + c_1 e^t\sin(t) + e^{2t}$

5.1: 1-6, 8, 21-24, 25-31, 33-37, 41-42, 49-53, 56-57

30) (a) $x = -e^{-t}\cos(2t)$

(b)

(c) Underdamped.

31) (a) $x = 3te^{-3t}$

(b) Critically damped.

(c) $y'(t) = 0 \Rightarrow t = \frac{1}{3}.y\left(\frac{1}{3}\right) = \frac{1}{e}$.

(d)

32) (a) $y = 4e^{-2t} + 5e^{-t} + 3\sin(t) + \cos(t)$.

(b) Transient: $4e^{-2t} + 5e^{-t}$. Steady-state: $3\sin(t) + \cos(t)$.

7.1: 1-16, 19-36, 37-38

33) (a) $F(s) = \frac{1}{(s-2)^2}$

(b) $F(s) = \frac{1}{s-1}\left(e^{-3} - e^{-3s}\right) + \frac{1}{s}e^{-3s}$

(c) $F(s) = \frac{1}{s^2}\left(4s^2 - 2 + (4s+2)e^{-2s}\right).$

7.2: 1-30, 35-44, 45-46

34) (a) $f(t) = t^2 + 2t - 1 + e^{-2t}$.

(b) $f(t) = 2\sin(2t) - e^{-2t} + e^{2t}$

(c) $f(t) = \cos(t) - 3\sin(t) - \cos(3t) + \sin(3t)$.

35) (a) $y = e^{-3t} + e^{3t} - 2$.

(b) $y = -\cos(t) + \sin(t) + e^{2t}$

(c) $y = e^{-2t} + 1 - \cos(t) + 2\sin(t)$

7.3: 1-8, 11-18, 21-30, 37-48, 49-54, 55-57, 59, 61-62, 63-65

36) (a) $y = e^{-t}\cos(2t) + 2e^{-t}\sin(2t) - 2e^{-2t}$.

(b) $y = 2e^{2t} - e^{-t} - 2t + 1$

(c) $y = 2e^{t} - 2e^{-t} - te^{-t}$

(d) $y = -2\cos(2t) - \frac{3}{2}\sin(2t) + 5te^{-t} + 2e^{-t}$

37) Write in terms of unit step functions. Then find the Laplace transform.

(a) $f(t) = 1 + u(t-2) + 2u(t-3)$. $F(s) = \frac{1}{s}(1 + e^{-2s} + 2e^{-3s})$.

(b) $f(t) = u(t-3) - u(t-5)$. $F(s) = \frac{1}{s}(e^{-3s} - e^{-5s})$.

(c) $f(t) = t - tu(t-3) = t - (t-3)u(t-3) - 3u(t-3)$. $F(s) = \frac{1}{s^2}(1 - e^{-3s}) - \frac{3}{s}e^{-3s}$.

7.4: 1-8, 45, 49-50, 51-52

38) (a) $F(s) = \frac{12s^2 - 16}{(s^2+4)^3}$ $\quad F(s) = \frac{s^2 - 2s}{(s^2 - 2s + 2)^2}$

39) $y = e^{-t} - 2\sin(2t) + 4\cos(2t)$ $\quad y = 3\cos(t) + 2\sin(t) + 1$

7.5: 1-12

40) (a) $y = e^{-t}\cos(2t) + 2e^{-t}\sin(2t)$.

(b) $y = 2e^{-t} + e^{t} + (e^{-(t-3)} + e^{t-3})u(t-3)$.

8.1: 1-6, 7-10, 11-16, 17-20, 21-23

41) (a) Check that $\vec{X}_1(t)$ and $\vec{X}_2(t)$ solve the matrix equation. The determinant of the fundamental solution matrix is e^{-2t} which is never 0.

(b) Check that $\vec{X}_1(t)$ and $\vec{X}_2(t)$ solve the matrix equation. The determinant of the fundamental solution matrix is -6 which is never 0.

8.2: 1-6, 13, 21-24, 31, 35-40, 48

42) (a) $\vec{X}'(t) = \begin{bmatrix} 1 & -1 \\ 4 & 5 \end{bmatrix} \vec{X}(t)$

(b) $\vec{X}(t) = c_1 e^{3t} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + c_2 e^{3t} \left(t \begin{bmatrix} 1 \\ -2 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} \right).$

(c) $x(t) = c_1 e^{3t} + c_2 t e^{3t}$, $y(t) = -2c_1 e^{3t} - c_2(2t+1)e^{3t}$.

43) (a) $\vec{X}'(t) = \begin{bmatrix} 8 & 6 \\ -3 & -1 \end{bmatrix} \vec{X}(t), \quad \vec{X}(0) = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

(b) $\vec{X}(t) = c_1 e^{2t} \begin{bmatrix} \vec{1} \\ -1 \end{bmatrix} + c_2 e^{5t} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = -3e^{2t} \begin{bmatrix} \vec{1} \\ -1 \end{bmatrix} + e^{5t} \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

(c) $x(t) = -3e^{2t} + 2e^{5t}$, $y(t) = 3e^{2t} - e^{5t}$.

41) (a) $\vec{X}'(t) = \begin{bmatrix} 1 & -2 \\ 5 & -1 \end{bmatrix} \vec{X}(t)$

(b) $\vec{X}(t) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \cos(3t) + \begin{bmatrix} -1 \\ 1 \end{bmatrix} \sin(3t).$

(c) $x(t) = \cos(3t) - \sin(3t)$, $y(t) = 2\cos(3t) + \sin(3t)$.