MATH 250: FINAL REVIEW, FALL 2019 ANSWERS

13.5) Lines and Planes in Space. 1-10, 11-26, 27-30, 31-37, 43-58, 61-64, 65-68, 71-72, 73-76, 77-80.

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1. (a) $\vec{r}(t) = (1 + 2t, -1 + 3t, 2 - t)$
    (b) $\vec{r}(t) = \langle 2t, 2 - t, 3 - 2t \rangle$.
    (c) $\vec{r}(t) = \langle -1 - 2t, 2 + t, t \rangle$.

2. (a) $5(x - 2) - (y - 3) + 3(z - 1) = 0$ or $5x - y + 3z = 10$.
    (b) $(x - 1) + (y - 2) - 2z = 0$ or $x + y - 2z = 3$.
    (c) $x - y + z = 2$.

13.6) Cylinders and Quadric Surfaces. 1-6, 7-12, 15-20, 21-28, 29-51, 54-58, 60.

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1. (a) Elliptic cylinder parallel to y=axis.
    (b) Parabolic cylinder parallel to x-axis.

2. (a) xy: hyperbola, xz: hyperbola, yz: no solution. Hyperboloid of two sheets.
    (b) xy: hyperbola, xz: hyperbola, yz: ellipse. Hyperboloid of one sheet.
    (c) $xy : y = \pm 3x$. $xz : (0, 0)$, $yz : y = \pm 3z$. Cone.

15.1) Graphs and Level Curves. 25-33, 34, 35, 36-43, 74-77.

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1. (a) plane.
    (b) elliptic paraboloid.
    (c) hyperbolic paraboloid.

2. (a) lines.
    (b) $c = 0, y = \pm x$. $c = -1, 1, 2$, hyperbolas.
    (c) $c = 2, (0, 0)$. $c = -1, 0, 1$ circles.

15.2) Limits and Continuity. 10-12, 13-27, 29-34, 35-50, 52-53, 62-67, 71.

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1. (a) Function is continuous at $(-1, 2)$ so $\lim\limits_{(x,y) \to (-1,2)} f(x, y) = f(-1, 2) = 0$.

    (b) $\lim\limits_{(x,y) \to (3,-1)} \frac{(x + 3y)(x - 3y)}{y(x + 3y)} = \lim\limits_{(x,y) \to (3,-1)} \frac{x - 3y}{y} = -6$.

2. (a) $\displaystyle\lim_{(t,0)\to(0,0)} f(x,y) = \frac{1}{2}$, $\displaystyle\lim_{(0,t)\to(0,0)} f(x,y) = 2$.

(b) $\displaystyle\lim_{(t,0)\to(0,0)} f(x,y) = 0$, $\displaystyle\lim_{(t,t)\to(0,0)} f(x,y) = \frac{1}{2}$.

15.3) Partial Derivatives. 1-9, 11-14, 15-30, 32-34, 38-46, 48-53, 54-59. |

1. (a) $\displaystyle\lim_{h\to0} \frac{(3(x+h)-y)-(3x-y)}{h} = 3$, $\displaystyle\lim_{h\to0} \frac{(3x-(y+h))-(3x-y)}{h} = -1$.

(b) $\displaystyle\lim_{h\to0} \frac{(x+h)y^2-xy^2}{h} = y^2$, $\displaystyle\lim_{h\to0} \frac{x(y+h)^2-xy^2}{h} = 2xy$.

2. (a) $f_{xx} = -y^2\sin(xy)$, $f_{xy} = \cos(xy) - xy\sin(xy)$, $f_{yx} = \cos(xy) - xy\sin(xy)$, $f_{yy} = -x^2\sin(xy)$.

(b) $f_{xx} = \frac{2(y^3-x^2)}{(x^2+y^3)^2}$, $f_{xy} = f_{yx} = \frac{-6xy^2}{(x^2+y^3)^2}$, $f_{yy} = \frac{6x^2y-3y^4}{(x^2+y^3)^2}$

15.4) The Chain Rule. 9-18, 19-26, 27-28, 29-30, 35-40, 57-59, 65, 67-69, 72-73, 75. |

1. $\frac{dw}{dt} = 3x^2y^2(2t) + 2x^3y(1 + e^{2-t}) = 3(25)(1)(4) + 2(5^3)(1)(2) = 800$.

2. $\frac{\partial w}{\partial u} = \frac{1}{x}(2) - \frac{1}{y}\frac{v^2}{u^2} = 2 - \frac{1}{2} = \frac{3}{2}$.
   $\frac{\partial w}{\partial v} = \frac{1}{x}(3) + \frac{1}{y}\frac{2v}{u} = 3 - 2 = 1$.

3. (a) $\frac{\partial F}{\partial x}(0) + \frac{\partial F}{\partial y}(\frac{\partial y}{\partial z}) + \frac{\partial F}{\partial z}(1) = 0 \Rightarrow \frac{\partial y}{\partial z} = -\frac{\partial F}{\partial z}/\frac{\partial F}{\partial y}$.

(b) $\frac{\partial y}{\partial z} = \frac{-xye^{xz}-xye^{yz}}{e^{xz}+xze^{yz}}$

4. $\frac{\partial z}{\partial x} = \frac{-2xy-z^3}{y^2+3xz^2}$, $\frac{\partial z}{\partial y} = \frac{-2yz}{y^2+3xz^2}$.

5. $\frac{dE}{dt} = \frac{1}{2}2v\frac{dv}{dt} + g\sin(\theta)\frac{d\theta}{dt} = -g\sin(\theta)v + g\sin(\theta)v = 0$.

15.5) Directional Derivatives and Gradient. 1-10, 11-12. 13-20, 21-30, 31-36, 43-44, 47-50, 59-64, 69-72, 74, 75-78, 81, 82, 85.

1. $(\nabla f)(-1,2,1) = (-4,1,3)$. $(D_{\vec{u}}f)(-1,2,1) = \frac{8}{3} + \frac{2}{3} + 1$.

2. (a) $\sqrt{145}, \langle\frac{8}{\sqrt{145}}, \frac{-9}{\sqrt{145}}\rangle$.

(b) $\langle 9,8\rangle$.

3. $f(x,y) = xy$.

(a) Graph the level set of f through the point $(2,1)$.

(b) Include the vector $\nabla f(2,1)$ on your graph.

(c) $(x-2) + 2(y-1) = 0 \Rightarrow y = -\frac{1}{2}x + 2$.

(d) The vector $\nabla f(2,1)$ is perpendicular to the level set of f at $(2,1)$.

1. Find the tangent plane to the surface at the given point.

   (a) $-4(x+1) - 1(x-2) + 4(x-2) = 0$.

   (b) $1(x-3) + 6(y-0) + 0(z-2) = 0$ or $x + 6y = 3$.

   (c) $\frac{1}{2}(x-3) - 2(y-1) - \frac{1}{2}(z+1) = 0$.

2. Find the tangent plane to the surface at the given point.

   (a) $\frac{5}{3}(x-5) - \frac{4}{3}(x-4) - (z-3) = 0$.

   (b) $0(x-2) + 2(y-0) + 1(z-3) = 0$ or $2y - 3z = 3$.

   (c) $3(x-2) - (y-1) + 4(z+1) = 0$.

1. Find any critical points and classify each as relative maximum, relative minimum, or saddle.

   (a) Local max at $(0,0)$.

   (b) local min at $(0,1)$

   (c) Saddle at $(1,2)$.

   (d) Local min $(\pm 1, 0)$. Local max $(0,0)$.

2. min $x^2 + y^2 + z^2 = 2$ at $(x,y) = (1,0)$.

3. max $V = 2$ at $(x,y) = (1,1)$.

1. max $= 3\sqrt{3}$, min $= -3\sqrt{3}$.

2. Area $= 32$.

3. $D = \sqrt{30}$.

1. $\frac{1}{4} \int_0^2 \int_1^3 2y - x \, dy dx = 3$.

2. (a) $\int_0^{\ln(3)} \int_0^1 xy e^{xy^2} dy dx = 1 - \frac{1}{2} \ln(3)$.

(b) $\int_0^3 \int_0^1 \frac{y}{\sqrt{1+xy}} dxdy = \frac{10}{3}.$

1. $\int_1^2 \int_{y^2+3}^{3y+1} 2y - 1 dxdy = \frac{1}{3}.$

2. (a) $\int_0^3 \int_0^{2y} e^{y^2} dxdy = e^9 - 1.$

   (b) $\int_{-2}^2 \int_{x^2}^4 e^{12x-x^3} dydx = \frac{1}{3}(e^{16} - e^{-16})$

1. Evaluate the integral by converting to polar.

   (a) $\int_0^4 \int_0^{\frac{\pi}{2}} (r\cos(\theta))(r\sin(\theta))r \, drd\theta = 32.$

   (b) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{(1+r^2)^2} r \, drd\theta = \frac{\pi}{10}.$

   (c) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \int_{\sec(\theta)}^{2\sec(\theta)} r \, drd\theta = 3.$

1. $\int_0^2 \int_0^{1-\frac{1}{4}x^2} \int_0^{4-4y-x^2} dzdydx$
   $\int_0^1 \int_0^{\sqrt{4-4y}} \int_0^{4-4y-x^2} dzdxdy$
   $\int_0^2 \int_0^{4-x^2} \int_0^{1-\frac{1}{4}z-\frac{1}{4}x^2} dydzdx$
   $\int_0^4 \int_0^{\sqrt{4-z}} \int_0^{1-\frac{1}{4}z-\frac{1}{4}x^2} dydxdz$
   $\int_0^4 \int_0^{1-\frac{1}{4}z} \int_{\sqrt{4-4y-z}} dxdydz$
   $\int_0^1 \int_0^{4-4y} \int_{\sqrt{4-4y-z}} dxdzdy$

2. (a) $\int_0^2 \int_{-\sqrt{4-2z}}^{\sqrt{4-2z}} \int_{x^2}^{4-2z} dydxdz = \frac{256}{30}.$

   (b) $\int_0^4 \int_{-\sqrt{y}}^{\sqrt{y}} \int_0^{2-\frac{1}{2}y} dzdxdy = \frac{256}{30}.$

1. $\int_0^{2\pi} \int_1^2 \int_0^{4-r^2} \frac{z}{r^{\frac{3}{2}}} rdzdrd\theta = 4.5\pi.$

2. $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^2 \rho^4 \cos(\theta) \sin^3(\phi) \cos(\phi) d\rho d\phi d\theta = \frac{32}{20}.$

3. (a) $\int_0^{2\pi} \int_0^3 \int_r^3 r \, dz dr d\theta = 9\pi$.

   (b) $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{3\sec(\phi)} \rho^2 \sin(\phi) d\rho d\phi d\theta = 9\pi$.

4. (a) $\int_0^{2\pi} \int_3^5 \int_0^{\sqrt{25-r^2}} r \, dz dr d\theta = \frac{128\pi}{3}$.

   (b) $\int_0^{2\pi} \int_{\cot^{-1}(4/3)}^{\frac{\pi}{2}} \int_{3\csc(\phi)}^5 \rho^2 \sin(\phi) \, d\rho d\phi d\theta = \frac{128\pi}{3}$.

16.7) Change of Variables. 5-11, 13-16, 17-22, 23-26, 27-30, 31-36, 37-39, 41-44, 46-47, 48, 50-52, 53, 56.

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1. $\int_1^2 \int_1^3 \frac{1}{2v} \, dv du = \frac{1}{2} \ln(3)$.

2. $\int_0^2 \int_1^3 (v - 2u)(\frac{1}{2}) \, du dv = -6$.

3. $\int_0^{\pi} \int_0^1 10 r^2 \sin(\theta) \, dr d\theta = \frac{20}{3}$.

17.1) Vector Fields. 2, 8-15, 18, 24, 25-30, 35-42, 43-45, 47-48, 49-52. |

1.

2.

17.2) Line Integrals. 4-10, 12-16, 17-34, 35-36, 39-40, 41-46, 47-48, 49-56, 57-60, 62, 64-65, 68, 70-72, 73.

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1. Evaluate the line integrals.

   (a) $\int_0^2 t^3(0) + t(2t) \, dt = \frac{16}{3}$.

   (b) $\int_0^1 (2t)(-1 + 2t)(3) \, dt = 1$.

   (c) $\int_{-1}^2 (t^4 - t)(4t^3) + 2t \, dt = 104.1$.

17.3) Conservative Vector Fields. 7, 9-16, 17-30, 31-34, 39-42, 44, 45-50, 51-52, 54-56, 59-62, 63-64. |

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1. (a) $\phi(x, y) = \sqrt{x^2 + y^2}$.

   (b) $\phi(3, 4) - \phi(1, 0) = 4$.

2. $\vec{F} = \langle 1 + \cos(y), 2y - x\sin(y) \rangle$.

   (a) $\frac{\partial g}{\partial x} = \frac{\partial f}{\partial y} = -\sin(x)$.

   (b) $\phi(x, y) = x + x\cos(y) + y^2$.

   (c) $\phi(2, \pi) - \phi(1, 0) = \pi^2 - 2$.

3. $\nabla \times \vec{F} = \vec{0}, \phi(x, y, z) = x^2 + xy + 2y + z^3$.

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1. $\int_0^2 \int_0^1 2x - 2x\, dy dx = 0$.

2. (a) $\int_{-1}^1 t - t^2 + (t + t^2)(2t)\, dt + \int_0^1 (1 - 2t - 1)(-2)\, dt = \frac{2}{3} + 2 = \frac{8}{3}$.

   (b) $\int_{-1}^1 \int_{x^2}^1 2\, dy dx = \frac{8}{3}$.

3. $\int_0^1 \int_0^{2x} 4\, dy dx = 4$.

4. $\int_0^{2\pi} 16\cos^4(t) + 16\sin^4(t)\, dt = 24\pi, \int_0^{2\pi} \int_0^2 3r^3\, dr d\theta = 24\pi. (The first integral is a little too hard/tedious t$

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1. $\vec{F}$ should be $\langle x^3, x^2 y, yz^3 \rangle$. $\nabla \cdot \vec{F} = 4x^2 + 3yz^2, \nabla \times \vec{F} = \langle z^3, 0, 2xy \rangle$.

2. $\nabla \times \langle \frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z} \rangle = \vec{0}$.

3. $\nabla \times \vec{F} = \langle -xy, -ze^{xz}, yz - 2y \rangle \neq \vec{0}$.

> *: For 29-34, 35-38, 43-48, you **must** parametrize the surface by
> $r(u, v)$ and use the parametric form to do the integrals, rather
> than the 'explicit' form indicated in the book here.

1. $\int_0^{2\pi} \int_0^1 \sqrt{1 + r^2}\, r\, dr d\theta = \frac{\pi}{6}(5^{\frac{3}{2}} - 1)$

2. $\int_0^{2\pi} \int_0^2 (-\cos(\theta) + 2)r\, dr d\theta = 8\pi$

3. $\int_0^3 \int_{u^2}^{3u} \sqrt{6}\, dv du = 4.5\sqrt{6}$

4. $d\vec{S} = \langle -f(v)f'(v), f(v)\cos(u), f(v)\sin(u) \rangle\, du dv, dS = f(v)\sqrt{1 + (f'(v))^2}\, du dv$

5. (a) $\int_0^\pi \int_0^1 \int u\cos(v)\sin(v) - u\cos(v)\sin(v) + uv\, du dv = \frac{\pi^2}{4}$.

   (b) $\int_0^\pi \int_0^1 u\sin(v)\sqrt{1 + u^2}\, du dv = \frac{2}{3}(2^{\frac{3}{2}} - 1)$.

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1. $\int_0^{2\pi} 4\sin^2(t) + 4\cos^2(t)\,dt = 8\pi$, $\int_0^{2\pi}\int_0^2 2\,r dr d\theta = 8\pi$.

2. $\int_0^{2\pi} 9\sin^2(t) + 9\cos^2(t)\,dt = 18\pi$.

3. $\int_0^1\int_0^{1-u}\int 2\,dv du = 1$

17.8) Divergence Theorem. 9-12, 13-16, 17-24, 25-27, 30.

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1. $\int_0^{2\pi}\int_0^2\int_{r^2}^2 r\,dz dr d\theta = \frac{128\pi}{3}$. $- \int_0^{2\pi}\int_0^2 r^5 dr d\theta + \int_0^{2\pi}\int_0^2 16r dr d\theta = \frac{128\pi}{3}$.

2. Use the divergence theorem to evaluate $\iint_S \vec{F}\cdot d\vec{S}$. The surfaces have outward pointing normal.

   (a) $\int_0^2\int_0^{3-1.5z}\int_0^{6-2y-3z} 3dxdydz = 18$.

   (b) $\int_0^{2\pi}\int_0^{\pi}\int_0^3 3\rho^5 \sin^3(\phi)\cos(\phi)\,d\rho d\phi d\theta = 0$.