2016-2017 Annual Program Assessment Report

Please submit report to your department chair or program coordinator, the Associate Dean of your College, and to james.solomon@csun.edu, Director of the Office of Academic Assessment and Program Review, by September 30, 2017. You may, but are not required to, submit a separate report for each program, including graduate degree programs, which conducted assessment activities, or you may combine programs in a single report. Please identify your department/program in the file name for your report.

College: Science and Mathematics

Department: Mathematics

Program: B.A. and B.S. and M.S., in addition to assessment of General Education Basic Skills SLOs in Mathematics.

Assessment liaison: Vladislav Panferov

1. **Please check off whichever is applicable:**
   A. ____X____ Measured student work within program major/options.
   B. ____X____ Analyzed results of measurement within program major/options.
   C. _________ Applied results of analysis to program review/curriculum/review/revision major/options.
   D. ____X____ Focused exclusively on the direct assessment measurement of General Education Basic Skills outcomes

2. **Overview of Annual Assessment Project(s).** On a separate sheet, provide a brief overview of this year's assessment activities, including:
   - an explanation for why your department chose the assessment activities (measurement, analysis, application, or GE assessment) that it enacted
   - if your department implemented assessment **option A**, identify which program SLOs were assessed (please identify the SLOs in full), in which classes and/or contexts, what assessment instruments were used and the methodology employed, the resulting scores, and the relation between this year's measure of student work and that of past years: (include as an appendix any and all relevant materials that you wish to include)
   - if your department implemented assessment **option B**, identify what conclusions were drawn from the analysis of measured results, what changes to the program were planned in response, and the relation between this year's analyses and past and future assessment activities
   - if your department implemented **option C**, identify the program modifications that were adopted, and the relation between program modifications and past and future assessment activities
   - if your program implemented **option D**, exclusively or simultaneously with **options A, B, and/or C**, identify the basic skill(s) assessed and the precise learning outcomes assessed, the assessment instruments and methodology employed, and the resulting scores
   - in what way(s) your assessment activities may reflect the university's commitment to diversity in all its dimensions but especially with respect to underrepresented groups
   - any other assessment-related information you wish to include, including SLO revision (especially to ensure continuing alignment between program course offerings and both program and university student learning outcomes), and/or the creation and modification of new assessment instruments

3. **Preview of planned assessment activities for 2017-18.** Include a brief description as reflective of a continuous program of ongoing assessment.

Overview of Assessment Projects
2016–2017 Academic Year

Prepared by the Assessment Committee
in Collaboration with the Graduate Committee
Department of Mathematics
California State, University Northridge

29 September 2017

Contents

Preface

This report describes all the assessment activities performed by the Department of Mathematics at California State University, Northridge for the 2016–2017 academic year. It is subdivided into three parts: Part I discusses assessment of General Education Basic Skills Student Learning Outcomes in Mathematics, Part II discusses assessment of Student Learning Outcomes for the undergraduate programs in mathematics, and Part III discusses assessment of Student Learning Outcomes for the graduate programs in mathematics.

Part I

General Education Assessment

Chapter 1

Introduction

We conducted an assessment of all four of the General Education Basic Skills SLOs in Mathematics:

- **SLO #1 Represent, understand and explain mathematical information symbolically, graphically, numerically and verbally.**
- **SLO #2 Develop mathematical models of real-world situations and explain the assumptions and limitations of those models.**
- **SLO #3 Use models to make predictions, draw conclusions, check whether the results are reasonable, and find optimal results using technology when necessary and appropriate.**
- **SLO #4 Demonstrate an understanding of the nature of mathematical reasoning, including the ability to prove simple results and/or make statistical inferences.**

1. In Fall of 2016 we measured student work in the Math 102 course (College Algebra) to evaluate General Education Basic Skills SLOs 1 and 3 in Mathematics.
2. In Fall of 2016 we measured student work in the Math 105 course (Precalculus) to evaluate General Education Basic Skills SLOs 1 and 4 in Mathematics.
3. In Fall of 2016 we measured student work in the Math 255A course (Calculus for the Life Sciences I) to evaluate all four of the General Education Basic Skills SLOs in Mathematics.
4. In Spring 2017 we measured student work in selected sections of the Math 140 course (Introductory Statistics) to evaluate all four of the General Education Basic Skills SLOs in Mathematics.
5. We analyzed all of our measurements for the academic year 2016–2017 and made recommendations to the department.

The Math 102 (College Algebra, 2 units) course provides a grounding in the algebraic concepts necessary to continue on to trigonometry and analytic geometry (Math 104, 3 units), and thence to the beginning of the Calculus sequences (Math 150A or Math 255A). Students with more extensive preparation in algebra may begin with the Math 105 course (Precalculus, 5 units), which, when completed, also brings one to the beginning of the Calculus sequences. These courses provide good places to assess the General Education Basic Skills SLOs in Mathematics, especially SLO 1.

The Math 255A course (Calculus for the Life Sciences I) provides a great opportunity to assess all four General Education Basic Skills SLOs in Mathematics at the same time. This gives us a panoramic view of how we are doing on these four SLOs within a single population. As biology continues its rapid development, mathematical models of increasing sophistication are playing ever greater roles in our understanding of the highly complex systems we encounter there. The Math 255A course equips students of biology with some of the basic tools that are used in these models.

The Math 140 course (Introductory Statistics) allows us to examine a different route that students may take in Mathematics General Education. Students do not need to take College Algebra (Math 102) to enroll in Math 140. They do need a level of competency in algebra equivalent to the completion of Math 093 (Developmental Mathematics II), which should normally occur in their high school curriculum. Many of the students who take Math 140 are in majors that do not require calculus. We focused especially in the sections of Math 140 that serve students in the College of Business and Economics. Some of these sections have instruction in a traditional classroom, one section provides the instruction online, and some sections offer a hybrid of the two approaches. This provides a good opportunity to assess how these different modalities of instruction correlate to student achievement of the SLOs. We examined all four of the General Education Basic Skills SLOs in Mathematics in the classes.

The four classes assessed cover a wide cross-section of CSUN students with very different career goals. The Mathematics Department maintains a commitment to equip each student in this very diverse group with a mathematical education that supports the student's major area of studies.

Each of the measurements summarized in the above synopsis is now examined in its own section, which includes analysis of the findings.

Chapter 2

Fall 2016: Measurement of Math 102 Class and Analysis

2.1 SLOs Assessed

The following two General Education Basic Skills SLOs in Mathematics were assessed:

- **SLO #1 Represent, understand and explain mathematical information symbolically, graphically, numerically and verbally.**
- **SLO #3 Use models to make predictions, draw conclusions, check whether the results are reasonable, and find optimal results using technology when necessary and appropriate.**

2.2 Course Description

Math 102 (College Algebra) is a 3 unit algebra course available for General Education credit in Basic Skills: Mathematics. Its catalog description read thus:

> *A preparation for the algebra necessary for calculus. This course is intended for computer science, engineering, mathematics, and natural science majors. It builds on students familiarity with linear, quadratic, and rational expressions to achieve fluent proficiency in analyzing the local and global behavior of functions involving such expressions.*

The textbook used in the Fall 2016 was a compilation of both course notes and a workbook prepared by CSUN faculty. Math 102 provides a grounding in the algebraic concepts necessary to continue on to trigonometry and analytic geometry (Math 104, 3 units), and thence to the beginning of the Calculus sequences (Math 150A or Math 255A).

2.3 Signature Assignment

One problem included in the Fall 2016 Math 102 Final Exam, of which there were the following two versions:

Version 1:

A rocket launched vertically upward from the surface of Mars with an initial speed of 84 meters per second has a height of $h(t) = 84t - 6t^2$ meters after t seconds. (a) How long does it take for the rocket to reach the maximum height? (b) What is the maximum height reached by the rocket? (c) Write a sentence to explain what $h(2) = 144$ means.

Version 2:

A rocket launched vertically upward from the surface of Jupiter with an initial speed of 280 meters per second has a height of $h(t) = 280t - 40t^2$ meters after t seconds. (a) How long does it take for the rocket to reach the maximum height? (b) What is the maximum height reached by the rocket? (c) Write a sentence to explain what $h(1) = 240$ means.

2.4 Assessment Procedure

The following rubric was by devised by two full time faculty for scoring, which identifies particular aspects of the SLOs and relates them to the specific parts of the question.

	0	1	2	3
SLO 1 (numerical): Demonstrate the ability to perform numerical computations to obtain solution of the problem (parts (a) and (b))	No relevant work shown	Major flaws	Minor, noticeable flaws	Computation performed correctly, with only negligible flaws if at all
SLO 1 (verbal): Represent mathematical information verbally (part (c))	No attempt is made or interpretation is clearly incorrect	Major flaws (units of velocity are used instead of units of length)	Mostly correct, with noticeable flaws (incorrect or missing units)	Clear and complete, with only negligible flaws if at all
SLO 3: Conceptualize the model and form a mathematically sound solution strategy (parts (a), (b) and (c))	No relevant work shown	Major flaws	Minor, noticeable flaws	Essentially correct interpretation of the model is demonstrated

2.5 Results, Analysis, and Recommendations

Student responses were collected for a sample of 120 students using simple random sampling from a MATH 102 class list of about 1200 students. 16 of the sampled students (13.3%) did not show up for the final. 10 of these students were from 2 large (110+) sections of MATH 102. Even considering the random nature of the sampling process this figure is significant for interpretation of the data presented below (the scores do not show students who did not show up for the final exam).

Student responses were scored according to the rubric outlined above; scoring was done independently by two members of the Assessment Committee, and is summarized in the bar graphs that follow.

The results for the averages were consistent within a few percentage points. The average overall score was 1.81 (59.9%) for SLO 1 (numerical), 2.28 (75.7%) for SLO 1 (verbal), and 1.95 (64.6%) for SLO 3.

The problem chosen for assessment included elements of modeling, numerical and symbolic computation, and verbal representation of mathematical information. A typical solution for parts (a) and (b) could proceed as follows: realize that the equation $h(t) = 84t - 6t^2$ represents a quadratic function, and that the point of maximum height corresponds to the vertex of a parabola drawn in the (t, h) plane. Since the location of the vertex corresponds to the mid-point between the two roots, it is helpful to find the roots, for instance

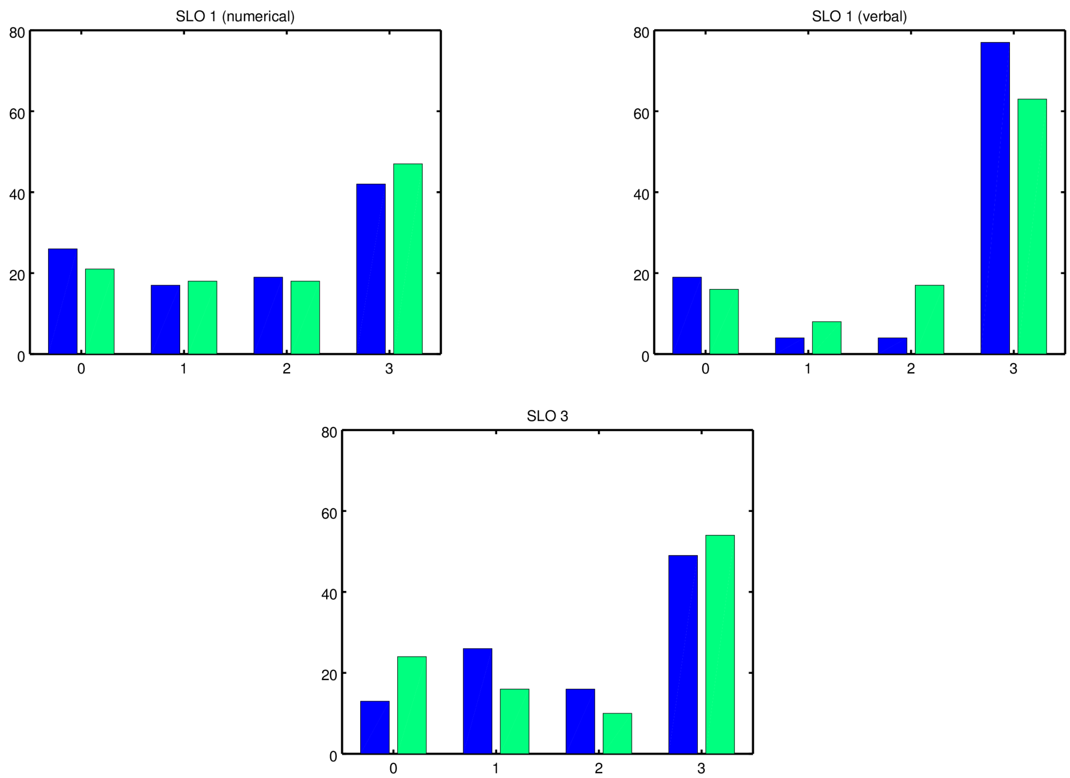


Figure 2.1: Score distribution for assessment of General Education Basic Skills Math SLOs #1 (numerical and verbal) and SLO #3 in Math 102, Fall 2016; data from two independent scorers plotted side-by-side.

by factoring

$$h(t) = 280t - 40t^2 = 40t\,(7 - t).$$

Then the roots are $t = 0$ and $t = 7$, and thus the location of the vertex is $t_* = \frac{7}{2}$. Further, using the factored form,

$$h\left(\frac{7}{2}\right) = 40\left(\frac{7}{2}\right)\left(\frac{7}{2}\right) = 10 \cdot 7 \cdot 7 = 10 \cdot 49 = 490.$$

Thus, the maximal height is 490 meters, which is reached at $t_* = \frac{7}{2} = 3.5$ seconds.

Instead, a typical student solution proceeded as follows:

Use the formula $t_* = -\frac{b}{2a}$ for the location of the vertex of the parabola (the modeling step is therefore bypassed, since the shortcut solution is more efficient). After computing

$$t_* = -\frac{b}{2a} = -\frac{84}{2 \cdot (-6)} = \frac{7}{2}$$

we obtain the maximal height

$$h\left(\frac{7}{2}\right) = 280\left(\frac{7}{2}\right) - 40\left(\frac{7}{2}\right)^2 = \frac{1960}{2} - \frac{1960}{4} = 980 - 490 = 490.$$

The details of the numerical computation may be performed differently, and frequently would take half a page of multi-digit multiplications and subtractions, since calculators are not allowed on the exam.

In this form, most of the effort is spent on arithmetic calculations (which are not part of the material taught in the course), and this is where most of the numerical mistakes done by the students occur.

In view of these comments, our assessment of SLO 3 in particular should be seen as rather crude, since not every student who provided a correct solution to the problem and received a score '3' may have fully mastered the modeling aspect of the problem. The same comment goes to our results on SLO 1 (numerical) since even if the arithmetic calculations were performed correctly, the apparent inefficiency of the method used raises questions of whether the student have achieved the appropriate level of mastery of the numerical aspects that would allow them to succeed in Precalculus II and Calculus classes.

We note that in Version 1 of the problem the time of highest elevation was integer $t_* = 7$, which made it an easier problem for the students, since the arithmetic calculations proceed a little more easily, and correct answers can be obtained by just "trial and error" method of computing $h(1)$, $h(2)$, $h(3)$, $h(4)$, etc., that was used by some students. It is recommended that in future versions of the exams non-integer values of t_* are used for all versions.

We recommend that, form now on, the issue of efficiency of performing numerical calculations be specifically addressed in the instruction of the course.

Chapter 3

Fall 2016: Measurement of Math 105 Class and Analysis

3.1 SLOs Assessed

The following two General Education Basic Skills SLOs in Mathematics were assessed:

- **SLO #1 Represent, understand and explain mathematical information symbolically, graphically, numerically and verbally.**
- **SLO #4 Demonstrate an understanding of the nature of mathematical reasoning, including the ability to prove simple results and/or make statistical inferences.**

3.2 Course Description

Math 105 is a 5 unit Precalculus course available for General Education credit in Basic Skills: Mathematics. Its catalog description reads thus:

> *A preparation for the trigonometric, exponential, and logarithmic functions used in calculus. This course is intended for computer science, engineering, mathematics, and natural science majors. This course builds on students familiarity with exponential, logarithmic, and trigonometric expressions to achieve proficiency in analyzing the local and global behavior of functions involving such expressions.*

Course contents have changed from the Fall 2016 to Spring 2017. The Textbook used in the Fall 2016 was *Precalculus: Graphical, Numerical, Algebraic* (Eighth Edition) by Demana, Waits, Foley, and Kennedy (Addison-Wesley, 2011). Math 105 provides a grounding in algebra, trigonometry, and analytic geometry necessary to continue on to the beginning of the Calculus sequences (Math 150A or Math 255A).

3.3 Signature Assignment

One problem included in the Fall 2016 Math 105 Final Exam.

Problem. Let $f(x) = -2x^2 + 12x - 7$.

(a) Express the function in the form $f(x) = a(x-h)^2 + k$ **by completing the square**.

(b) What is the vertex of $f(x)$?

(c) Is the vertex a maximum or a minimum value of $f(x)$?

(d) Sketch the graph of $f(x)$. Label the vertex and the y-intercept.

3.4 Assessment Procedure

Parts (a) and (c) serve to assess SLO 4. Part (a) requires students to demonstrate understanding to the algorithm for completing the square. Part (c) asks the to interpret the ordinate of the vertex as a maximum value or a minimum value of the quadratic function.

Parts (b) and (d) serve to assess SLO 1. Part (d) requires students to graphically represent the information obtained from the vertex form equation. Part (b) requires them to identify the vertex coordinates (h, k).

Two scorers from the full-time faculty of the Mathematics Department assigned numerical scores from 0 to 3 to each of the four parts of the Problem using the following rubric:

(0) No relevant work shown. No attempt is made or interpretation is clearly incorrect.

(1) Major flaws (e.g. vertex is drawn as minimum instead of maximum).

(2) Mostly correct, with noticeable flaws.

(3) Computation performed correctly, with only negligible flaws, if any.

	0	1	2	3
SLO 1: Student performs symbolic and numerical computation correctly (part (a))	No relevant work shown	Major flaws	Minor, noticeable flaws	Computation performed correctly
SLO 1: Student presents a clear and legible graph that matches the information obtained in part (a) (part (d))	No relevant work shown	Major flaws (e. g. vertex drawn as minimum instead of maximum)	Mostly correct, with noticeable flaws (e. g. vertex placed incorrectly)	Clear and complete, with only negligible flaws if at all
SLO 3 and 4: Interprets the model correctly, argues logically to obtain correct answers (parts (a), (b) and (c))	No attempt is made or interpretation is clearly incorrect	Major flaws	Minor, noticeable flaws	Essentially correct interpretation is demonstrated

More details of the scoring criteria discussed by the two grades are as follows:

(a) We awarded 1 if some work was done showing some kind of idea of what is involved in completing the square (what "amount or work" was at the discretion of each grader). A score of 2 was awarded if a vertex form was obtained with a correct value for the h coordinated of the vertex but an incorrect value for the k coordinate (typical error), and a score of 3 if both h and k were obtained.

(b) A score of 3 was awarded if identifying (h, k) correctly from the vertex form $f(x) = a(x-h)^2 + k$, even if this form was not correct in Part (a). A score of 2 was given if $-h$ instead of h, or if missing parenthesis in the expression (h, k).

(c) This was a yes or no question. A score of 3 was given for "maximum" even if no explanation (most). If an explanation was given but this was incorrect, then a score of 2 was awarded. Typical explanations found where listing some values for $f(x)$ (incorrect), or using the first derivative test to determine increasing/decreasing through the vertex h-value (pertaining to Math 150A).

(d) We interpreted "labeling" as meaning "write down the pair of coordinates" of the vertex and of the vertical intercept. We also wanted a graph that crossed the horizontal axis (as it should), and exhibited some degree of care in its execution.

3.5 Results and Analysis

There were three sections (with $35(-4), 32(-2), 29(-1)$) totaling $96(-7) = 89$ exams. The table below gives the average and median scores for each of the four parts of the problem:

	Average Score	Median Score
Part (a)	1.95	2
Part (b)	2.36	3
Part (c)	2.47	3
Part (d)	1.79	2

In Figure 3.1 we give the distributions of the scores on the problems.

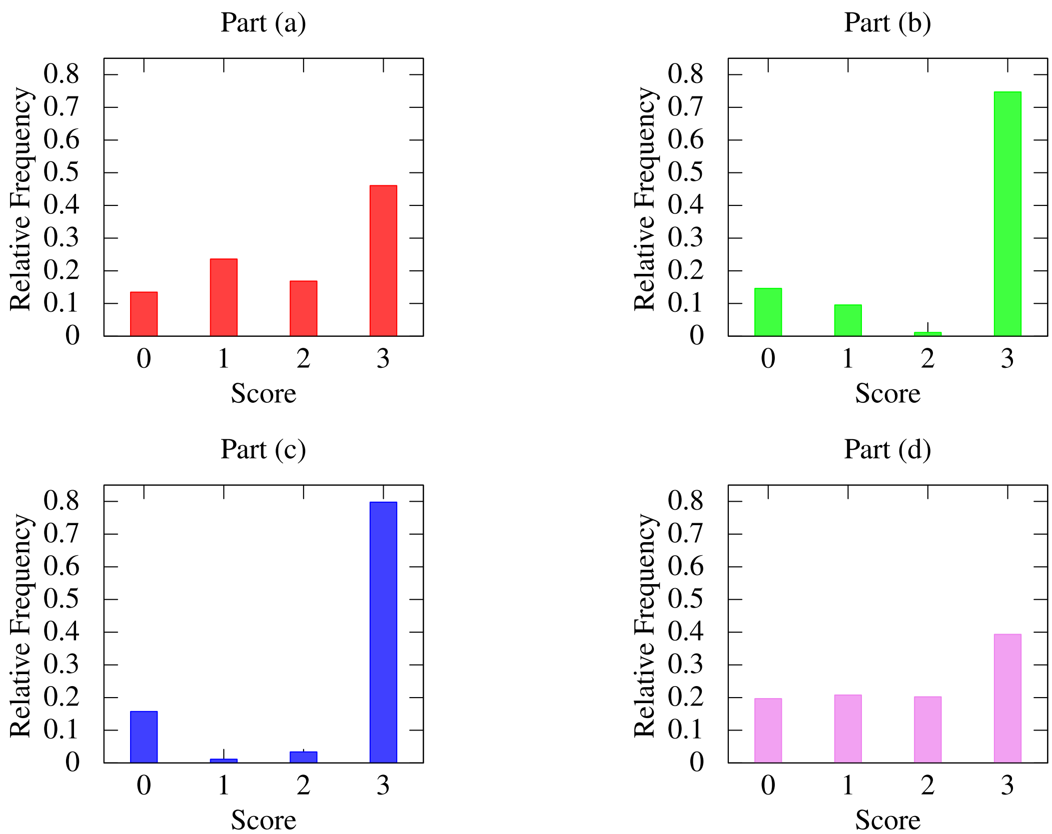


Figure 3.1: Score distribution for assessment of General Education Basic Skills Math SLO #1 in Math 105, Fall 2016

We note that Parts (b) and (c) have higher average score because they are essentially a yes/no question. Part (b) only required that students write the pair (h,k) from the answer obtained in Part (a) (even if this was not correct). Part (c) only required them to write maximum/minimum but not to justify the answer. We may guess that the correct answer (max) was selected because the negative leading coefficient of the original equation. But this is debatable: some students attempted to give a justification, but this was either incorrect, or incomplete, or used mathematical concepts beyond of what is covered in this course.

3.6 Observations and Recommendations

(a) We recommend that a clear (and homogeneous across multiple sections) algorithm for completing the square be presented to the students, for instance:

$$
\begin{aligned}
f(x) &= -2x^2 + 12x - 7 && \text{(Original equation)}\\
f(x) + 7 &= -2x^2 + 12x && \text{(Subtract independent term)}\\
\frac{f(x)+7}{(-2)} &= x^2 - 6x && \text{(Divide by coefficient of } x^2\text{)}\\
\frac{f(x)+7}{(-2)} + 9 &= x^2 - 6x + 9 && \text{(Add square of } 1/2 \text{ the coefficient of } x\text{)}\\
\frac{f(x)+7}{(-2)} + 9 &= (x-3)^2 && \text{(Binomial theorem)}
\end{aligned}
$$

———— (Retrace previous steps) ————

$$
\begin{aligned}
\frac{f(x)+7}{(-2)} &= (x-3)^2 - 9 && \text{(Subtract 9)}\\
f(x) + 7 &= -2(x-3)^2 + 18 && \text{(Multiply by } (-2)\text{)}\\
f(x) &= -2(x-3)^2 + 18 - 7 && \text{(Subtract 7)}\\
f(x) &= -2(x-3)^2 + 11 && \text{(Vertex form equation)}
\end{aligned}
$$

This method has the virtue of emphasizing the algorithmic nature of completing the square and therefore it has the virtue of making students aware of the algorithmic nature of many mathematical problems at a very early stage in the mathematical endeavors. It is in fact a wonderful opportunity to showcase one of the beautiful aspects of mathematics to beginning students.

It also has the value of being equally applicable to solving quadratic equations by completing the square. Indeed, it is not uncommon that students think that there are two methods, one for quadratic functions and one for quadratic equations, and then we find the aberration of rewriting the original problem in the form $-2x^2 + 12x - 7 = 0$ and going on from there, which makes no mathematical sense.

We recommend that instructors insist on students developing the habit of checking the correctness of the expression $f(x) = a(x-h)^2 + k$ just obtained. This is very easily done by plugging $x = 0$ into the expression $f(x) = -2(x-3)^2 + 11$ just obtained and verifying that the number is precisely the independent term of the original equation, which is also the y intercept.

(b) The vertex. Given vertex form $f(x) = a(x-h)^2 + k$, most students get the vertex value (h, k) correct, even if the vertex form obtained is incorrect. There are however some that write $(-h, k)$ (misunderstanding) or h, k (sloppiness).

(c) Students should be taught how to explain why the y coordinate of the vertex is a maximum value or a minimum value for the function. We want to note that the question should be precisely formulate: it is not whether the "vertex is a maximum or minimum value of f" but whether the "vertex ordinate is a maximum or a minimum value of f."

(d) Students are asked to label the vertex and the vertical intercept. Labeling means to identify the point(s) on the graph by writing down their coordinates.

This is another place for checking the correctness of Part (a), as one can both find the vertical intercept by using the independent term of the original equation, or by substituting $x = 0$ into the vertex form $-ah^2 + k$.

It is also important to teach the students to graph the parabola with certain degree of smoothness at the vertex (especially because most student will go into Math 150A), and also to be sure to draw the graph crossing the horizontal axes whenever if applicable, and not to graph "vertical asymptotes."

Chapter 4

Fall 2016: Measurement of Math 255A Class and Analysis

4.1 SLOs Assessed

All four of the General Education Basic Skills SLOs in Mathematics were assessed:

- **SLO #1 Represent, understand and explain mathematical information symbolically, graphically, numerically and verbally.**
- **SLO #2 Develop mathematical models of real-world situations and explain the assumptions and limitations of those models.**
- **SLO #3 Use models to make predictions, draw conclusions, check whether the results are reasonable, and find optimal results using technology when necessary and appropriate.**
- **SLO #4 Demonstrate an understanding of the nature of mathematical reasoning, including the ability to prove simple results and/or make statistical inferences.**

4.2 Course Description

Math 255A (Calculus for the Life Sciences I) is a 3 unit calculus course available for General Education credit in Basic Skills: Mathematics. Its catalog description reads thus:

> *First semester of a short course in calculus. Topics in calculus of functions of one variable including techniques of differentiation, applications to graphing, extreme problems and an introduction to integration. Applications to life sciences are emphasized.*

The textbook used in the Fall 2016 was *Calculus for the Life Sciences* by Schreiber, Smith, and Getz (Wiley, 2014).

Math 255A serves as an introduction to calculus for students in biological sciences. It covers standard topics in single-variable calculus (graphing, differentiation, extreme value problems, and integration) with an emphasis on how these techniques are used in mathematical models of biological systems and processes. The question chosen here explores how calculus techniques can be used to model the concentration of a drug in a patient's bloodstream as a function of time.

Data were collected from all three class sections: 16996, 17451, and 17452. These three sections were randomly labeled Sections 1–3.

4.3 Signature Assignment

One question from the final exam for the Fall 2016 course Math 255A (Calculus for the Life Sciences I) was designated for assessment.

There were two versions of the exam, and each had a slightly different numerical constant in the expression for $C(t)$ below.

Problem. Version 1

The following function has been suggested as a model of how acetaminophen concentration in patients bloodstream changes with time:

$$C(t) = 28.6(e^{-0.3t} - e^{t}) \quad \text{[mg/l]},$$

where t is in hours after taking the dose. Describe the following features of the function (if applicable). Fill in all the blanks. Use the next page to show your work.
Biologically meaningful domain: ____________
Vertical asymptote(s): ____________
Horizontal asymptote(s): ____________
Critical point(s) (t values): ____________
Interval(s) of increase: ____________
Interval(s) of decrease: ____________
Global maximum: ____________
Global minimum: ____________
Sketch a graph of the function. Label all intercepts, local extrema, and show the asymptote(s) if applicable.
According to this model, the concentration $C(t)$ is ____________when $t = 0$, reaches its ____________at ____________and ____________as $t \to \infty$.

Problem. Version 2

The following function has been suggested as a model of how acetaminophen concentration in patients bloodstream changes with time:

$$C(t) = 30.2(e^{-0.3t} - e^{t}) \quad \text{[mg/l]},$$

where t is in hours after taking the dose. Describe the following features of the function (if applicable). Fill in all the blanks. Use the next page to show your work.

Biologically meaningful domain: ____________
Vertical asymptote(s): ____________
Horizontal asymptote(s): ____________
Critical point(s) (t values): ____________
Interval(s) of increase: ____________
Interval(s) of decrease: ____________
Global maximum: ____________
Global minimum: ____________
Sketch a graph of the function. Label all intercepts, local extrema, and show the asymptote(s) if applicable.
According to this model, the concentration $C(t)$ is ____________when $t = 0$, reaches its ____________at ____________and ____________as $t \to \infty$.

4.4 Mathematical Background

To do all the parts of this problems (filling in twelve blanks and drawing and labeling a graph) requires many of the basic skills of single-variable calculus, including use of the derivative to find the critical points and thus determine extrema. The sum total of all that is done in this problem made it an ideal place to measure all four of the General Education Basic Skills SLOs in Mathematics.

4.5 Assessment Procedure

The formal assessment of the student work was done based on the following four **scoring rubrics**, one for each of the SLOs:
SLO #1: Represent, understand and explain mathematical information symbolically, graphically, numerically and verbally.

Student will sketch and label features on the graph in the box.

Scoring will be based on how well the student sketches and labels features on the graph in the box.

Score	0	1	2	3
Description	Does not present any correct information.	Presents at least some correct information, but falls far short of what was requested.	Presents substantial correct information, but with some omissions or flaws.	Presents all of the information requested. There should be at most minor omissions or flaws.

SLO #2: Develop mathematical models of real-world situations and explain the assumptions and limitations of those models.

Student will determine the biologically meaningful domain and the asymptotes of the model.

Scoring will be based on how well the student determines the biologically meaningful domain and the asymptotes of the model. The function must only be read where it makes sense, and a model will be unreasonable if it has vertical asymptotes or a horizontal asymptote that is other than $y = 0$.

Score	0	1	2	3
Description	Does not present any correct information about the domain where one can reasonably interpret the model and the reasonableness of the model.	Presents at least some correct information about reasonableness, but falls far short of what was requested.	Presents substantial correct information about reasonableness, but with some omissions or flaws.[1]	Presents all of the information about reasonableness requested. There should be at most minor omissions or flaws.

[1]This could be because the student only successfully filled in information for two of three parts, or because the student attempted all three parts with some success, but has some conceptual flaw in execution or presentation.

SLO #3: Use models to make predictions, draw conclusions, check whether the results are reasonable, and find optimal results using technology when necessary and appropriate.

Student will fill in information about initial concentration, maximum concentration, and asymptotic concentration in the long term.

Scoring will be based on the last two lines of the problem, which requires the student to fill in information about initial concentration, maximum concentration, and asymptotic concentration in the long term.

Score	0	1	2	3
Description	Does not present any valid conclusions or predictions of the model.	Presents at least one valid conclusion or prediction, but falls far short of what was requested.	Presents substantial correct conclusions or predictions, but with some omissions or flaws.[1]	Presents correctly all of the conclusions and predictions requested. There should be at most minor omissions or flaws.

[1]This could be because the student only successfully filled in information about two of three parts, or because the student attempted all three parts with some success, but has some conceptual flaw in execution or presentation.

SLO #4: Demonstrate an understanding of the nature of mathematical reasoning, including the ability to prove simple results and/or make statistical inferences.

Student will determine the critical point and intervals of increase and decrease via differentiation.

Scoring will be based on the student's ability to determine the critical point and intervals of increase and decrease via differentiation.

Score	0	1	2	3
Description	Is not able to use mathematical reasoning to determine any of the requested information about critical points or intervals of increase or decrease.	Presents at least one valid mathematical inference about the requested information, but falls far short of what was requested.	Presents substantial correct inferences, but with some omissions or flaws.[1]	Correctly infers all the requested information. There should be at most minor omissions or flaws.

[1]This could be because the student only successfully solved two of three parts, or because the student attempted all three parts with some success, but has some conceptual flaw in execution or presentation.

4.6 Results and Analysis, and Recommendations

The scoring for formal assessment, based on the above scoring rubric, was performed by two full-time faculty of the Department of Mathematics. The distribution of scores for the entire class and for individual sections is presented in Figures 4.1–4.4 (one for each SLO).

The following are the average scores for each SLO, measured for the whole class and by individual section.

	All Sections	Section 1	Section 2	Section 3
SLO #1	1.33	1.59	1.22	1.26
SLO #2	1.72	2.10	1.52	1.69
SLO #3	1.38	1.51	1.24	1.50
SLO #4	1.37	1.53	1.30	1.32

For each SLO, the highest average was always that for Section 1, the middle average was for Section 2, and the lowest was for Section 3. In this particular problem, students seem to be having less difficulty with SLO #2 (develop mathematical models of real-world situations and explain the assumptions and limitations of those models) than the others. This may be more due to the limited scope of the portions of the question that were used to evaluate SLO #2 than due to any difference in inherent difficulty of the SLOs themselves.

The overall results show that the students are finding the Math 255A course quite challenging. The scorers noticed that often students left many of the questions within the problem unanswered. Problems such as this efficiently test the students in many areas without them needing to do as many individual calculations. But such problems still take a long time to complete. Perhaps many students could have done this question but ran out of time. We recommend that future instructors of this class carefully consider the number of problems they place on exams, keeping in mind that problems with an extensive number of parts (such as this one) take a lot of time to complete. We also note that the last part of the problem, which involves filling in four blanks within a sentence, can be answered in many ways that are correct, and students often filled in the blanks to form a true statement that was less informative than what the instructor was probably seeking. We encourage instructors to write exams that do not allow such ambiguities.

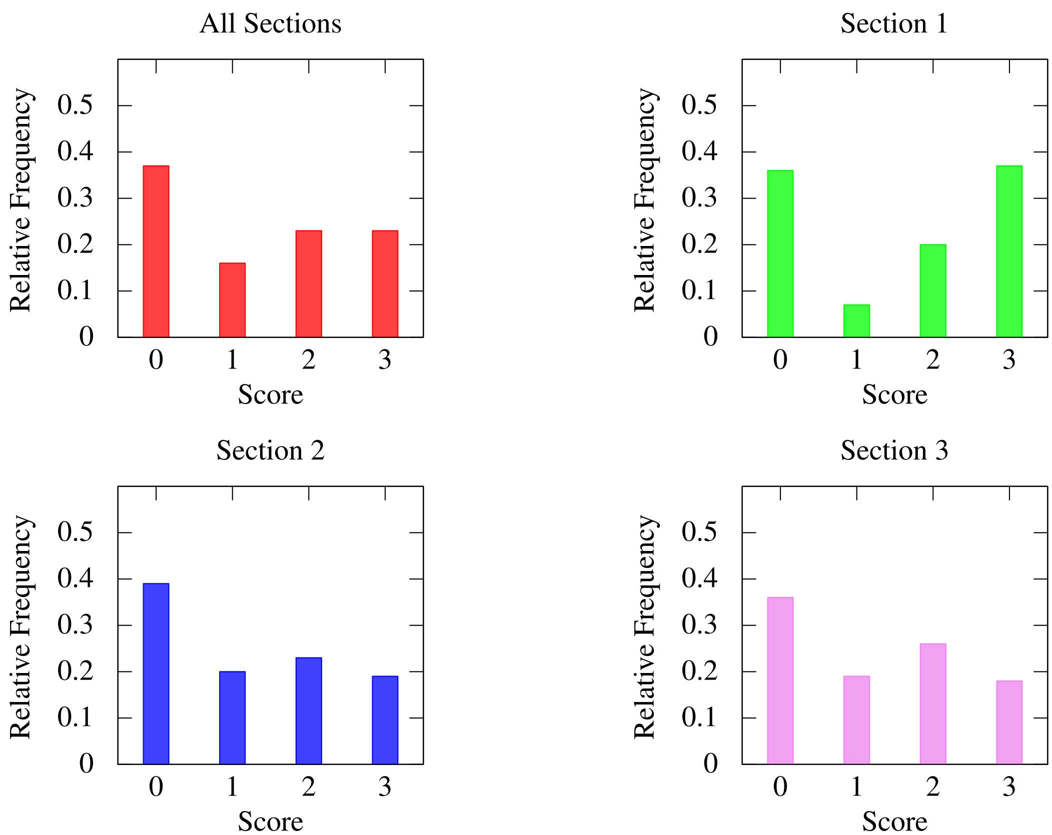


Figure 4.1: Score distribution for assessment of General Education Basic Skills Math SLO #1 in Math 255A, Fall 2016

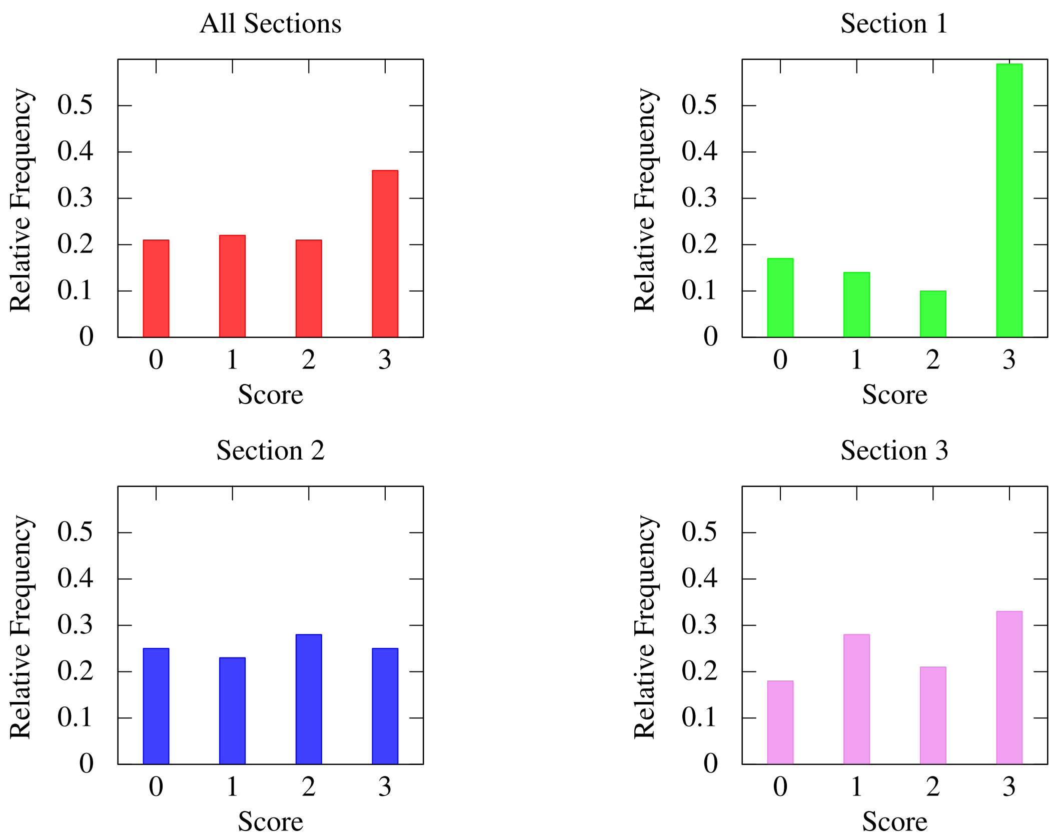


Figure 4.2: Score distribution for assessment of General Education Basic Skills Math SLO #2 in Math 255A, Fall 2016

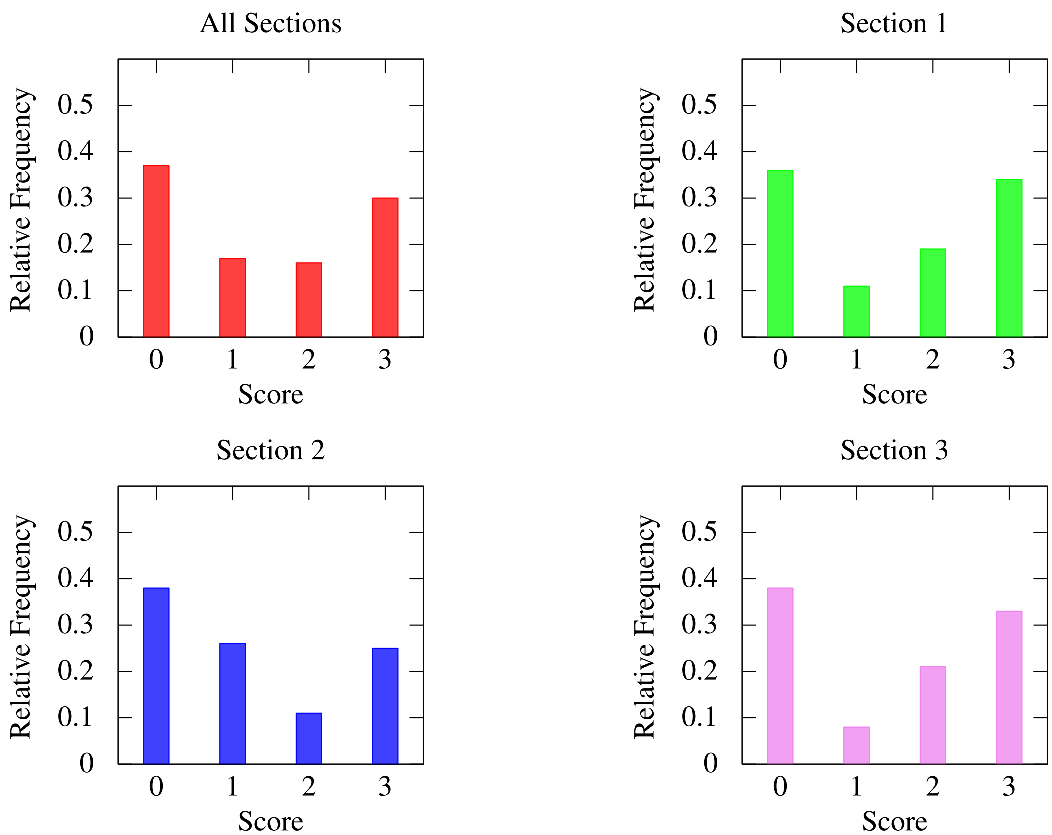


Figure 4.3: Score distribution for assessment of General Education Basic Skills Math SLO #3 in Math 255A, Fall 2016

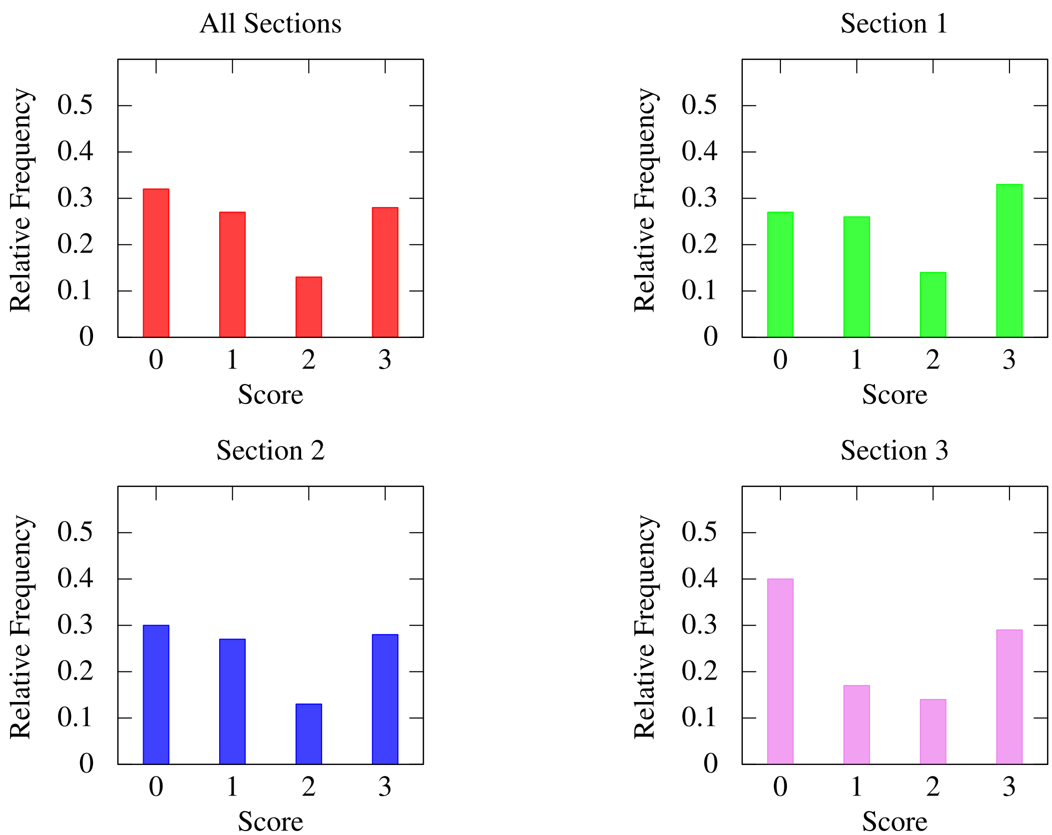


Figure 4.4: Score distribution for assessment of General Education Basic Skills Math SLO #4 in Math 255A, Fall 2016

Chapter 5

Spring 2017: Measurement of Math 140 Class and Analysis

5.1 SLOs Assessed

All four of the General Education Basic Skills SLOs in Mathematics were assessed:

- **SLO #1 Represent, understand and explain mathematical information symbolically, graphically, numerically and verbally.**
- **SLO #2 Develop mathematical models of real-world situations and explain the assumptions and limitations of those models.**
- **SLO #3 Use models to make predictions, draw conclusions, check whether the results are reasonable, and find optimal results using technology when necessary and appropriate.**
- **SLO #4 Demonstrate an understanding of the nature of mathematical reasoning, including the ability to prove simple results and/or make statistical inferences.**

5.2 Course Description

Math 140 (Introductory Statistics) is a 5 unit statistics course available for General Education credit in Basic Skills: Mathematics. Its catalog description reads thus:

> *Methods for displaying, describing and producing data. Normal distribution. Correlation and regression. Sampling distributions and probability. Statistical inference for means and proportions.*

The textbook used in the Spring 2017 was *Statistics in Practice*, by Moore, Notz and Flinger (W. H. Freeman, 2014).

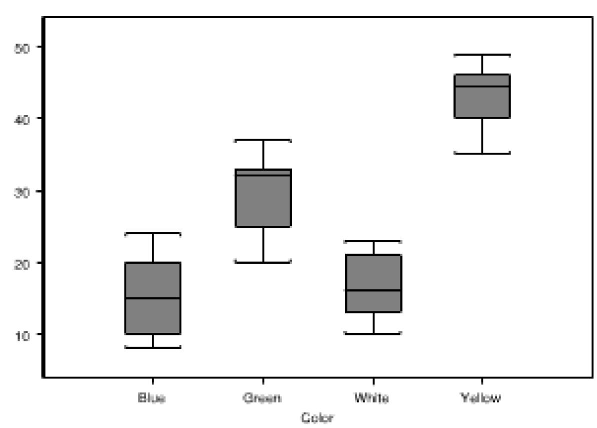
Math 140 serves as an introduction to statistics for students in various disciplines. The course covers methods of presenting and describing data, sampling, linear regression and correlation, confidence intervals, and some elementary probability theory. The signature assignment here consists of many questions that collectively cover these areas in some breadth. Different sections are specially tailored for different undergraduate majors. We focused especially on the sections of Math 140 that serve students in the College of Business and Economics (sections 16433, 16434, 16613, 17020, 17341, 17342, and 17511). Some of these sections have instruction in a traditional classroom, one section provides the instruction online, and some sections offer a hybrid of the two approaches. This provides a good opportunity to assess how these different modalities of instruction correlate to student achievement of the SLOs. All students come to campus to take exams in person, so even the section which we call "online" is not done completely remotely. The sections were grouped and labeled as follows:

1. One regular-sized traditional section (40 enrolled),
2. Two regular-sized traditional sections (50 and 53 enrolled) with the same instructor
3. One large traditional section (89 enrolled),
4. Two hybrid sections (34 and 25 enrolled), and
5. One online section (107 enrolled).

5.3 Signature Assignment

Twenty-four multiple choice questions from the final exam for the Spring 2017 course Math 140 (Introductory Statistics) were designated for assessment. To make this report more easy to read, we grouped the twenty four multiple-choice questions into four groups of six, according to which SLO they were used for (so Problem X concerns SLO X for $X = 1, 2, 3$ or 4). This is not the original order or numbering that was used in the exam.

Problem 1. Questions (i), (ii) and (iii) refer to the following situation: A clothing manufacturer tracked the number of its T-shirts that were sold in each of the 40 retail outlets in which they were available. The boxplots below show the number of T-shirts of each color that were sold in these 40 stores:



(i) The median number of Blue T-shirts sold was approximately:

(a) 8

(b) 10

(c) 15

(d) 20

(ii) Approximately how many of the 40 stores sold less than 20 Blue T-shirts?

(a) 10

(b) 20

(c) 25

(d) 30

(iii) Judging by the interquartile ranges, which color had the least variability from store to store in the numbers of T-shirts sold?

(a) Blue

(b) Green

(c) White

(d) Yellow

(iv) A research study in Africa looked at various characteristics of the people with HIV in a certain set of villages. One of the variables measured was education level, which was recorded as illiterate, primary, or secondary. What type of variable is education level?

(a) a quantitative variable

(b) a categorical variable

(c) a response variable

(d) a lurking variable

(v) The distribution of salaries for Major League Baseball players is strongly skewed to the right. If the median salary is $\$ 500,000$ per year, then the mean salary is:

(a) greater than $\$ 500,000$ per year.

(b) equal to $\$ 500,000$ per year.

(c) less than $\$ 500,000$ per year.

(d) Any of the above are possible.

(vi) A simple random sample of size $n = 25$ produced a sample standard deviation of 7.5. What is the standard error of the sample mean $\bar{x}$?

(a) 1.5

(b) 7.5

(c) 15

(d) It is impossible to say without knowing the sample mean.

Problem 2. (i) A correlation r between two variables shows that a cause and effect relationship exists between the variables if:

(a) it is close to 0.

(b) it is close to 1.

(c) it is close to 1 or -1.

(d) A correlation by itself never establishes a cause and effect relationship.

(ii) The minimum wage varies from state to state. A business group looked at the minimum wage in all 50 states, as well as the percentage rate of profit of companies in each state. A least squares regression on the data they studied gave the following equation:

$$\hat{y} = .034 - 0.12x$$

where y represents the percentage rate of profit and x is the minimum wage in dollars. Which of the following is a valid conclusion?

(a) The least square line points downhill, therefore the correlation between the percentage rate of profit and the minimum wage is negative.

(b) Increasing the minimum wage in a state tends to reduce the percentage rate of profit in that state.

(c) The minimum wage in a state is a poor predictor of the percentage rate of profit in that state.

(d) The minimum wage in a state is a good predictor of the percentage rate of profit in that state.

(iii) The Department of Education wishes to estimate the proportion of all college students who have a job off-campus. It surveyed 1600 randomly selected students; 451 had such jobs. The population of interest to the Department of Education is:

(a) All 1600 students surveyed.

(b) The 451 students in the survey who had off-campus jobs.

(c) All college students.

(d) All college students who have off-campus jobs.

(iv) A randomized comparative experiment must have:

(a) random assignment of the subjects to the treatment and control groups.

(b) random sampling of the subjects from the population.

(c) random assignment of the placebo to subjects in each group.

(d) separate random samples of the population for the control group and the treatment group.

(v) A sampling distribution:

(a) shows how many different random samples are possible when sampling from a population.

(b) describes how a statistic's value can vary from sample to sample.

(c) is any distribution that is sampled from the population.

(d) shows where different samples are taken from in the population.

(vi) A sociologist wanted to assess whether two large banks hire minorities at different rates. He determined the number of minority hires made at twelve randomly selected branches of Bank A, and the number of minority hires made at twelve randomly selected branches of Bank B. He found the sample means to be 3.5 hires for Bank A and 5.2 hires for Bank B. Let μ_A μ_B be the mean numbers of minority hires made at all branches of Banks A and B, respectively. The appropriate null and alternative hypotheses for the sociologist to use are:

(a) $H_0\colon \bar{x}_A = \bar{x}_B$ versus $H_a\colon \bar{x}_A < \bar{x}_B$

(b) $H_0\colon \bar{x}_A = \bar{x}_B$ versus $H_a\colon \bar{x}_A \neq \bar{x}_B$

(c) $H_0\colon \mu_A = \mu_B$ versus $H_a\colon \mu_A < \mu_B$

(d) $H_0\colon \mu_A = \mu_B$ versus $H_a\colon \mu_A \neq \mu_B$

Problem 3. (i) Suppose that the length of time a certain brand of battery operates follows a normal distribution with mean of 25 days and a standard deviation of 6 days. The first quartile of this distribution is closest to:

(a) 4 days

(b) 21 days

(c) 29 days

(d) 37 days

(ii) If the mean score on an exam is 45 and the standard deviation is 11, about what percent of the scores would you expect to fall between 34 and 56, assuming that the data distribution is approximately normal?

(a) 22%

(b) 50%

(c) 68%

(d) 95%

(iii) For the situation described in the previous question, about what grade would be needed to fall into the top 5% of scores? (You may need to do a calculation.)

(a) At least 56
(b) At least 63
(c) At least 67
(d) At least 959.

(iv) The number of people sitting at a randomly selected table in a library has the probability distribution given below. Fill in the missing entry:

No. of people	0	1	2	3	4 or more
Probability	0.15	0.20		.20	.05

(a) 0.10
(b) 0.15
(c) 0.20
(d) 0.40

(v) A 99% confidence interval will typically capture:

(a) 99% of the data.
(b) 99% of the sampling distribution.
(c) 99% of the population.
(d) the value of the parameter of interest, or the difference between two parameters.

(vi) A sample of 41 randomly selected patients at a hospital emergency room gave a mean waiting time of 11.3 minutes and a sample standard deviation of 6.5 minutes. A 95% confidence interval for the mean waiting time of all patients is:

(a) $11.3 \pm 2.000(6.5)$
(b) $11.3 \pm 1.960(6.5)$
(c) $11.3 \pm 1.960\left(\frac{6.5}{\sqrt{41}}\right)$
(d) $11.3 \pm 2.021\left(\frac{6.5}{\sqrt{41}}\right)$

Problem 4. (i) Least squares regression is often used to:

(a) measure how strong the relationship is between two quantitative variables.
(b) measure how strong the relationship is between two categorical variables.
(c) predict the value of one quantitative variable from the value of another quantitative variable.
(d) predict the value of a quantitative variable from the value of a categorical variable.

(ii) A study of darts players in a bar looked at the relationship between the number of alcoholic drinks each consumed and the percentage of times they were successful in hitting the particular part of the dart board they were aiming for. The correlation came out to be $r = 0.08$. Which of the following is the best conclusion?

(a) There is almost no linear relationship between the number of alcoholic drinks a player consumed and the percentage of times their dart throws were successful.

(b) An increase in alcohol intake caused players to play worse.

(c) Players play their best before they have consumed any alcohol.

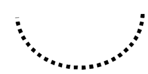
(d) None of the above.

(iii) For which of the following cases will the correlation coefficient not be close to 1?

(a) When there is a very strong positive linear association between x and y.

(b) When the least squares regression line tilts upward and passes close to all the points in the scatterplot.

(c) When the least squares regression line tilts upward and all of the residuals are small.

(d) When there is a very strong nonlinear (curved) association between x and y, like this: 

(iv) The Central Limit Theorem is important because:

(a) it tells us that the sample mean will be close to the center of the data when n is large enough.

(b) it allows us to use Normal probability calculations to solve problems involving sample means even when the population distribution is not Normal, if n is large enough.

(c) it tells us that the sample mean will equal the population mean if n is large enough.

(d) it tells us that the shape of the sampling distribution of $\bar{x}$ will approach the shape of the population distribution as n increases.

(v) Suppose that we take a random sample of size n from a large population and compute the proportion of individuals who are left-handed. Which of the following statements is true?

(a) The larger the sample size, the smaller the bias.

(b) The larger the sample size, the smaller the margin of error of a confidence interval for the proportion of left-handed individuals.

(c) The larger the sample size, the larger the length of a 99 percent confidence interval for the proportion of left-handed individuals.

(d) All of the above are true.

(vi) A hypothesis test rejected the null hypothesis at a significance level of .05. Thus:

(a) The test would have rejected H_0 at a significance level of $\alpha = .10$ but not at $\alpha = .01$.

(b) The test would have rejected H_0 at a significance level of $\alpha = .01$ but not at $\alpha = .10$.

(c) The test would have rejected H_0 at $\alpha = .10$, but may or may not have at $\alpha = .01$.

(d) It is not possible to say whether the test would have rejected H_0 at any other significance levels.

5.4 Statistical Background

- In Problem 1, parts (i), (ii), and (iii) students must be able to read graphical representations of statistical information from the box plot, demonstrate that they understand verbal descriptions such as "median" and "interquartile range," and then indicate the correct numerical description of the data. Part (iv) again tests their ability to correctly use verbal descriptions of data. Part (v) tests their understanding of the verbal description "skewed" as applied to a distribution. Students must understand that skewness of itself does not imply a particular relationship between the median and the mean. Part (vi) requires the students to describe a data set numerically using the notion of standard error when given numerical data about the sample standard deviation.

- In Problem 2, parts (ii) and (vi) students must understand how one formulates and validly speaks about the statistical models for situations that could be encountered in the real world. In parts (iii), (iv), and (v), students show that they understand the importance of sampling and how it ought to be done when collecting data in the real-world, which indicates whether students understand the assumptions that underlie statistical models. (If the sampling plan does not match these assumptions, then the models do not apply.) In part (i), students must demonstrate knowledge of the limitation of the concept of mathematical correlation in determining causal relationships.

- In Problem 3, parts (i), (ii), and (iii), students must make predictions about a data set when given the information that the data is well-modeled by a normal distribution. In parts (iv), students must use probability theory to draw a conclusion about missing information in a data set. In parts (vi) and (v) students must show that they understand how to calculate a confidence interval and what conclusions one can draw from it.

- In Problem 4, parts (i)–(iii), students must demonstrate mathematical insight into what linear regression and correlation do and do not enable us to infer about data. In part (iv), students must demonstrate an understanding of how the Central Limit Theorem from probability impacts statistical reasoning and practice. In part (v), the students must reason about how sample size will affect our statistical inferences. In part (vi), the students must understand the concept of significance level in hypothesis testing, and must reason to infer from one situation what would have happened in an alternative situations.

5.5 Assessment Procedure

All the multiple choice questions were marked correct (1) or incorrect (0) with a ScanTron machine. Each Problem above is a group of six of the multiple choice questions relevant to a particular SLO (recall that

Problem X concerns SLO X for $X = 1, 2, 3$ or 4). For each Problem, the average number of correct answers (from 0 to 6) was computed as a score for each section. Also the average was computed for all sections combined. These average scores were interpreted using the following rubrics, one for each of the SLOs:

SLO #1: Represent, understand and explain mathematical information symbolically, graphically, numerically and verbally.

Student will answer six multiple choice questions about graphical, numerical, and verbal representations of statistical information.

Scoring is based on the number of correct replies (from 0 to 6).

Score	0.0 to 1.5	1.5 to 3.0	3.0 to 4.5	4.5 to 6.0
Description	Demonstrates negligible ability to understand and communicate statistical information.	Demonstrates low ability to understand and communicate statistical information.	Demonstrates substantial but imperfect ability to understand and communicate statistical information.	Demonstrates strong ability to understand and communicate statistical information.

SLO #2: Develop mathematical models of real-world situations and explain the assumptions and limitations of those models.

Student will answer six multiple choice questions about statistical sampling and models, how they are used practically, and their underlying assumptions and limitations.

Scoring is based on the number of correct replies (from 0 to 6).

Score	0.0 to 1.5	1.5 to 3.0	3.0 to 4.5	4.5 to 6.0
Description	Demonstrates negligible knowledge of statistical modeling.	Demonstrates low knowledge of statistical modeling.	Demonstrates substantial but imperfect knowledge of statistical modeling.	Demonstrates strong ability of statistical modeling

SLO #3: Use models to make predictions, draw conclusions, check whether the results are reasonable, and find optimal results using technology when necessary and appropriate.

Students will answer six multiple choice questions where they must make predictions and draw conclusions from models of data.

Scoring is based on the number of correct replies (from 0 to 6).

Score	0.0 to 1.5	1.5 to 3.0	3.0 to 4.5	4.5 to 6.0
Description	Demonstrates negligible ability to make predictions or conclusions	Demonstrates low ability to make predictions or conclusions	Demonstrates substantial but imperfect ability to make predictions or conclusions	Demonstrates strong ability to make predictions or conclusions

SLO #4: Demonstrate an understanding of the nature of mathematical reasoning, including the ability to prove simple results and/or make statistical inferences.

Students will answer six multiple choice questions where they must show understanding into what linear regression and correlation allow them to infer, about the role of the Central Limit Theorem in statistical theory and practice, about how sample size impacts statistical inference, and about significance level in hypothesis testing.

Scoring is based on the number of correct replies (from 0 to 6).

Score	0.0 to 1.5	1.5 to 3.0	3.0 to 4.5	4.5 to 6.0
Description	Demonstrates negligible understanding of statistical reasoning and inference.	Demonstrates low understanding of statistical reasoning and inference.	Demonstrates substantial but imperfect understanding of statistical reasoning and inference.	Demonstrates strong understanding of statistical reasoning and inference.

5.6 Results, Analysis, and Recommendations

For each of the four SLOs (each scored based on six multiple choice questions, so scored from 0 to 6), the average scores for the entire cohort and for individual sections is presented in the below table and in the histograms in Figure 5.1 (one for each SLO).

	All Sections	Section 1	Section 2	Section 3	Section 4	Section 5
SLO #1	4.00	4.18	3.86	4.00	4.35	3.86
SLO #2	2.35	2.18	2.43	2.17	2.65	2.33
SLO #3	2.91	2.97	2.80	2.92	3.46	2.63
SLO #4	2.53	2.53	2.67	2.19	2.81	2.53
Average	2.95	2.96	2.94	2.82	3.32	2.84

We also show the percentage of students (for all sections put together) that answered each question right in the following table:

Problem	Part	Percentage Answering Correctly
1	(i)	94.58
1	(ii)	28.31
1	(iii)	44.88
1	(iv)	83.43
1	(v)	80.72
1	(vi)	67.77
2	(i)	17.77
2	(ii)	43.98
2	(iii)	23.80
2	(iv)	60.54
2	(v)	27.11
2	(vi)	61.75
3	(i)	62.35
3	(ii)	75.00
3	(iii)	37.05
3	(iv)	60.84
3	(v)	34.94
3	(vi)	20.48
4	(i)	37.35
4	(ii)	60.54
4	(iii)	79.52
4	(iv)	29.22
4	(v)	21.99
4	(vi)	24.10

Overall student performance shows that a typical student is close to the boundary between low and substantial knowledge of statistical concepts. There is no pronounced difference in performance among different sections or different styles (traditional versus online versus hybrid courses). The hybrid sections tend to have somewhat higher average scores, but in interpreting this, one must be careful to note that a slightly lower proportion of the enrolled students took the final exam as compared to the traditional sections.

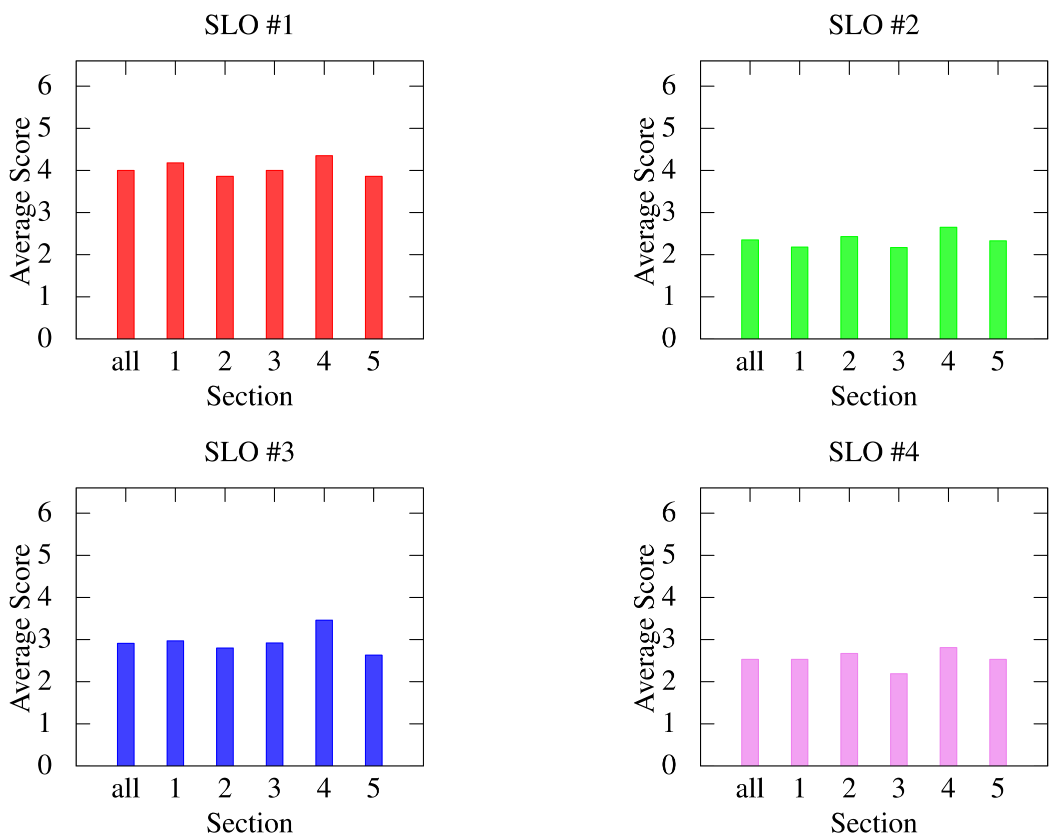


Figure 5.1: Average scores for assessment of General Education Basic Skills Math SLOs #1–#4 in Math 140, Fall 2017

The percentage of enrolled students who took the final exam for each of the sections is indicated on the following table:

Section	Description	Percentage of enrolled students who took the final exam
1	one regular-sized traditional section	85.0%
2	two regular-sized sections	94.2%
3	one large traditional section	93.3%
4	two regular-sized hybrid sections	81.4%
5	one large online section	65.4%

The rate of completion for online courses is lower than for other sections. We suggest that the department track when students stop actively participating in the online section as a function of time to see if students who do not take the final are persisting in class assignments and exams up to the final or if they cease activity earlier in the term.

In addition to the difference in format among the sections, there are differences in the modes of additional support provided by the Mathematics Department. The department provides the large section (Section 3) and the two hybrid sections (grouped into Section 4 here) with supplemental instruction. For the online section (Section 5), the department makes an important investment in providing a student assistant to help monitor the online question forums. Students from every section can use the Mathematics Tutoring Center organized by the department. Some students in the online section have said that they found the help provided by the tutors inadequate due to their unfamiliarity with the learning management system used for the online course. If funds were available, the online instructor believes that a couple dedicated tutors for Math 140 would be immensely helpful, and in particular, recommends that they should independently work through the pertinent online home and quiz assignments and be familiar with the expected format for answers. Another possible intervention would be to institute *online* tutoring specifically for the online section, so that students could receive assistance without having to come to campus.

Students have a fairly good grasp of SLO #1 (representing and understand mathematical information symbolically, graphically, numerically and verbally), and a rudimentary grasp of SLO #3 (using models), but are weak on SLOs #2 and #4, which ask for a deeper philosophical knowledge of the underlying assumptions and reasoning behind the statistical models. This is not unexpected, and it has been noted that the General Education SLOs for Basic Skills in Mathematics were written when the faculty were not yet experienced in the process of formal assessment. As such, they reflect ideal aspirations rather than realistic expectations of what students can accomplish in their first exposure to mathematical statistics. Therefore, we recommend revisiting the General Education SLOs and revising them to be more realistic and precisely measurable, rather than idealistic and aspirational.

For SLO #1 (Problem 1), the questions that gave the students the most difficulty were parts (ii) and (iii). These required the students to understand how the box plots represent the quartiles in the data, so perhaps this lesson should be better emphasized.

For SLO #2 (Problem 2), the question that gave the students the most difficulty was (i), where most students failed to answer that correlation does not imply causation. This point, which is critical for people to become educated interpreters of statistical data both in their profession and in everyday life, should be stressed more. Students also had difficulty with parts (iii) and (v), which concern the correct understanding of statistical sampling from a larger population.

For SLO #3 (Problem 3), the students did fairly well with the parts dealing with the normal distribution (i)–(iv), except for part (iii), which required them to calculate how many standard deviations one would need to be above the mean to reach the 95th percentile. Students had much greater difficulty with the theory and practice of confidence intervals (parts (v) and (vi)), so perhaps more attention needs to be given to confidence intervals.

For SLO #4 (Problem 4), the students had difficulty with part (iv), which is philosophical: it concerns the role that the Central Limit Theorem from probability plays in statistical practice. They also had difficulty with parts (v) and (vi), which require them to do thought experiments to reason from one scenario to another: they must understand the effects of changing sample size or significance level. It is not surprising that these caused the most difficulty for students.

Part II

Mathematics Undergraduate Program Assessment

Chapter 6

Introduction

We conducted an assessment of the Mathematics Bachelor's program

- **SLO #3: Demonstrate facility with the objects, terminology and concepts of linear algebra.**

1. In Fall of 2016 we measured student work in the Math 262 course (Introduction to Linear Algebra) to assess this SLO in the lower division course where the concepts of this SLO are introduced and practiced.
2. In Spring of 2017 we measured student work in the Math 462 course (Advanced Linear Algebra) to assess this SLO in the upper division course where the concepts of this SLO are practiced and demonstrated.
3. We analyzed these measurements for this year and made recommendations to the department.

The assessment of the Mathematics Bachelor's program SLO #3 continues our assessment of the SLOs that we revised and implemented in the 2015–16 academic year. We plan to continue assessing these SLOs until we assemble a full picture of the current state of the bachelor's program. Linear algebra is especially important in the physical sciences, engineering, and all other branches of mathematics. In teaching this subject, we are serving a diverse population that includes both majors and non-majors. Their ability to understand linear algebra is vital to their success, both here at CSUN and later in their careers. Math 262 (Introduction to Linear Algebra) is our first course in linear algebra, so it is natural that we begin the assessment there. The Math 462 course (Advanced Linear Algebra) is a more advanced, theoretical course taken by many of our math majors, and so it is important to assess it as a follow-up to evaluate how students are progressing in their knowledge.

Each of the measurements summarized in the above synopsis is now examined in its own section, which includes analysis of the findings.

Chapter 7

Fall 2016: Measurement of Math 262 Class and Analysis

7.1 SLO Assessed

The following SLO for the Mathematics Bachelor's Program was assessed:

- **SLO #3: Demonstrate facility with the objects, terminology and concepts of linear algebra.**

7.2 Course Description

Math 262 (Introduction to Linear Algebra) is a 3 unit linear algebra course required for all mathematics majors. Its catalog description reads thus:

> *Systems of linear equations, matrices, determinants, eigenvalues, vector spaces and linear transformations, as well as introduction to inner products on Rn and spectral theorem for symmetric matrices.*

The textbook used in the Fall 2016 was *Linear Algebra with Applications (9th ed.)* by Leon (Pearson, 2014).

Math 262 is a sophomore-level course that serves as a student's first introduction to linear algebra. It focuses on systems of linear equations, and vector and matrix algebra, as well as on the basic theory of span, independence, basis, and linear transformations and their eigenvectors and eigenvalues. The signature assignment chosen here explores the concept of forming a matrix representation of a linear transformation and using it to show where the transformation maps a specified vector.

Data were collected from all four class sections: 16747, 16896, 16897, and 17501. These four sections were randomly labeled Sections 1–4.

7.3 Signature Assignment

One question from the final exam for the Fall 2016 course Math 262 (Introduction to Linear Algebra) was designated for assessment.

Problem. We are given that $E = \{\mathbf{v}_1, \mathbf{v}_2\}$ is an ordered basis for V and $F = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ is an ordered basis for W. Suppose $L\colon V \to W$ is a linear transformation such that $L(\mathbf{v}_1) = 2\mathbf{w}_1 + 4\mathbf{w}_2 - 3\mathbf{w}_3$ and and $L(\mathbf{v}_2) = 3\mathbf{w}_1 - 2\mathbf{w}_2 + 5\mathbf{w}_3$.

(a) Write down the matrix representation of L relative to E, F.

(b) If $\mathbf{v} = 4\mathbf{v}_1 + 3\mathbf{v}_2$, use the matrix from part(a) to find $[L(\mathbf{v})]_F$.

(c) For $\mathbf{v} = 4\mathbf{v}_1 + 3\mathbf{v}_2$, use your answer to part b to find $L(\mathbf{v})$ in terms of $\mathbf{w}_1$, $\mathbf{w}_2$, and $\mathbf{w}_3$.

7.4 Mathematical Background

Part (a) of the question requires the student to remember how the data of the linear transformation are encoded in matrix form. Part (b) requires the student to understand how to encode the data about the given abstract vector $\mathbf{v}$ as a list of numbers (column vector), and then multiply the matrix from part (a) by this column vector to obtain another column vector, which is then interpreted again as an abstract vector in part (c). Thus this problem emphasizes techniques that serve as bridges between abstract linear algebra and concrete matrix algebra.

7.5 Assessment Procedure

The formal assessment of the student work was done based on the following **scoring rubric**:

Student will answer a three part problem on the representation of a linear transformation by a matrix.

Score	0	1	2	3
Description	Demonstrates no relevant idea about how to use linear algebraic ideas to solve any part the problem.	Presents at least one relevant idea, but falls far short of what is needed to solve all three parts of the problem.	Presents a solution that contains substantial correct ideas, but not a fully correct solution.[1]	Presents valid and clear solutions to all three parts, with no flaws or very minor flaws (e.g., simple calculation error).

[1]This could be because the student only successfully solved two of three parts, or because the student attempted all three parts with some success, but has some conceptual flaw in execution or presentation.

7.6 Results, Analysis, and Recommendations

The scoring for formal assessment, based on the above scoring rubric, was performed by two full-time faculty of the Department of Mathematics. The distribution of scores for the entire class and for individual sections is presented in Figure 7.1.

The following are the average scores overall and by individual section.

	All Sections	Section 1	Section 2	Section 3	Section 4
Average Score	1.64	2.18	1.43	1.23	1.69

There is considerable difference between the mean scores for the various sections. Section 1 has the majority of students with higher scores (2 or 3), while Sections 2 and 3 show the reverse (majority of scores among 0 and 1). Section 4 has a fairly flat distribution by comparison.

The overall results show that the procedure of *representing* abstract objects (vectors from abstractly defined vector spaces and their transformations) using concrete objects (matrices) is one that challenges Math 262 students. This skill is indispensable for anyone who wants to take real-world data and form a mathematical representation that can be manipulated by a computer. The results of Section 1 show that this concept is not inaccessible to our students, and we will consult with the instructor of that section about techniques for more effectively teaching this essential skill.

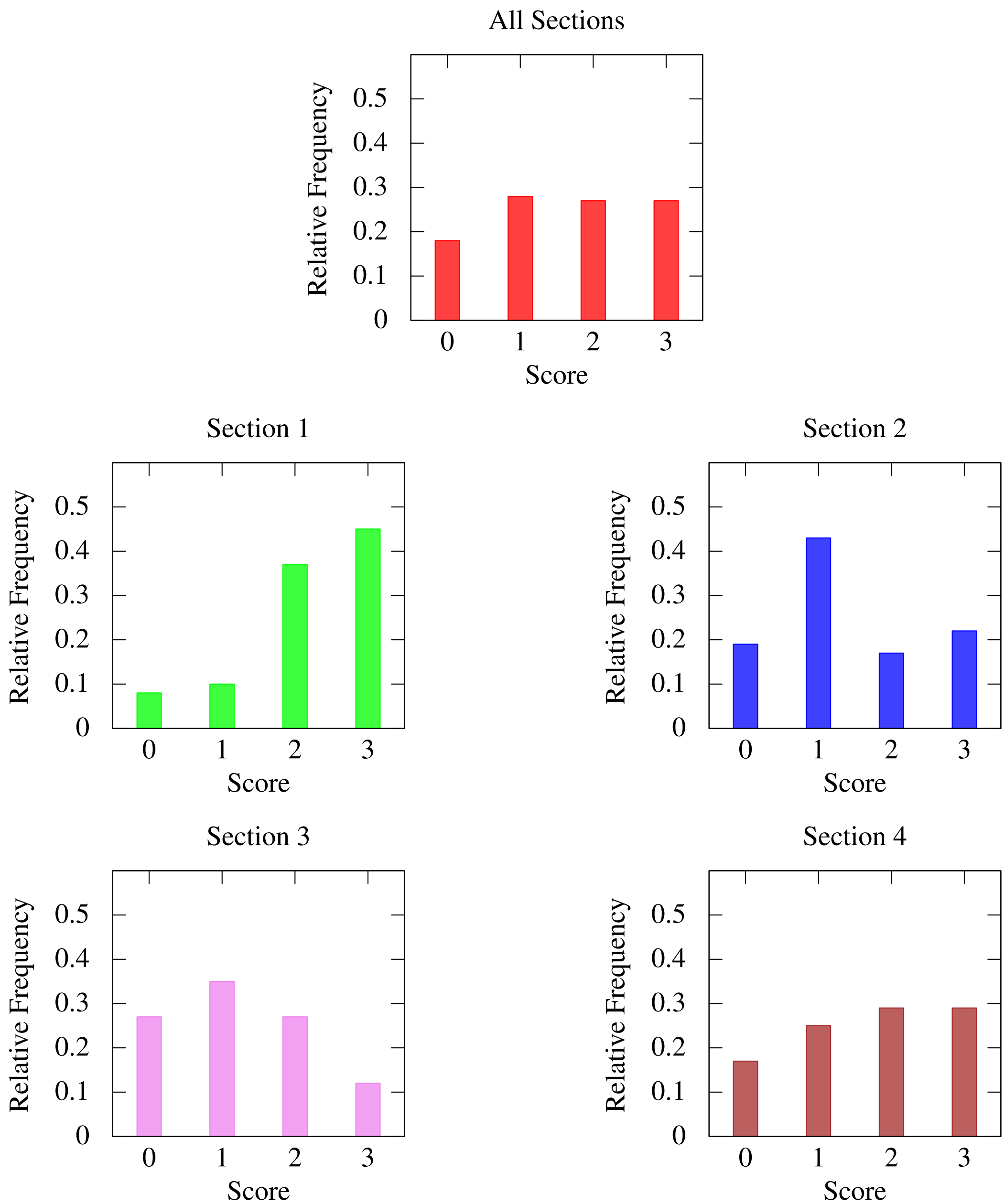


Figure 7.1: Score distribution for assessment of Math Bachelor's SLO #3 in Math 262, Fall 2016

Chapter 8

Spring 2017: Measurement of Math 462 Class and Analysis

8.1 SLO Assessed

The following SLO for the Mathematics Bachelor's Program was assessed:

- **SLO #3: Demonstrate facility with the objects, terminology and concepts of linear algebra.**

8.2 Course Description

Math 462 (Advanced Linear Algebra) is a 3 unit linear algebra course required for mathematics majors in the B.A. general option, the B.S. in mathematics option, and the B.S. statistics option. It can be used to fulfill the upper division math elective requirements for the other bachelor's program options (B.A. four-year integrated mathematics subject matter program for the single subject credential, B.A. junior-year integrated mathematics subject matter program for the single subject credential, and B.S. applied mathematics option). Its catalog description reads thus:

> *Finite dimensional vector spaces, linear transformations, matrix polynomials, canonical forms.*

The textbook used in the Spring 2017 was *Linear Algebra: An Introduction to Abstract Mathematics* by Valenza (Springer, 2012).

Math 462 is an upper division course that serves as a student's second exposure to linear algebra. It focuses on the abstract theory of vector spaces and linear transformations, and builds up to the theory of canonical forms for linear operators, including operators on inner product spaces. The signature assignments chosen here include some true/false questions on these topics and two longer questions. One of the longer questions explores the theory of forming a matrix representation of a linear transformation and then working out a concrete example. As such, it is very similar to the signature assignment for the assessment that was done for Math 262 (discussed in Section 7 above). The second longer question asks about how the rank and nullity of a linear transformation and the dimensions of its domain and codomain relate to injectivity and surjectivity of the transformation.

Data were collected from the single section of this class (17035) that ran in Spring 2017.

8.3 Signature Assignment

Extracts from one problem (amounting to four true/false questions) and two longer problem from the final exam for the Spring 2017 course Math 462 (Advanced Linear Algebra) were designated for assessment.

Problem 1. True of False?

(a) The function $C^1(\mathbb{R}) \to C^0(\mathbb{R}) : f(x) \mapsto f'(x) + 1$ is an $\mathbb{R}$-linear transformation.

(b) Suppose V, W are k-vector spaces of dimensions n, m, respectively, for some field k. Then $\mathrm{Hom}_k(V, W)$ is isomorphic to the k-vector space of $m \times n$ matrices over k.

(c) In an inner product space V with inner product $\langle \cdot \mid \cdot \rangle$ and where $\dim V \geq 1$, if $v, w \in V$ satisfy $\langle v \mid w \rangle = 0$, then either $v = 0$ or $w = 0$.

(d) The characteristic polynomial of the matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ equals $t^2 + 2$ (where t is the variable).

Problem 2. (a) Suppose V, V' are finite-dimensional vector spaces over a field k, and that $T\colon V \to V'$ is a k-linear transformation. Suppose also that $v_1, \ldots, v_n$ form a basis B for V, while $v'_1, \ldots, v'_m$ form a basis B' for V'. Write down the formula for the j-th column of $M_{B,B'}(T)$, the matrix of T with respect to the bases B and B'.

(b) Consider the $\mathbb{R}$-linear transformation

$$T : \mathbb{R}^3 \to \mathbb{R}^2$$
$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \mapsto \begin{pmatrix} x_1 - x_3 \\ x_2 - 2x_3 \end{pmatrix}.$$

Suppose B is the canonical basis for $\mathbb{R}^3$, and B' is the canonical basis for $\mathbb{R}^2$. Work out the matrix $M_{B,B'}(T)$. (Note: in class, we also denoted this matrix simply $M(T)$ since B, B' are the canonical bases.)

Problem 3. Suppose V, W are finite-dimensional k-vector spaces for some field k, and $T\colon V \to W$ is a k-linear transformation.

(a) State the rank-nullity theorem for T.

(b) Prove the following statement using the rank-nullity theorem: if V, W above have the same dimension and T is injective, then T must be a k-vector space isomorphism.

8.4 Mathematical Background

- Part (a) of Problem 1 requires the student to know the definition of a linear transformation and check if a specific example satisfies it.
- Part (b) of Problem 1 requires the student to be aware of one of the main facts of linear algebra: that linear transformations can be represented by matrices, and the student must also check that all the details of this formulation are correct.
- Part (c) of Problem 1 requires the student to have enough working experience with inner product spaces to recall that two nonzero vectors can have a vanishing inner product (i.e., be orthogonal).
- Part (d) of Problem 1 requires the student to be able to compute the characteristic polynomial of a matrix.
- Part (a) of Problem 2 requires the student to understand the theory of how a matrix is used to represent a linear transformation, and part (b) requires the student to apply this theory to compute the matrix representation in a concrete example.
- Part (a) of Problem 3 requires the student to remember the rank-nullity theorem for linear transformations, and part (b) requires the student to write a proof that uses this theorem in concert with two other concepts (that the nullity of an injective linear transformation is 0, and that a linear transformation whose rank is equal to the dimension of the codomain is surjective) that the student must recall.

8.5 Assessment Procedure

Problem 1 consisted of four true/false parts, and each was given a score of 0 (correct) or 1 (incorrect), and these individual scores were totalled to give an overall score of 0 to 4. The following scheme was used to interpret this score, and to understand the lowest bracket, note that answering four true/false questions at random will produce an average score of 2:

Student will answer four multiple choice questions on theoretical and practical linear algebra.

Score	0 to 2	3	4
Description	Demonstrates negligible knowledge linear algebra concepts	Demonstrates some knowledge of these linear algebra concepts	Demonstrates full knowledge of these linear algebra concepts

The formal assessment of the student work on Problem 2 was based on the following **scoring rubric**:

Student will answer a two part question on the theory and practice of representing a linear transformation by a matrix.

Score	0	1	2	3
Description	Demonstrates no relevant idea about how to use linear algebraic ideas to solve any part the problem.	Presents at least one relevant idea, but falls far short of what is needed to solve all parts of the problem.	Presents a solution that contains substantial correct ideas, but not a fully correct solution.	Presents valid and clear solutions to all parts, with no flaws or very minor flaws (e.g., simple calculation error).

The formal assessment of the student work on Problem 3 was based on the following **scoring rubric**:

Student will recall the rank-nullity theorem for linear transformations and apply it to prove a proposition relating the injectivity and the dimensions of the domain and codomain of a linear transformation to the bijectivity of the same transformation.

Score	0	1	2	3
Description	Demonstrates no relevant idea about how to use linear algebraic ideas to solve any part the problem.	Presents at least one relevant idea, but falls far short of what is needed to solve all parts of the problem.	Presents a solution that contains substantial correct ideas, but not a fully correct solution.	Presents valid and clear solutions to all parts, with no flaws or very minor flaws (e.g., simple calculation error).

8.6 Results, Analysis, and Recommendations

The scoring for formal assessment, based on the above scoring rubrics, was performed by two full-time faculty of the Department of Mathematics on the 16 exams that were turned in. There was also an additional make-up exam, but it had somewhat different questions, and it was decided only to mark the regular exams for the sake of consistency.

The distribution of scores for the three problems is presented in Figure 8.1 below. This figure also includes a breakdown of Problem 1 into its four parts, indicating the frequency of correct answers on each part, which is also tabulated here:

Problem 1	Part (a)	Part (b)	Part (c)	Part (d)
Rate of Correct Answer	56.25%	75.00%	68.75%	81.25%

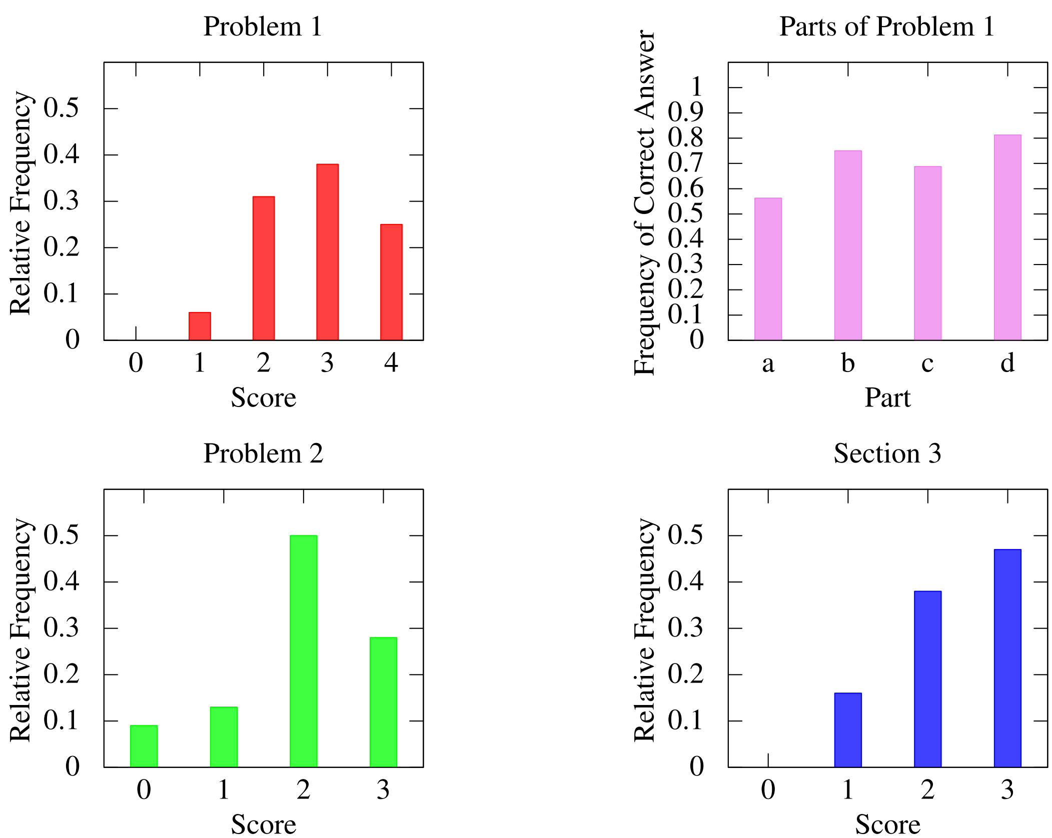


Figure 8.1: Score distribution for assessment of Math Bachelor's SLO #3 in Math 462, Spring 2017

The following are the average scores for each problem.

	Problem 1	Problem 2	Problem 3
Average Score	2.81	1.97	2.31

The results for Problem 2 confirm that the procedure of *representing* abstract objects (vectors from abstractly defined vector spaces and their transformations) using concrete objects (matrices) remains a challenge for Math 462 students, as it did for Math 262 students. (Cf. §7.6 above.) Part (b) of Problem 1 concerns this same topic: the representation of linear transformations by matrices. The students answered this true/false question correctly with a frequency of 75%. This indicates that the general notion is clear to most students, but they have difficulty with the details of representing a linear transformation by a matrix.

Students did somewhat better on Problem 3, which is about the abstract theory of linear transformations. This shows that students' difficulty is not necessarily with abstraction *per se*, but with the representation process that is the bridge between the abstract and the concrete.

Part (a) of Problem 1 caused the most difficulty for the students, as compared to the other parts. Only 56.25% answered correctly, indicating that student shave difficulty applying the definition of linear transformation to a concrete example. Problem (c) of Problem 1 also caused difficulty for many students, and this one is most simply approached by thinking of a counterexample to the claim. It suggests that students need to work more on remembering basic examples to illustrate the definitions that they memorize. Most students were able to correctly answer part (d) of Problem 1, which requires them to perform a straightforward calculation and check if their result matches the proposed result.

The overall results show that, on average, students are finishing Math 462 with substantial but imperfect knowledge of linear algebraic concepts. The students' performance on specific questions suggests that more emphasis should be placed on making the connection between abstract concepts of linear algebra and their concrete manifestations. This is best done by illustrating the theory with examples whenever a new concept is introduced.

Part III

Mathematics Graduate Program Assessment

Chapter 9

Introduction

We conducted an assessment of the Mathematics Master's program.

- **SLO #2 for M.S. in Mathematics, and at the same time SLO #3 for M.S in Applied Mathematics: Demonstrate proficiency in general topology, including metric spaces.**

1. In Fall of 2016 we measured student work in the Math 501 course (Topology) to assess these SLOs in the course where these concepts are introduced and practiced.
2. We analyzed these measurements for this year and made recommendations to the department.

The assessment of the Mathematics Master's program SLO #2 for the M.S. in Mathematics and SLO #3 for the M.S. in Applied Mathematics continues our assessment of the SLOs that we recently revised in the 2015–16 academic year. We plan to continue assessing these SLOs until we assemble a full picture of the current state of the program. Topology is a foundational course in the master's program, as many of our graduate courses require it as a prerequisite, and having a good background in topology makes many other courses easier for the student. For this reason, entering students are encouraged to take this course as soon as possible.

Chapter 10

Fall 2016: Measurement of Math 501 Class and Analysis

10.1 SLO Assessed

The following SLOs for the Mathematics Master's Program were assessed:

- **SLO #2 for M.S. in Mathematics, and at the same time SLO #3 for M.S in Applied Mathematics: Demonstrate proficiency in general topology, including metric spaces.**

10.2 Course Description

Math 501 (Topology) is a 3 unit course in general topology required for mathematics master's students in both the Mathematics and Applied Mathematics options. Its catalog description reads thus:

> *Metric spaces, topological spaces, compactness, completeness and connectedness. Introduction to function spaces, with emphasis on the uniform topology.*

The textbook used in the Fall 2016 was *Topology* (2nd edition) by Munkres (Pearson, 2000).

Math 501 is a graduate level course that serves as an introduction to general topology, with an emphasis on metric spaces. It provides many of the topological tools that are needed to study modern analysis. The signature assignments chosen here include two portions of problems that test the students' grasp of basic concepts such as open, closed, compact, and path-connected sets, both in a familiar metric topology and in an unfamiliar topology defined in a more abstract way.

Data were collected from the single section of this class (16475) that ran in Fall 2016.

10.3 Signature Assignment

Two portions of problems (comprising four individual parts) from the final exam for the Fall 2016 course Math 501 (Topology) were designated for assessment.

Problem 1. Consider $[0, 1)$ in the lower limit topology on $\mathbb{R}$ (that is $\mathbb{R}_\ell$)

(a) Explain why $[0, 1)$ is closed in $\mathbb{R}_\ell$

(b) Show $[0, 1)$ is not compact by producing an open cover of $[0, 1)$ with no finite subcover (justify)

Problem 2. (a) In a given topological space X for $x, y \in X$, define $x \sim y$ if there is a path in X connecting x and y. Prove $\sim$ defines an equivalence relation on X

(b) Let $X = \mathbb{R}^n$ with the standard Euclidean metric $d(x, y) = \|x - y\|$ (that is, the usual distance between the two points). Assuming the usual metric properties of this distance, prove that $B(x, \epsilon) = \{y \in X : d(x, y) < \epsilon\}$ is path connected.

10.4 Mathematical Background

- Part (a) of Problem 1 requires the student to recall what the lower limit topology is, and then apply the axioms of general topology to show that $[0, 1)$ is closed.
- Part (b) of Problem 1 requires the student to produce an open cover of $[0, 1)$ without a finite subcover.
- Part (a) of Problem 2 requires the student to remember the axioms of an equivalence relation (which the student should know from undergraduate level mathematics) and then show that the relation of path-connectedness satisfies all these axioms. This requires the student to recall the definition of a path, and to realize that constant functions are paths, traversing a path in reverse also gives a path, and the concatenation of two paths is a path.
- In Part (b) of Problem 2 the student should apply Part (a) to a familiar example: every point in the ball is path connected to the center by a radial path, hence every pair of points is connected by a path. Or the student may connect any pair of points in the ball with a straight path, but then must prove that this path lies entirely within the ball.

10.5 Assessment Procedure

Each of the four parts of the above problems was separately assessed by two independent scorers from the full time faculty using a single score determined by the following rubric:

Score	0	1	2	3
Description	No relevant work shown	Major flaws	Minor, noticeable flaws	Clear and complete, with only negligible flaws, if at all

10.6 Results, Analysis, and Recommendations

The scoring for formal assessment, based on the above scoring rubrics, was performed by two full-time faculty of the Department of Mathematics on the 22 exams that were turned in.

The distribution of scores for the three problems is presented in Figure 10.1 below.

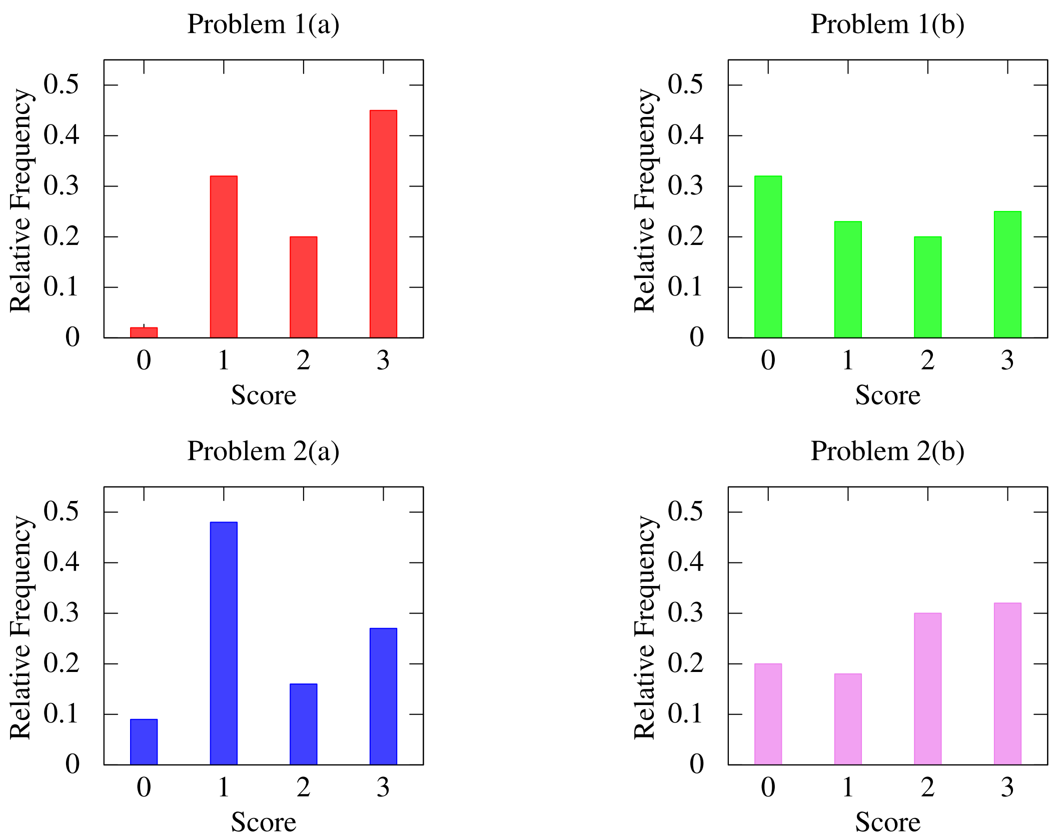


Figure 10.1: Score distribution for assessment of Mathematics M.S. SLO #2 and Applied Mathematics M.S. SLO #3 in Math 501, Fall 2016

The following are the average scores for each problem.

	Problem 1(a)	Problem 1(b)	Problem 2(a)	Problem 2(b)
Average Score	2.09	1.39	1.61	1.73

Of the four parts, students had the most success on Problem 1(a), and the least with Problem 1(b). Both of these are more basic than Problems 2(a) and 2(b): they demand that students know the relevant definitions, and once these are known, the answers are relatively short. Therefore it is interesting that Problem 1(b) gave the students the most trouble. Perhaps the students saw the construction for Problem 1(b) in class, and some of them tried to write down what they remembered, without checking whether it made sense. The lower limit topology, while certainly not commonly encountered elsewhere in mathematics, is a useful example in this course, and the students could reasonably be expected to recall it, and seemed that they had no problem recalling the definition. If anything they may have been too comfortable with it, and as a result often just quoted some facts that it might have been better to ask them to demonstrate (e.g., that $(-\infty, 0)$ is open in the lower limit topology). In fact, a student could just correctly give an example for Problem 1(b) consisting of an open cover of $[0, 1)$ consisting of open sets in the usual topology of $\mathbb{R}$, without needing to recall much about the lower limit topology other than it includes the usual topology. Because of these issues, Problem 1(b) left room for interpretation as to whether the student was correctly recalling relevant facts about the lower limit topology or just using facts about the usual topology without justifying why they would also hold in the lower limit topology. For this reason, it might have been preferable to ask the students to show that $[0, 1]$ is not compact in the lower limit topology, since this set is compact in the usual topology (while $[0, 1)$ is compact in neither topology).

For problem 2(a), almost all students correctly recalled the definition of an equivalence relation (which is undergraduate material), but some of them had a hard time to relate it to the concept of path-connectivity, which was covered in Math 501. The transitivity property of this equivalence relation is the most difficult to check. Some of the students named the key technique ("pasting lemma") used for concatenating two paths, but did not elaborate on the specifics; others made a simplifying assumption that the given paths are linear functions, which is not justified. These answers are incomplete.

One approach to Problem 2(b) required a calculation showing that every convex combination of two points in a ball lies in the same ball, which in turn requires familiarity with the triangle inequality over normed vector spaces. Setting up the right inequalities, and interpreting the results was a challenge for some of the students, although this material was clearly covered in class. The flaws were most likely due to students trying to remember the chain of inequalities, instead of deriving them in a logical manner.

Another approach to Problem 2(b) would have been to show that a radial path in an open ball from the center an arbitrary point is contained in the ball, and then use the transitivity of the path-connectedness relation (established in Problem 2(a)) to deduce that any two points in the ball are path-connected.

We recommend that Math 501 instructors train the students to be ready to use their topological tools for new situations. Memorizing the course material is insufficient for proficiency in topology. Our homework, midterm, and final examination questions should prepare them to apply the new definitions and theorems to unfamiliar examples.

Part IV

Future Assessment Activities

Chapter 11

Preview of Planned Assessment Activities for Next Year

In the 2015–2016 academic year, the Department of Mathematics revised the undergraduate program SLOs and created graduate program SLOs. We are now assessing these new undergraduate and graduate SLOs to obtain a comprehensive view of our programs. We plan to continue to assess these new SLOs over the next few years until we have covered the full set of SLOs for each program. So we plan to assess at least one SLO in the undergraduate program and to assess at least one SLO in the graduate program in 2017–2018. During our assessment of the General Education Basic Skills SLOs in 2016–2017, we mentioned to the Coordinator for Program Review and to the Director of the Office of Academic Assessment and Program Review that the General Education Basic Skills SLOs for Mathematics would benefit from revision. If the Office of Academic Assessment and Program Review is ready for us to begin this task, we may also be able to work on this revision in 2017–2018.