Math 150B Final Review, Fall 2019. Answers.

1. 7.3) 5, 23-33, 77-80, 85-86, 91-98. , 7.5) 49-69, 104-105.
   Find the Derivatives.

   (a) $2x\log_2(x^3) + \dfrac{3x}{\ln(2)}$

   (b) $\dfrac{1}{\ln(3)}\left(\dfrac{2}{x} - \dfrac{1}{x+1}\right)$

   (c) $\dfrac{\ln(3)x^3 3^x - 3x^2 3^x}{x^6}$

   (d) $\dfrac{2}{x\sqrt{x^4-1}}$

   (e) $-\dfrac{1}{x^2+1}$

   (f) $\dfrac{d}{dx}(\arccos(5^x))\ \dfrac{-\ln(5)5^x}{\sqrt{1-5^{2x}}}$

2. 7.3) 51-56, 57-64, 69.
   Find the derivatives.

   (a) $\left(\dfrac{\ln(x)}{2\sqrt{x}} + \dfrac{1}{\sqrt{x}}\right) x^{\sqrt{x}}$

   (b) $(\ln(\sin(x))$ + $x\cot(x))(\sin(x))^x$

   (c) $\left(\dfrac{6x}{2x+1} + 3\ln(2x+1)\right)(2x+1)^{3x}$

3. 7.3) 8-10, 38-44, 99-103. , 7.5) 77-89.
   Find the antiderivatives.

   (a) $\dfrac{5^{x^3-1}}{3\ln(5)} + C$

   (b) $\dfrac{1}{2\ln(2)}(\ln(x))^2 + C = \dfrac{\ln(2)}{2}(\log_2(x))^2 + C$

   (c) $\dfrac{1}{2}\arcsin(x^2) + C$

   (d) $-\arctan(\cos(x)) + C$

   (e) $\operatorname{arcsec}(|3x|) + C$

   (f) $\dfrac{1}{\ln(3)}\arcsin(3^x) + C$

4. 7.6) 7-10, 12-16, 18, 21-29, 44-52, 54-57.
   Find the limits

   (a) 1

   (b) e^{-3}

   (c) 7

   (d) $e^{.5}$

5. 7.6) 31-42.
   Evaluate $\lim_{x\to\infty} \frac{f(x)}{g(x)}$, and determine if $f(x)$ grows faster than $g(x)$, $g(x)$ grows faster than $f(x)$, or they grow at the same rate.

   (a) $\lim_{x\to\infty} \dfrac{2^x}{x^3} = +\infty$, f faster.

   (b) $\lim_{x\to\infty} \dfrac{\ln(x^3)}{(\ln(x))^2} = 0$. g faster.

6. 8.2) 2-3, 9-37, 39-40, 42-46, 48, 59-64.
   Find the antiderivatives

   (a) $\frac{1}{2}x^2\sin(2x)+\frac{1}{2}x\cos(2x)-\frac{1}{4}\sin(2x)+C$
   (b) $x\arcsin(x)+\sqrt{1-x^2}+C$
   (c) $-\frac{\ln(x)}{x}-\frac{1}{x}+C$
   (d) $x(\ln(x))^2-2x\ln(x)+2x+C$
   (e) $-\frac{1}{10}e^x\cos(3x)+\frac{3}{10}e^x\sin(3x)+C$
   (f) $-2\sqrt{x}\cos(\sqrt{x})+2\sin(\sqrt{x})+C$

7. 8.3) 9-24, 27-30, 33-40, 43-45, 53, 56, 62, 64, 67-70, 71.
   Find the antiderivatives

   (a) $\frac{1}{5}\sin^5(x)-\frac{2}{3}\sin^3(x)+\sin(x)+C$
   (b) $(\cos(x))^{-1}+\cos(x)+C$
   (c) $\frac{1}{8}x-\frac{1}{32}\sin(4x)+C$
   (d) $\frac{1}{3}\sec^3(x)-\sec(x)+C$

8. 8.4) 1-3, 7-15, 18-27, 30-33, 35-44, 60-63.
   Find the antiderivatives

   (a) $\ln\left(\frac{1}{x}-\frac{1}{\sqrt{x^2+1}}\right)+C$ or $-\ln\left(\frac{1}{x}+\frac{1}{\sqrt{x^2+1}}\right)+C.$
   (b) $\frac{x}{2}\sqrt{4-x^2}+2\arcsin\left(\frac{x}{2}\right)+C$

9. 8.5) 4, 5-12, 17-22, 23-41, 43-56, 58.
   Find the antiderivatives

   (a) $\ln(x+3)+\ln(x-3)-\ln(x)+C$
   (b) $\frac{1}{x+1}-\ln(x+1)+\ln(x-1)+C$
   (c) $\frac{1}{2}\ln(x^2-6x+10)+5\arctan(x-3)+C$
   (d) $\frac{1}{3}x^3+\frac{1}{2}x^2+3x+3\ln(x-1)+C$

10. More Review Integrals.

   (a) $x\tan(x)+\ln(\cos(x))+C$
   (b) $\frac{1}{3}\arctan(e^{3x})+C$
   (c) $\frac{1}{\ln(2)}(x\ln(x)-x)+C$
   (d) $2\ln(x)-\ln(x^2+1)+\arctan(x)+C$

11. 8.9) 2-4, 6, 7-28, 30-33, 35-55, 57-58.
   First express the improper integral as a limit. Then evaluate or show that the integral diverges.

(a) $\lim_{a\to-\infty}\int_a^0 \frac{x}{x^2+1}\,dx + \lim_{b\to\infty}\int_0^b \frac{x}{x^2+1}\,dx$. Diverges.

(b) $\lim_{b\to\infty}\int_1^b \frac{1}{x(x+2)}\,dx = \frac{\ln(3)}{2}$. (You need to combine logs before taking limit.)

(c) $\lim_{c\to 1^-}\int_0^c \frac{1}{\sqrt{1-x^2}}\,dx = \frac{\pi}{2}$.

(d) $\lim_{c\to 3^-}\int_0^c \frac{2}{(x-3)^2}\,dx + \lim_{c\to 3^+}\int_c^5 \frac{2}{(x-3)^2}\,dx$. Diverges.

12. 8.9) 65-76.

(a) $\pi\int_1^\infty (3^{-x})^2\,dx = \pi\int_1^\infty 3^{-2x}\,dx = \frac{\pi}{18\ln(3)}$.

(b) $2\pi\int_0^\infty x3^{-x}\,dx = \frac{2\pi}{(\ln(3))^2}$.

13. 10.1) 8-12, 35, 37, 39-40, 49-50.

10.2) 1-4, 6-12, 13-20, 22-42, 44-48, 55-56, 61-63, 65-67, 75-79.

Find the limits or show that they diverge.

(a) $\frac{1}{2}$

(b) 0

(c) e

(d) $+\infty$

14. 10.3) 21-42, 46-53, 54-55, 57, 59-62, 65-69, 75-77, 80-86, 87.

Find the sums.

(a) $\frac{7}{12}$

(b) $\frac{1}{2}$

(c) 1

(d) $\frac{1}{3}$

(e) $\frac{8}{5}$

(f) $\frac{1}{2}$

15. Find as a fraction by evaluating a geometric series.

(a) 2

(b) $\frac{70}{99}$

16. (10.4) 1-6, 9-16, 17-22, 23-33, 36-38, 49-63.
(10.5) 9-19, 23-28, 32, 34, 40-41, 43, 45-47, 50, 54, 59.
(10.7) 9-10, 13-17, 19-20, 28, 33, 37, 45, 47-48, 58-63.
Determine if the series converge or diverge.

(a) Converge. Integral test.
(b) Diverge. Regular comparison test.
(c) Converge. Integral test. (Or ratio test.)
(d) Diverge. Regular comparison test.
(e) Converge. Limit comparison test.
(f) Diverge. Ratio, regular comparison, or divergence test.

17. (10.6) 11-22, 27, 45-54, 57-63.
Determine if the series converge absolutely, converge conditionally, or diverge.

(a) Conditionally converges. Integral test.
(b) Diverges. Divergence test.
(c) Absolutely converges. Ratio test.
(d) Diverges. Divergence test.
(e) Absolutely converges. Limit comparison test.
(f) Conditionally converges. Limit comparison test.

18. (11.1) 9-16, 17-24, 25-28, 33-38, 47-52, 53-58.
Find the quadratic approximation to $f(x)$ around $x = a$ (2nd order Taylor approximation). Then use this to approximate the given value.

(a) $f(x) \approx \frac{1}{2} - \frac{1}{4}x + \frac{1}{8}x^2$. $\frac{1}{2.1} \approx \frac{1}{2} - \frac{1}{4}(.1) + \frac{1}{8}(.1)^2 = .47625$.
(b) $f(x) \approx 5 + \frac{1}{10}(x-25) + \frac{1}{1000}(x-25)^2$ $\sqrt{28} \approx 5 + \frac{1}{10}(3) - \frac{1}{1000}(3^2) = 5.291$.
(c) $f(x) \approx \frac{\pi}{4} + \frac{1}{2}(x-1) - \frac{1}{4}(x-1)^2$. $\arctan(2) \approx \frac{\pi}{4} + \frac{1}{2} - \frac{1}{4} = \frac{\pi+1}{4}$.

19. (11.2) 9-18, 20-24, 26-27, 29-32, 34, 36.
Find the radius of convergence of each power series

(a) $\rho = 0$
(b) $\rho = 1$
(c) $\rho = 3$

Find the interval of convergence of each power series.

(a) $[-1, 5)$
(b) $(0, 2)$
(c) $(-\infty, \infty)$

20. 11.2 41-46, 51-56, 57-59 , 11.3) 35-42, 68-69, 72

Know the power series for $\frac{1}{1-x}$, e^x, $\cos(x)$, and $\sin(x)$

Find a power series expansion of $f(x)$ from known series. (At least 3 nonzero terms.)

(a) $f(x) = x - x^3 + x^5 + ...$

(b) $f(x) = \dfrac{2}{3} + \dfrac{2}{9}x + \dfrac{2}{27}x^2 + ...$

(c) $f(x) = 1 - \dfrac{1}{2}x + \dfrac{1}{24}x^2 + ...$

(d) $f(x) = x + \dfrac{1}{2}x^3 + \dfrac{1}{6}x^5 + ...$

(e) $x^3 - \frac{1}{6}x^7 + \frac{1}{120}x^{11} - ...$

(f) $x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - ...$

21. 11.2) 67-71 , 11.4) 55-61.

Find $f(x)$ by evaluating the series.

(a) $f(x) = \dfrac{2}{3-x}$

(b) $f(x) = \dfrac{1}{1+x^2}$

(c) $f(x) = \dfrac{e^{-x}}{1-e^{-x}}$

22. 11.3) 9-13, 16-18, 20-22, 25-26, 27-33, 76-78

Find the Taylor series for $f(x)$ at $x = a$, up to the first four nonzero terms.

(a) $1 - (\ln(2))x + \dfrac{1}{2}(\ln(2))^2x^2 - \dfrac{1}{6}(\ln(2))^3x^3 + ...$

(b) $32 + 20(x-4) + \dfrac{15}{4}(x-4)^2 + \dfrac{5}{32}(x-4)^3 + ...$

(c) $1 - 2\left(x - \dfrac{\pi}{4}\right)^2 + \dfrac{2}{3}\left(x - \dfrac{\pi}{4}\right)^4 + ...$

23. 11.4) 25-32, 37-39, 41-42, 44

Estimate $\int_a^b f(x)\,dx$ using a power series expansion for $f(x)$ with two nonzero terms.

(a) $\frac{11}{18}$

(b) $\frac{17}{9}$

24. 12.1) 11-14, 15-30, 31-35, 37-46, 49, 67-68, 73-76, 81-85.

Eliminate the parameter t. Then graph the parametric curve.

(a) $x = y^2 - 2y$

(b) $y = \frac{1}{(x-3)^2}$

(c) $x^2 + (y-2)^2 = 9$

(d) $y = 1 - x^2$

25. Find the tangent line to the parametric curve at the given value of t.

(a) $y = 3x - 4$.

(b) $y = \frac{1}{2}x + \frac{\pi}{4}$

26. (12.2) 9-14, 15-22, 25-30, 31-36, 37-43, 47-48, 49-52, 53-54, 56, 57-58, 63, 97-98.
Convert the polar equation to cartesian. Then graph.

(a) $x^2 + (y-3)^2 = 9$ (b) $2x + y = 3$

27. (12.3) 11-15, 20, 33-39, 41, 43, 45-50, 53-56, 63-69.
Find the tangent line to the polar curve at the given value of θ.

(a) $y = \frac{1}{3}(x-1)$ (b) $y = -x + 2\sqrt{2}$

Find the arclength of the polar curves.

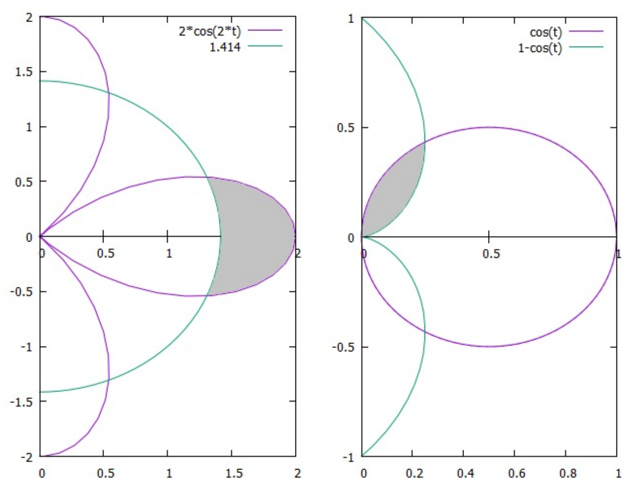
(a) $\frac{19}{3}$.

(b) $7\frac{\sqrt{1+(\ln(2))^2}}{\ln(2)}$.

Find the areas.

(a) $\frac{1}{2}$

(b) $\frac{\pi}{24} - \frac{\sqrt{3}}{16} + \frac{\pi}{4} - \frac{7\sqrt{3}}{16}$



28. 13.1) 21-23, 24-27, 28-31, 32-40, 41-46, 49-52, 60-63.

$\langle 3, 4\rangle$, $\langle \frac{3}{5}, \frac{4}{5}\rangle$.

29. 13.2) 13-14, 15-17, 19, 23-28, 29-38, 39-43, 45-49, 54, 70-73.

(a) Center $(1, 0, -2)$, radius 4.

(b) $(x-2)^2 + (y-2)^2 + (z-3)^2 = 9$.

30. 13.3)19-28, 30, 35-40, 52-55, 62-65.

(a) $\theta = \frac{\pi}{6}$.

(b) $\theta = \frac{3\pi}{4}$.

31. 13.4) 5-7, 23-28, 29-32, 33-38, 42-44

(a) $\langle \frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}, -\frac{2}{\sqrt{14}}\rangle$, $\langle -\frac{3}{\sqrt{14}}, -\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}\rangle$.

(b) $A = \sqrt{14}$.

32. $A = \frac{\sqrt{11}}{2}$.

33. 14.1: 9-14, 15-23.

(a) $\langle 1+2t, 2-t, 3+3t\rangle$, $0 \leq t \leq 1$.

(b) $\langle 3+t, -1+2t, 1-t\rangle$.

(c) $\langle 1-5t, -1, 2+3t\rangle$.

34. 14.2: 9-16, 17-22, 23-28, 29-32, 53-56, 59-64, 65-70, 71-76, 80-83.
$\vec{r}(t) = \langle \ln(2+t), 2t, t^2\rangle$.

(a) $\vec{T}(t) = \frac{1}{\sqrt{(2+t)^{-2}+4+4t}} \quad \frac{1}{2+t}, 2, 2t \quad .$

(b) $T(-1) = (1/3)\left\langle 1, 2, -2\right\rangle$.

(c) $\vec{\ell}(t) = \quad \frac{1}{3}t, -2+\frac{2}{3}t, 1-\frac{2}{3}t$ or $\vec{\ell}(t) = \langle t, -2+2t, 1-2t\rangle$.

35. 14.3: 9-20, 29-32, 35-40, 47-50.

A hummingbird has acceleration $\vec{a}(t) = \cos(t)\vec{i} + \sin(t)\vec{j} + 6t\,\vec{k}\ m/s^2$. Suppose it has an initial velocity of $\vec{v}(0) = 2\,\vec{k}\ m/s$ and an initial position of $\vec{2}\,i - \vec{k}\ m$.

(a) $\vec{v}(t) = \langle \sin(t), -\cos(t) + 1, 3t^2 + 2 \rangle$.

(b) $s(t) = \sqrt{\sin^2(t) + (-\cos(t) + 1)^2 + (3t^2 + 2)^2}$.

(c) Find the position of the hummingbird at time t. $\vec{r}(t) = \langle -\cos(t) + 3, -\sin(t) + t, t^3 + 2t - 1 \rangle$.

36. 14.4: 5-8, 9-20, 22, 23-25, 33-38, 41-42.

Find the arclength of the parametric curves.

(a) 72.

(b) 10.