Math 150B Final Review, Fall 2019.

1. 7.3) 5, 23-33, 77-80, 85-86, 91-98. , 7.5) 49-69, 104-105.
Find the Derivatives.

(a) $\frac{d}{dx} x^2 \log_2(x^3)$
(b) $\frac{d}{dx} \log_3\left(\frac{x^2}{x+1}\right)$
(c) $\frac{d}{dx}\left(\frac{3^x}{x^3}\right)$
(d) $\frac{d}{dx}(\operatorname{arcsec}(x^2))$
(e) $\frac{d}{dx} \arctan\left(\frac{1}{x}\right)$
(f) $\frac{d}{dx}(\arccos(5^x))$

2. 7.3) 51-56, 57-64, 69.
Find the derivatives.

(a) $\frac{d}{dx} x^{\sqrt{x}}$
(b) $\frac{d}{dx}(\sin(x))^x$
(c) $\frac{d}{dx}(2x+1)^{3x}$

3. 7.3) 8-10, 38-44, 99-103. , 7.5) 77-89.
Find the antiderivatives.

(a) $\int x^2 5^{x^3-1}\,dx$
(b) $\int \frac{\log_2(x)}{x}\,dx$
(c) $\int \frac{x}{\sqrt{1-x^4}}\,dx$
(d) $\int \frac{\sin(x)}{1+\cos^2(x)}\,dx$
(e) $\int \frac{1}{x\sqrt{9x^2-1}}\,dx$
(f) $\int \frac{3^x}{\sqrt{1-9^x}}\,dx$

4. 7.6) 7-10, 12-16, 18, 21-29, 44-52, 54-57.
Find the limits

(a) $\lim_{x\to+\infty} x^{\frac{1}{x}}$
(b) $\lim_{x\to+\infty} \left(1-\frac{1}{x}\right)^{3x}$
(c) $\lim_{x\to+\infty} (2+7^x)^{\frac{1}{x}}$
(d) $\lim_{x\to0^+} \left(\sqrt{x}\right)^{\frac{1}{\ln(x)}}$

5. 7.6) 31-42.
Evaluate $\lim_{x\to\infty} \frac{f(x)}{g(x)}$, and determine if $f(x)$ grows faster than $g(x)$, $g(x)$ grows faster than $f(x)$, or they grow at the same rate.

(a) $f(x) = 2^x$; $g(x) = x^3$
(b) $f(x) = \ln(x^3)$; $g(x) = (\ln(x))^2$

6. 8.2) 2-3, 9-37, 39-40, 42-46, 48, 59-64.
   Find the antiderivatives

   (a) $\int x^2 \cos(2x)\,dx$
   (b) $\int \arcsin(x)\,dx$
   (c) $\int \frac{\ln(x)}{x^2}\,dx$
   (d) $\int (\ln(x))^2\,dx$
   (e) $\int e^{-x}\cos(3x)\,dx$
   (f) $\int \sin(\sqrt{x})\,dx$

7. 8.3) 9-24, 27-30, 33-40, 43-45, 53, 56, 62, 64, 67-70, 71.
   Find the antiderivatives

   (a) $\int \cos^5(x)\,dx$
   (b) $\int \frac{\sin^3(x)}{\cos^2(x)}\,dx$
   (c) $\int \sin^2(x)\cos^2(x)\,dx$
   (d) $\int \sec(x)\tan^3(x)\,dx$

8. 8.4) 1-3, 7-15, 18-27, 30-33, 35-44, 60-63.
   Find the antiderivatives

   (a) $\int \frac{1}{x\sqrt{x^2+1}}\,dx$
   (b) $\int \sqrt{4-x^2}\,dx$

9. 8.5) 4, 5-12, 17-22, 23-41, 43-56, 58.
   Find the antiderivatives

   (a) $\int \frac{x^2+9}{x^3-9x}\,dx$
   (b) $\int \frac{x+3}{(x+1)^2(x-1)}\,dx$
   (c) $\int \frac{x+2}{x^2-6x+10}\,dx$
   (d) $\int \frac{x^3+2x}{x-1}\,dx$

10. More Review Integrals.

   (a) $\int x\sec^2(x)\,dx$
   (b) $\int \frac{e^{3x}}{1+e^{6x}}\,dx$
   (c) $\int \log_2(x)\,dx$
   (d) $\int \frac{x+2}{x(x^2+1)}\,dx$

11. 8.9) 2-4, 6, 7-28, 30-33, 35-55, 57-58.
   First express the improper integral as a limit. Then evaluate or show that the integral diverges.

(a) $\displaystyle\int_{-\infty}^{\infty} \frac{x}{x^2+1}\,dx$

(b) $\displaystyle\int_{1}^{\infty} \frac{1}{x(x+2)}\,dx$

(c) $\displaystyle\int_{0}^{1} \frac{1}{\sqrt{1-x^2}}\,dx$

(d) $\displaystyle\int_{0}^{5} \frac{2}{(x-3)^2}\,dx$

12. 8.9) 65-76.

    (a) Find the volume given by revolving the region between $y = 3^{-x}$ and $y = 0$, from $x = 1$ to $x = \infty$ around the x-axis.

    (b) Find the volume given by revolving the region between $y = 3^{-x}$ and $y = 0$, from $x = 0$ to $x = \infty$ around the y-axis.

13. 10.1) 8-12, 35, 37, 39-40, 49-50.
    10.2) 1-4, 6-12, 13-20, 22-42, 44-48, 55-56, 61-63, 65-67, 75-79.
    Find the limits or show that they diverge.

    (a) $\displaystyle\lim_{n\to\infty} \frac{n}{\sqrt{4n^2+1}}$

    (b) $\displaystyle\lim_{n\to\infty} (.9)^n$

    (c) $\displaystyle\lim_{n\to\infty} \left(\frac{n+1}{n}\right)^n$

    (d) $\displaystyle\lim_{n\to\infty} \frac{n!}{3^n}$

14. 10.3) 21-42, 46-53, 54-55, 57, 59-62, 65-69, 75-77, 80-86, 87.
    Find the sums.

    (a) $\displaystyle\sum_{k=3}^{\infty} \left(\frac{1}{k} - \frac{1}{k+2}\right)$

    (b) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k(k+1)}$

    (c) $\displaystyle\sum_{k=0}^{\infty} \frac{2}{3^{k+1}}$

    (d) $\displaystyle\sum_{k=1}^{\infty} \frac{2}{3^{k+1}}$

    (e) $\displaystyle\sum_{k=0}^{\infty} \frac{3^k}{2^{3k}}$

    (f) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{4}{3^k}\right) - \left(\frac{3^k}{5^k}\right)$

15. Find as a fraction by evaluating a geometric series.

    (a) $3 - \frac{3}{2} + \frac{3}{4} - \frac{3}{8} + \frac{3}{16} - ...$

    (b) .70707070...

16. 10.4) 1-6, 9-16, 17-22, 23-33, 36-38, 49-63.

10.5) 9-19, 23-28, 32, 34, 40-41, 43, 45-47, 50, 54, 59.

10.7) 9-10, 13-17, 19-20, 28, 33, 37, 45, 47-48, 58-63.

Determine if the series converge or diverge.

(a) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k(\ln(k))^2}$

(b) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{\ln(k)}$

(c) $\displaystyle\sum_{k=1}^{\infty} \frac{k}{e^{2k}}$

(d) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{1+\sqrt{k}}$

(e) $\displaystyle\sum_{k=1}^{\infty} \frac{k-1}{k^3+5}$

(f) $\displaystyle\sum_{k=1}^{\infty} \frac{k^k}{k!}$

17. 10.6) 11-22, 27, 45-54, 57-63.

Determine if the series converge absolutely, converge conditionally, or diverge.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^k \frac{1}{k\ln(k)}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^k \sec\left(\frac{1}{k}\right)$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^k \frac{k^2}{2^k}$

(d) $\displaystyle\sum_{n=1}^{\infty} (-1)^k \frac{k}{\sqrt{k^2+1}}$

(e) $\displaystyle\sum_{n=1}^{\infty} (-1)^k \frac{1}{\sqrt{k^3+1}}$

(f) $\displaystyle\sum_{n=1}^{\infty} (-1)^k \frac{k}{\sqrt{k^3+1}}$

18. 11.1) 9-16, 17-24, 25-28, 33-38, 47-52, 53-58.

Find the quadratic approximation to $f(x)$ around $x = a$ (2nd order Taylor approximation). Then use this to approximate the given value.

(a) $f(x) = \frac{1}{x+2}$ at $x = 0$. $\frac{1}{2.1}$.

(b) $f(x) = \sqrt{x}$ at $x = 25$. $\sqrt{28}$.

(c) $f(x) = \arctan(x)$ at $a = 1$. $\arctan(2)$.

19. 11.2) 9-18, 20-24, 26-27, 29-32, 34, 36.

Find the radius of convergence of each power series

(a) $\displaystyle\sum_{k=0}^{\infty} \frac{k!}{3^k}(x-2)^k$

(b) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{\sqrt{k+3}}(x+1)^k$

(c) $\displaystyle\sum_{k=0}^{\infty} \frac{kx^{2k}}{9^k}$

Find the interval of convergence of each power series.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{(x-2)^k}{3^k\sqrt{k}}$

(b) $\displaystyle\sum_{k=0}^{\infty} k^2(x-1)^k$

(c) $\displaystyle\sum_{k=0}^{\infty} \frac{3^k}{k!}x^{k+2}$

20. 11.2 41-46, 51-56, 57-59 , 11.3) 35-42, 68-69, 72

Know the power series for $\frac{1}{1-x}$, e^x, $\cos(x)$, and $\sin(x)$

Find a power series expansion of $f(x)$ from known series. (At least 3 nonzero terms.)

(a) $f(x) = \dfrac{x}{1+x^2}$

(b) $f(x) = \dfrac{2}{3-x}$

(c) $f(x) = \cos(\sqrt{x})$

(d) $f(x) = \dfrac{e^{x^2}-1}{x}$

(e) $f(x) = x\sin(x^2)$

(f) $f(x) = \ln(1+x)$
(Hint: Integrate something.)

21. 11.2) 67-71 , 11.4) 55-61.

Find $f(x)$ by evaluating the series.

(a) $f(x) = \displaystyle\sum_{k=0}^{\infty} \frac{(x-1)^k}{2^k}$ (b) $f(x) = \displaystyle\sum_{k=0}^{\infty} (-1)^k x^{2k}$ (c) $f(x) = \displaystyle\sum_{k=1}^{\infty} e^{-kx}$

22. 11.3) 9-13, 16-18, 20-22, 25-26, 27-33, 76-78

Find the Taylor series for $f(x)$ at $x = a$, up to the first four nonzero terms.

(a) $f(x) = 2^{-x}$, $a = 0$.

(b) $f(x) = x^{\frac{5}{2}}$, $a = 4$.

(c) $f(x) = \sin(2x)$, $a = \dfrac{\pi}{4}$.

23. 11.4) 25-32, 37-39, 41-42, 44

Estimate $\int_a^b f(x)\,dx$ using a power series expansion for $f(x)$ with two nonzero terms.

(a) $\int_1^2 \frac{\sin(x)}{x}\,dx$

(b) $\int_{-1}^1 \cos(x^4)\,dx$

24. 12.1) 11-14, 15-30, 31-35, 37-46, 49, 67-68, 73-76, 81-85.

Eliminate the parameter t. Then graph the parametric curve.

(a) $x = t^2 - 1$, $y = t + 1$.

(b) $x = e^{-t} + 3$, $y = e^{2t}$.

(c) $x = 3\sin(t)$, $y = 3\cos(t) + 2$.

(d) $x = \cos(t)$, $y = \sin^2(t)$.

25. Find the tangent line to the parametric curve at the given value of t.

(a) $x = t^2$, $y = t^3$, $t = 2$.

(b) $x = \ln(t)$, $y = \arctan(t)$, $t = 1$.

26. 12.2) 9-14, 15-22, 25-30, 31-36, 37-43, 47-48, 49-52, 53-54, 56, 57-58, 63, 97-98.
Convert the polar equation to cartesian. Then graph.

(a) $r = 6\sin(\theta)$. (Complete the square after converting to cartesian.)

(b) $r = \frac{3}{2\cos(\theta)+\sin(\theta)}$.

27. 12.3) 11-15, 20, 33-39, 41, 43, 45-50, 53-56, 63-69.
Find the tangent line to the polar curve at the given value of θ.

(a) $r = e^{3\theta}$, $\theta = 0$.

(b) $r = 2\sin(2\theta)$, $\theta = \frac{\pi}{4}$.

Find the arclength of the polar curves.

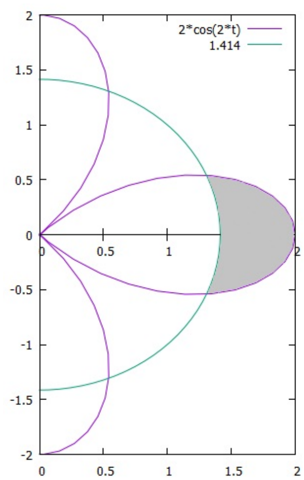
(a) $r = \theta^2$ from $\theta = 0$ to $\theta = \sqrt{5}$.

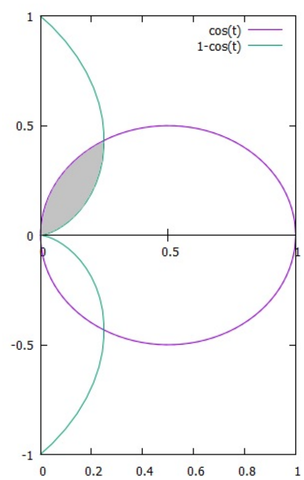
(b) $r = 2^\theta$ from $\theta = 0$ to $\theta = 3$.

Find the areas.

(a) Find the area between $r = 2\cos(2\theta)$ and $r = \sqrt{2}$ indicated in the picture (left).

(b) Find the area between $r = \cos(\theta)$ and $r = 1 - \cos(\theta)$ indicated in the right picture (right).





28. 13.1) 21-23, 24-27, 28-31, 32-40, 41-46, 49-52, 60-63.

    $P = (2, -1)$, $Q = (5, 3)$. Find $\overrightarrow{PQ}$ and a unit vector parallel to $\overrightarrow{PQ}$.

29. 13.2) 13-14, 15-17, 19, 23-28, 29-38, 39-43, 45-49, 54, 70-73.

    (a) Find the center and radius of the sphere by completing the square. $x^2 - 2x + y^2 + z^2 + 4z = 11$.

    (b) Find an equation for the sphere with center at the midpoint of P and Q, which passes through both P and Q. $P = (3, 4, 5)$, $Q = (1, 0, 1)$.

30. 13.3)19-28, 30, 35-40, 52-55, 62-65.

    (a) Find the angle between the vectors. $\vec{v} = 2\vec{i} - \vec{j} + \vec{k}$, $\vec{w} = \vec{i} + \vec{k}$.

    (b) For the triangle PQR, use vectors to find the angle formed at the vertex P. $P = (1, 2)$, $Q = (8, 1)$, $R = (-2, 6)$.

31. 13.4) 5-7, 23-28, 29-32, 33-38, 42-44

    (a) Find two unit vectors orthogonal to both $\vec{v} = \langle 1, 1, 2\rangle$ and $\vec{w} = \langle 1, -1, 1\rangle$.

    (b) Find the area of a parallelogram with side lengths $\vec{v}$ and $\vec{w}$.

32. Find the area of the triangle with vertices at $P = (0, 1, 2)$, $Q = (2, 0, 1)$, $R = (1, 1, 3)$.

33. 14.1: 9-14, 15-23.

    (a) Find the equation for the line segment from $(1, 2, 3)$ to $(3, 1, 6)$.

    (b) Find the equation for a line through $(3, -1, 1)$, parallel to the line $\vec{r}(t) = (7 + t)\vec{i} + 2t\vec{j} + (2 - t)\vec{k}$.

    (c) Find the equation for a line through $(1, -1, 2)$, perpendicular to the line $\vec{r}(t) = \langle 2 + 3t, 1 + 2t, -3 + 5t\rangle$ and the y-axis.

34. 14.2: 9-16, 17-22, 23-28, 29-32, 53-56, 59-64, 65-70, 71-76, 80-83.
    $\vec{r}(t) = \langle \ln(2 + t), 2t, t^2\rangle$.

    (a) Find the unit tangent vector $T(t)$.

    (b) Find the unit tangent vector at $t = -1$.

    (c) Find the tangent line to $\vec{r}(t)$ at $t = -1$.

35. 14.3: 9-20, 29-32, 35-40, 47-50.

A hummingbird has acceleration $\vec{a}(t) = \cos(t)\vec{i} + \sin(t)\vec{j} + 6t\,\vec{k}\ m/s^2$. Suppose it has an initial velocity of $\vec{v}(0) = 2\,\vec{k}\ m/s$ and an initial position of $\vec{2}\,i - \vec{k}\ m$.

(a) Find the velocity of the hummingbird at time t.

(b) Find the speed of the hummingbird at time t.

(c) Find the position of the hummingbird at time t. $\vec{r}(t) = \langle -\cos(t) + 3, -\sin(t) + t, t^3 + 2t - 1\rangle$.

36. 14.4: 5-8, 9-20, 22, 23-25, 33-38, 41-42.

Find the arclength of the parametric curves.

(a) $\vec{r}(t) = \langle e^{2t} + 1, 2e^{2t} - 1, 2e^{2t}\rangle$ from $t = 0$ to $t = \ln(5)$.

(b) $\vec{r}(t) = \langle 4\sin(t), 3\sin(t), 5\cos(t)\rangle$ from $t = 0$ to $t = 2$.