Math 150A. Final Review Answers, Fall 2019.

- Definition of Limits. 2, 3-6, 15, 17-18, 19-23, 25-26, 34, 45-49, 50-52.

  1. Find the values, or state they do not exist.

     (a) 3 (b) 1 (c) DNE (d) 1 (e) 2 (f) 2 (g) 2 (h) 4

  2. $\lim_{x\to 0^-} f(x) = -2$, $\lim_{x\to 0^+} f(x) = 2$, $\lim_{x\to 0} f(x) = DNE$, $f(0) = 1$, and $f(1) = 2$.

- Computing limits. 7-12, 15-16, 19-41, 45-51, 53, 55-56, 59-60, 62, 71, 81-83, 85, 87-89, 90.

  1. Find the limits (Show work.)

     (a) $\frac{1}{5}$ (b) 12 (c) $\frac{1}{4}$

  2. $\lim_{x\to 3^-} f(x) = \sqrt{25-9} = 4$, $\lim_{x\to 3^+} f(x) = 6 - 2 = 4$. So $\lim_{x\to 3} f(x) = 4$.

- Infinite Limits. 6-10, 17-18, 19-20, 21-28, 31-36, 39-42, 47-50.

  1. Find the limits (Show work).

     (a) $-\infty$ (b) $+\infty$ (c) $+\infty$ (d) $-\infty$.

- Limits at Infinity. 3-10, 17-19, 25-31, 33-34, 35, 37-44, 46, 48, 74-75.

  1. Find the limits (Show work).

     (a) 0 (b) 3 (c) $-\infty$ (d) 2

- Continuity. 2.6) 5-8, 21-24, 25-30, 31-38, 61-64, 65-68, 81, 82-83.

  1. $A = \frac{2}{3}$.
  2. $f(1) = 1 - 1 = 0$ and $f(2) = 8 - 2 = 6$. Since $f(x)$ is continuous on $[1, 2]$, $f(1) < 3$, and $f(2) > 3$, the Intermediate Value Theorem guarantees there is c in $(1, 2)$ with $f(c) = 3$.

- Precise Definition of Limit. 2.7) 5, 9-12, 15-16, 19-24, 27-30, 32-35.

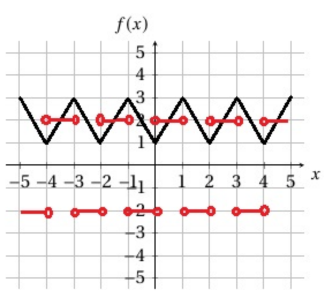
1. Let $f(x) = 2x - 1$. Then $|f(x) - 3| = |(2x-1) - 3| = |2x - 4| = 2|x-2|$. Choose $\delta = \frac{\varepsilon}{2}$. Then $|x - 2| < \delta$ implies $|f(x) - 3| < \varepsilon$.

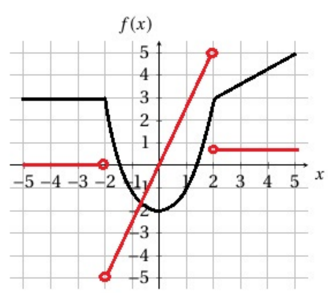
- Introducing the Derivative. 3.1) 6-10, 15, 17, 21-31, 33-42, 43-46, 56-61.

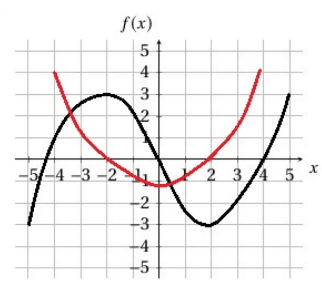
  1. Find $f'(a)$ from the limit definition of derivative, and the tangent line through $(a, f(a))$.
     (a) $f'(1) = 5$, $y = 5x - 1$.
     (b) $f'(3) = 1$, $y = x + 5$.
  2. Find $f(x)$ and a such that the given limit is $f'(a)$.
     (a) $f(x) = \sqrt{x+1}\,,\ a = 3$.
     (b) $f(x) = \cos(x)\,,\ a = 0$.

- The Derivative as a Function. 3.2) 5-10, 17-20, 21-29, 35-40, 45-46, 48-51, 55-56.

  1. Find $f'(x)$ from the limit definition of derivative.
     (a) $f'(x) = 3x^2 - 1$.
     (b) $f'(x) = \frac{-3}{(x+1)^2}$.
  2. Graph $f'(x)$ from the graphs of $f(x)$ below. Where is $f(x)$ not differentiable?
     (a) f not differentiable at integers $x = n$.
     (b) f not differentiable at $x = \pm 2$.
     (c) f differentiable for all x.







- Basic Rules of Differentiation. 3.3) 7-8, 19-36, 44-52, 55-58, 59-63, 64-68, 69, 78-81.

  1. Find the derivatives.
     (a) $f'(x) = -x^{-\frac{3}{2}} + 3x^{-4}$ (b) $f'(x) = 0$
  2. Find the limit by evaluating a derivative at an appropriate value of x.

(a) $f(x) = x^4$, $f'(2) = 32$.
(b) $f(x) = x^5$, $f'(2) = 80$.
(c) $f(x) = \sqrt{x}$, $f'(1) = \frac{1}{2}$.

- The product and quotient rules. 3.4) 7-12, 16, 18, 19-43, 47-51, 57-60, 66-69.

  1. Find $f'(x)$.

     (a) $f'(x) = x^{-\frac{1}{2}}(x^5 - 2 + \frac{3}{x}) + (1 + 2x^{\frac{1}{2}})(5x^4 - 3x^{-2})$.
     (b) $f'(x) = \frac{-x^4+4x^3+2x-2}{(x^3+1)^2}$

  2. $f''(x) = \frac{18}{(x+3)^3}$.

- Derivatives of Trigonometric Functions. 3.5) 11-18, 21-22, 23-49, 52-54, 57-64, 67-70, 72-75, 83.

  1. Find the limits.

     (a) 2
     (b) 1
     (c) 1
     (d) 0

  2. Find $f''(x)$.

     (a) $f''(x) = (-x^3+6x)\sin(x)+6x^2\cos(x)$.
     (b) $f''(x) = 2\sec^2(x)\tan(x)$.

  3. $y = 2\sqrt{3}x - \frac{2\pi\sqrt{3}}{3} + 2$.

- Derivatives and Rates of Change. 3.6) 4, 15-20, 21-27, 36-37.

  1. A ball is thrown into the air and has vertical position $s(t) = -16t^2 + 16t + 4$.
     (a) $t = \frac{1}{2}$, $s(t) = 8$.
     (b) $16\sqrt{2}$ downward.

- The Chain Rule. 3.7) 1-5, 8-11, 13-22, 25-51, 55-62, 74-77, 78, 81-84.

  1. Find the derivatives.

     (a) $f'(x) = \frac{3}{2}\frac{x^2}{\sqrt{x^3-9}}$
     (b) $f'(x) = 2x\cos(5x) - 5(x^2+1)\sin(5x)$
     (c) $f'(x) = -15\frac{(x+1)^4}{(2x-1)^6}$
     (d) $f'(x) = \frac{3}{2\sqrt{x}}\sin^2(\sqrt{x})\cos(\sqrt{x})$.

- Implicit Differentiation. 3.8) 5-22,25-30,55-59,72-74.

  1. Find $\frac{dy}{dx}$.

     (a) $\frac{dy}{dx} = \frac{\sin(x)+y}{\cos(y)-x}$
     (b) $\frac{dy}{dx} = \frac{2y}{(x+y)^2+2x}$

  2. Find the tangent lines at the given points.

(a) $y = \frac{2}{3}x - \frac{1}{3}$. (b) $y = \frac{3}{2}x + \frac{1}{2}$.

- Related Rates. 3.9) 4-10, 11-27, 29, 31-33, 35-37, 39, 41-44, 46, 50.

  1. $\frac{dh}{dt} = \frac{2}{9\pi}$ *in/sec.*
  2. (a) 64 *ft/sec.*

     (b) $-\frac{12}{25}$ *rad/sec.*

- Maxima and Minima. 4.1) 11-18, 19-22, 23-40, 41-55, 59-62.

  1. Find the absolute maximum and absolute minimum on the given interval.

     (a) Min $f(2) = -3$, Max $f(3) = 1$. (b) Min $f(-1) = -\frac{1}{2}$, Max $f(1) = \frac{1}{2}$.

- The Mean Value Theorem. 4.2) 1-4, 8-10, 11-18, 19-20, 21-22, 24-26, 29-32, 36, 41-43, 47-49.

  1. If f is continuous on $[1,4]$ and differentiable on $(1,4)$ then there is $c \in (1,4)$ with $f'(c) = (f(4) - f(1))/3$. If $f(x) = x^2$ then $f'(x) = 2x$. $2c = 15/3$ gives $c = 2.5$, which is in $(1,4)$.
  2. $f'(x) = -1$ for $-1 < x < 0$ and $f'(x) = 1$ for $0 < x < 2$, but $(f(2) - f(-1))/(2 - (-1)) = \frac{1}{3}$. The MVT doesn't apply because f is not differentiable at 0, which is in $(-1, 2)$.
  3. If $f(x) = \sec^2(x) - \tan^2(x)$ then $f'(x) = 2\sec^2(x)\tan(x) - 2\tan(x)\sec^2(x) = 0$. Since $f'(x) = 0$, $f(x) = C$. But $f(0) = \sec(0) - \tan(0) = 1$. So $f(x) = 1$ for all x. $\sec^2(x) - \tan^2(x) = 1$.

- What Derivatives Tell Us. 4.3) 9-12, 19-39, 41-48, 53-56, 57-64, 66-68, 69-75, 79-84.

  1. Use the first derivative test to find all relative maxima and minima.

     (a) Rel max $f(8) = 4$,
     Rel min $f(0) = 0$.

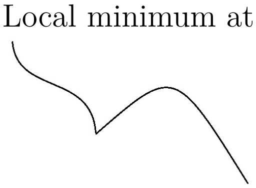
     (b) Rel max $f(2) = 4$,
     Rel min $f(-2) = -4$.

  2. Find the inflection points. Use the second derivative test to find any relative maxima or minima.

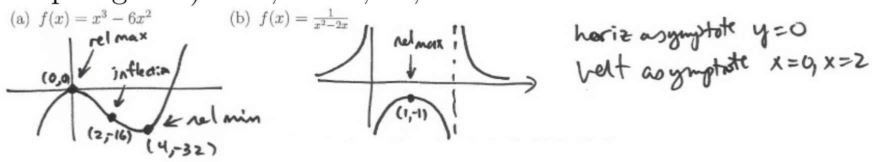
     (a) Rel max at $x = 0$, Rel min at $x = 2$,
     inflection at $x = 1$.

     (b) Rel min at $x = 0$, Rel max at $x = 4$,
     inflection at $x = 4 \pm 2\sqrt{2}$.

  3. Local minimum at $x = 0$, local maximum at $x = 3$, inflection point at $x = -2$.



- Graphing. 4.4) 7-12, 15-42, 44, 46.



- Optimization. 4.5) 5-10, 11-22, 25, 32-35, 37-38, 40-41.

  1. For each question, find the extremum and **prove that it is a local extremum using the first derivative test**.
     (a) $x = 30$, $y = 40$. \$ 720.
     (b) $x = 2$, $y = 5$. 20 in^3.

- Linear Approximations and Differentials. 4.6) 19-24, 25-36, 37-46, 55-58.

  1. $\sqrt{100} + \frac{1}{2\sqrt{100}}(103 - 100) = 10.15$.
  2. $4\pi(3)^2(.2) = 7.2\pi$ in^3.

- Antiderivatives. 4.9) 2-5, 11-22, 23-37, 39-43, 45-47, 55-60, 61-66, 73-78, 79-84, 89-92, 94-96, 98, 101-104.

  1. Find the antiderivatives.

     (a) $8\sqrt{x} + \frac{1}{x} + C$. (b) $x^2 + 4x^3 + \frac{9}{2}x^4 + C$. (c) $\frac{1}{2}\sec(2x) + C$.

  2. (a) $y = \tan(x) + 2$. (b) $y = 2x^2 - \frac{1}{x} - 3$.

- Approximating Area. 5.1) 23-24, 25-30, 48-49, 50i, 61.

  1. (a) Left: $\frac{\pi}{4}(1 + \sqrt{2}2 + 0 - \sqrt{2}2) = \frac{\pi}{4}$.
     Right: $\frac{\pi}{4}(\sqrt{2}2 + 0 - \sqrt{2}2 - 1) = -\frac{\pi}{4}$.
     (b) Left: $.5(\ln(3) + \ln(3.5) + \ln(4) + \ln(4.5))$.
     Right: $.5(\ln(3.5) + \ln(4) + \ln(4.5) + \ln(5))$.

- Definite Integrals. 5.2) 7-11, 25-28, 29-32, 33-36, 37-43, 50-51, 57-64, 75-81, 82-83.

  1. (a) $\int_0^2 x^3\,dx = 4$.
     (b) $\int_1^4 x^2 - 1\,dx = 18$.
  2. Find the definite integrals. Do not use the Fundamental Theorem of Calculus.

     (a) $\frac{5}{2}$ (b) $\frac{7}{2}$ (c) $\frac{25}{4}\pi$.

- Fundamental Theorem of Calculus. 5.3) 7-11, 25-28, 29-32, 33-36, 37-43, 50-51, 57-64, 75-81, 82-83.

  1. Find the derivative.

(a) $\cos(x)\sqrt{1+\sin^4(x)}$.

(b) $2x\cos(x^4) - 3\cos(9x^2)$.

2. Find the definite integrals.

(a) $\frac{8}{3}$

(b) $\frac{1}{3}$

(c) 2

- Working with Integrals. 5.4) 9, 13-22, 25-28, 29-54, 71-84, 101-103.

1. Evaluate the integrals using symmetry.

(a) $\int_{-\pi}^{\pi} \sin(|x|)\, dx = 4$.

(b) $\int_{-1}^{1} \tan(x^3)\, dx = 0$.

2. Find the average value of the function on the interval. Then verify the Mean Value Theorem for Integrals.

(a) Avg $= 4$, $c = 2$.
Note that $-1 < 2 < 5$.

(b) Avg $= \frac{4}{3}$. $c = \frac{17}{9}$.
Note that $1 < \frac{17}{9} < 3$.

- Substitution. 5.5) 7-10, 11-14, 17-26, 28-32, 34-38, 41-66, 74-82.

1. Evaluate the integrals.

(a) $-\frac{1}{3}(3-2x)^{\frac{3}{2}} + C$

(b) $\frac{1}{2}\tan^2(x) + C$ or $\frac{1}{2}\sec^2(x) + C$.

(c) $\cos(x^{-1}) + C$

(d) $\int -\frac{1}{4}(x^2+2x)^{-2} + C$

(e) $\frac{1}{(2-x)^2} - \frac{1}{2-x} + C = \frac{x-1}{(2-x)^2} + C$.

(f) $2(x+2)^{\frac{5}{2}} - 6(x+2)^{\frac{3}{2}} + C$.

- Regions Between Curves. 6.2) 9-32, 39-43, 45-48, 55-58.

1. Area $= 9$.
2. Area $= 4 + 4 = 8$.

- Volume by slicing. 6.3) 11-16, 17-38, 45-50, 62, 65-66.

1. 18 m^3.
2. $V = \pi \int_{-R}^{R} R - x^2\, dx$.
3. Let R be the region between $y = 4 - x^2$ and $y = 4 - 2x$.

(a) Vol $= \frac{32\pi}{5}$.

(b) Vol $= \frac{64\pi}{15}$.

- Volume by Shells. 6.4) 5-8, 9-21, 35-40, 45, 48, 49-58, 60-63.

1. Volume $= \frac{16\pi}{3}$.
2. Let R be the region between $y = 2x$ and $y = x^2$.

(a) $2\pi \int_0^2 x(2x - x^2)\, dx = \frac{8\pi}{3}$.

(b) $2\pi\int_0^2 (3-x)(2x-x^2)\,dx = \frac{16\pi}{3}$.

(c) $2\pi\int_0^2 (x+2)(2x-x^2)\,dx = 8\pi$.

- Length of Curves. 6.5) 3-6, 7-8, 9-16.

  1. $\frac{1}{4}(e^2+1)$.

- Surface Area. 6.6) 7-13, 15-16, 18, 21-22, 32-33.

  1. $2\pi\int_0^1 x^3\sqrt{1+9x^4}\,dx = \frac{\pi}{27}\left(10\sqrt{10}-1\right)$.
  2. Find the surface area of $y = 2x^{\frac{3}{2}}$ when revolved around the y-axis, from $x=0$ to $x=1$. $2\pi\int_0^2 x\sqrt{1+4x}\,dx = \frac{149\pi}{15}$.

- Physical Applications. 6.7) 21-25, 27-29, 31-34.

  1. (a) $k=5$. $\int_0^2 5x\,dx = 10$ N-m.

     (b) $k=4$. $\int_3^5 4x\,dx = 32$ N-m.
  2. $9.8\int_0^{10} 20+.5x\,dx = 2205$ N-m or $9.8\int_0^{10} 20+.5(10-x)\,dx = 2205$ N-m.

- Inverse Functions. 7.1) 39-44, 45-48, 49-52, 53-54.

  1. $f(2)=5$, so $1/(f'(2)) = \frac{1}{11}$.
  2. $f(2)=0$, so $1/(f'(2)) = \frac{1}{3}$. Find the derivative of $f^{-1}(x)$ at $x=0$. $f(x) = \int_2^x \sqrt{1+t^3}\,dt$. (Hint: When is $f(x)=0$? Then use FTC.)

- Logarithms and Exponentials. 7.2) 17-38, 39-40, 41-46, 48, 51-60, 62, 63-70, 73, 76-91, 93-94, 96, 98-99.

  1. Find $f'(x)$.

     (a) $f'(x) = \frac{1}{x} + \cot(x)$

     (b) $f'(x) = 3x^2e^{x^3}\ln(\sec(x)) + e^{x^3}\tan(x)$

     (c) $f'(x) = \frac{1}{2(x-1)} - \frac{3}{x+1}$.

     (d) $f'(x) = \frac{10e^{2x}-9^{3x}-e^{5x}}{(e^{3x}+5)^2}$

  2. Use logarithmic differentiation to find $f'(x)$.

     (a) $f'(x) = \frac{x^3}{\sqrt{x^2+1}}\left(\frac{3}{x} - \frac{x}{2(x^2+1)}\right)$.

     (b) $f'(x) = \left(2\ln(3x+1) + \frac{6x}{3x+1}\right)(3x+1)^{2x}$.

  3. Find the integrals.

     (a) $\frac{1}{2}\ln(5)$

     (b) $\frac{1}{3}\ln(x^3+1)+C$

     (c) $\tan(\ln(x))+C$

     (d) $\frac{1}{6}$

     (e) $\frac{2}{3}e^{6x} + \frac{4}{3}e^{3x} + x + C$

     (f) $\ln(e^x+e^{-x})+C$

     (g) $\frac{14}{3}$

     (h) $e-1$

4. Find the limits

   (a) 0.

   (b) $\frac{5}{2}$.

5. (a) $\frac{\pi}{2}(e^2 + 4e - 3)$.

   (b) π.