Things Aren't Always What They Seem

Careful statistical analysis reveals that some well-known truths are in fact in doubt or even false.

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As educated individuals, we have amassed a great body of knowledge about the world we live in—information we accept as valid, that can significantly affect how we conduct our lives. Yet statistical analysis reveals that some well-known truths are in fact in doubt or even false. In this column I will touch upon a few such items that might be called common knowledge, but that careful statistical investigation calls into question or even refutes.

Body Temperature

Nearly everyone knows that normal human body temperature is 98.6° Fahrenheit. There is even a mark on thermometers at that value. The credit for 98.6° F becoming the standard for "normal" is due to Carl Wunderlich, who published a paper in 1868 summarizing the results of measurements on fully 25,000 adults.

In the early 1990's some modern researchers decided to revisit the question of normal body temperature. They measured the temperatures of healthy volunteer subjects aged 18 to 40, orally, several times a day for three consecutive days. They found that normal temperatures varied by as much as 4.8° F within individual subjects (Wunderlich also found that temperatures varied, but this part of the story was lost to history), that women tend to be warmer than men (hold your comments), and that body temperature tends to be higher in the afternoon than in the early morning. The overall mean temperature in the study was found to be 98.2° F, but the primary conclusion was that all temperatures between about 97.0° F and 99.5° F occur frequently in healthy individuals.

It should not really be surprising that there is so much variation in normal human body temperature considering that virtually all other body parameters (height, weight, blood pressure, pulse rate, etc.) vary greatly—yet the 98.6° "fact" has become ingrained in our knowledge base. The statistical lesson here is that variation is even more prevalent than we think. (This is an essential concept in modern industry.)

Women's Salaries

Most people are aware that, in the United States at least, the salaries of working women are on the whole much lower than those of men. From time to time articles and news reports appear stating that women earn only 77 cents on the dollar (or thereabouts) as compared to men. This is often accompanied by calls for "equal pay for equal work." But is there really such a large salary bias against women in the work force?

In actuality the number given above does not represent a comparison of "equal work," but rather is the weekly median earnings of all working women divided by the weekly median earnings of all working men. How the salaries of men and women *in the same or equivalent jobs* compare cannot be determined from this figure. There are generally far fewer women than men in high paying professions. The reasons for this (recent mass entrance of women into the work force, gender bias in hiring and promotion in many occupations, etc.) have been discussed and debated widely, but their effect is to depress the median earnings for women as compared to men. Careful studies that attempt to contrast earnings of men and women in comparable jobs would surely show a much smaller discrepancy in most occupations than is reflected by a ratio of the type given above.

Cancer Rates

Progress in the war against cancer has been difficult. However, five-year survival rates are increasing for most forms of cancer, which at least is an encouraging sign. Or is it?

The five-year survival rate is the proportion of people alive five years after being diagnosed with cancer. Table 1 gives a comparison of the five-year survival rates for the leading types of cancer in 1950–1954 vs. 1989–1995.

The survival rates have increased for each of these cancers, dramatically for

MathFest Winners

The MAA summer meeting, Mathfest, was held in Madison, Wisconsin in August. Attendees enjoyed the book sales, panel discssions, fish boil and many, many fantastic lectures by students and faculty. One of the highlights from our point of view was the awards ceremonies. Each year the MAA awards cash prizes to those students who give especially noteworthy talks at Mathfest and to those authors who write especially noteworthy articles for MAA publications. This year the Trevor Evans Prizes for expository writing in *Math Horizons* were awarded to two of our favorite and most frequent contributors: Jim Tanton, our dozenal correspondent, for "A Dozen Areal Maneuvers" (September 2000) and Ira Rosenholtz for "One Point Determines a Line" (November 2000). Our congratulations and our thanks for their many wonderful contributions to *Horizons* go to Ira and Jim.

The Mathfest program included 46 talks involving 49 students from 32 colleges and universities. Many of those students spoke of results obtained at REUs including those at: Williams College, Rutgers University, Indiana University, Mount Holyoke College, Lafayette College, and the University of Idaho. Our congratulations to the award winners:

Aliyah Ali—Rutgers University, Jarod Alper—Brown University, John Bryk—Williams College, Eva Kashat and Daniela Silva (jointly)—Wayne State University, Eric Katerman—Williams College, Nicholas Leger—University of Texas at Austin, John Meth—Indiana University, Ellen Panofsky—Millersville University, Cody Patterson—Texas A&M University, D. Jacob Wildstrom—Massachusetts Institute of Technology.

Also, a special research prize from the Council on Undergraduate Research was awarded to Paul Holt of Williams College.

Cancer	1950–1954	1989–1995
Prostate	43%	93%
Melanoma	49%	88%
Breast	60%	86%
Colon	41%	62%
Ovary	30%	50%
Stomach	12%	19%
Lung	6%	14%
Liver	1%	6%
Pancreas	1%	4%

Table 1. Five-year survival rates for leading types of cancer

many. The five-year survival rates for all other common types of cancer (brain, esophagus, oral cavity, cervix, uterus, larynx, thyroid, rectum, bladder, testicles and kidney) have increased as well.

Unfortunately these figures present quite a misleading picture of the degree of progress in the fight against cancer. Modern diagnostic techniques allow doctors to discover many cancers in an earlier stage than in the past. Many cancers detected today are asymptomatic. Some are even nonprogressive; that is, they do not lead to symptoms or result in death. (Prostate cancer is a good example of one which for many men is nonprogressive: it is said that virtually all men eventually get prostate cancer if they live long enough—but many are not treated for it, and eventually die from something else.) Consequently, the denominators of the five-year survival rates for 1989–1995 represent individuals who, on the whole, had less advanced cancer when their five-year period began than was the case for the 1950–1954 group.

It may well be that more people are beating cancer than ever before. Certain other forms of statistical evidence support this claim. But increases in five-year survival rates, so often reported by the news media as measuring success in the war against cancer, do not demonstrate that people who contract cancer are surviving longer than before. These increases may be merely an artifact due to improved detection procedures. Even without advances in treatment effectiveness, earlier detection tends to produce survival rates that are higher than those of the past.

Gender Bias in Graduate Admissions

In the 1970's, the University of California at Berkeley was charged with favoring men over women in admissions to its graduate schools. Data for the 1973 fall quarter showed that 44% of all male applicants were admitted, while only 35% of female applicants were admitted.

Looking at just the six largest departments or programs combined, the admission rates were 44.6% for males and 30.5% for females. The much higher admission rate for male applicants must surely be because the admission rate was substantially higher for males for at least one—if not most—of these six departments—right?

Table 2 shows a breakdown of the data for the six largest departments. Only in Departments 3 and 5 is the proportion of males admitted higher than the proportion of females admitted, and only by small amounts at that. What's more, if we exclude those two departments the female admission rate is higher than the male admission rate in each remaining department, yet the combined admission rates of the remaining departments still show males much higher than females, at 47.2% vs. 30.9%! (You can easily check the numbers yourself.)

So, surprisingly, the common belief that a trend that goes consistently in one direction within subsets of a population must go the same way when these subsets are combined is *untrue*. This phenomenon is known as *Simpson's paradox*.

We have not addressed the *reason* that the graduate admissions data have this seemingly bizarre structure, as the point of the example is merely to illus-

Department/ Program	Male applicants	No. admitted	% admitted	Female applicants	No. admitted	% admitted
1	825	511	**61.9%**	108	89	**82.4%**
2	560	352	**62.9%**	25	17	**68.0%**
3	325	120	**36.9%**	593	202	**34.1%**
4	407	137	**33.7%**	375	132	**35.2%**
5	191	53	**27.7%**	393	95	**24.2%**
6	373	22	**5.9%**	341	24	**7.0%**
Total	**2681**	**1195**	**44.6%**	**1835**	**559**	**30.5%**

Table 2. 1973 Graduate School Admissions data from the University of California at Berkeley

trate that such counterintuitive situations can arise. You may wish to look carefully at the data to see if you can figure this out yourself. The answer is given in Endnote 2 below.

Conclusion

The examples above provide a lesson that all statisticians are well versed in: never take any data at face value. One needs to be very cautious when drawing conclusions from numerical information. A central element of the science of statistics is to learn how to untangle real relationships (or lack of them) from apparent ones.

Endnotes

1. The fact that the mean of Wunderlich's measurements of human body temperature was 98.6° F and not something closer to 98.2° F is likely due in part to the inferior measuring instruments of his time—it took 15 to 20 minutes to obtain temperatures, which were recorded while the thermometers were kept in place in the subjects' armpits.

2. To see why the UC Berkeley admission rates come out the way they do, notice that males tended to apply in greater numbers to departments 1 and 2, programs that had relatively high rates of admission—for both sexes. Females tended to apply to other departments. The overall admission rate for each sex is a weighted average of the departmental admission rates, with the male overall admission rate putting higher weight on the higher departmental admission rates of departments 1 and 2 than does the female overall admission rate.

The fact that males and females applied unequally to departments with different acceptance rates represents a *lurking variable* that has the potential to skew the ultimate conclusion. Both the male/female salary comparison and the cancer example also contain such lurking variables. You may wish to reflect upon what these lurking variables are. ■

Exponents

(To the tune of "Jingle Bells")

Exponents! Exponents!
Powerful things.
Raise you up,
And root you down,
Are what logarithms bring.

Exponents! Exponents!
Powerful things.
Raise you up,
And root you down,
Are what logarithms bring.

Natural is *e*.
Common is 10.
e to the zero's one;
10 to the zero's one again.

To multiply, just add;
To divide, simply subtract—
Just be sure the bases match,
Or you're on the wrong track.

Exponents! Exponents!
Powerful things.
Raise you up,
And root you down,
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The Conic Section Carol

(To the tune of "God Rest Ye Merry Gentlemen")

God rest ye merry ellipses
And hyperbolas, too.
Parabolas and circles
All curves of degree two.
Degenerate and involute, the mathematician's toy.
O sections of comfort and joy, etc.

O symmetry and focuses
And fun directrices.
Both smooth and sharply pointed curves
Can have some vertices.
To prove this fact, derivatives are such a clever ploy.
O sections of comfort and joy, etc.

SUSAN JANE COLLEY
Oberlin College