Problem 1.11.
A cattle farmer expects to have 120,000 pounds of live cattle to sell in three months. The live-cattle futures contract on the Chicago Mercantile Exchange is for the delivery of 40,000 pounds of cattle. How can the farmer use the contract for hedging? From the farmer’s viewpoint, what are the pros and cons of hedging?

The farmer can short 3 contracts that have 3 months to maturity. If the price of cattle falls, the gain on the futures contract will offset the loss on the sale of the cattle. If the price of cattle rises, the gain on the sale of the cattle will be offset by the loss on the futures contract. Using futures contracts to hedge has the advantage that it can at no cost reduce risk to almost zero. Its disadvantage is that the farmer no longer gains from favorable movements in cattle prices.

Problem 1.13.
Suppose that a March call option on a stock with a strike price of $50 costs $2.50 and is held until March. Under what circumstances will the holder of the option make a gain? Under what circumstances will the option be exercised? Draw a diagram showing how the profit on a long position in the option depends on the stock price at the maturity of the option.

The holder of the option will gain if the price of the stock is above $52.50 in March. (This ignores the time value of money.) The option will be exercised if the price of the stock is above $50.00 in March. The profit as a function of the stock price is shown in Figure S1.1.

![Figure S1.1](image-url)

Figure S1.1 Profit from long position in Problem 1.13

Problem 1.14.
Suppose that a June put option on a stock with a strike price of $60 costs $4 and is held until June. Under what circumstances will the holder of the option make a gain? Under what circumstances will the option be exercised? Draw a diagram showing how the profit on a short position in the option depends on the stock price at the maturity of the option.

The seller of the option will lose if the price of the stock is below $56.00 in June. (This ignores the time value of money.) The option will be exercised if the price of the stock is below $60.00 in June. The profit as a function of the stock price is shown in Figure S1.2.
Problem 1.20.
A trader enters into a short forward contract on 100 million yen. The forward exchange rate is $0.0080 per yen. How much does the trader gain or lose if the exchange rate at the end of the contract is (a) $0.0074 per yen; (b) $0.0091 per yen?

a) The trader sells 100 million yen for $0.0080 per yen when the exchange rate is $0.0074 per yen. The gain is $100 \times 0.0006 = 60,000$ million dollars or $60,000. (Dollar appreciates.)

b) The trader sells 100 million yen for $0.0080 per yen when the exchange rate is $0.0091 per yen. The loss is $100 \times 0.0011 = 110,000$ million dollars or $110,000. (Dollar depreciates.)

Problem 2.11.
A trader buys two July futures contracts on frozen orange juice. Each contract is for the delivery of 15,000 pounds. The current futures price is 160 cents per pound, the initial margin is $6,000 per contract, and the maintenance margin is $4,500 per contract. What price change would lead to a margin call? Under what circumstances could $2,000 be withdrawn from the margin account?

There is a margin call if more than $1,500 is lost on one contract. This happens if the futures price of frozen orange juice falls by more than 10 cents to below 150 cents per lb. $2,000 can be withdrawn from the margin account if there is a gain on one contract of $1,000. This will happen if the futures price rises by 6.67 cents to 166.67 cents per lb.
Problem 2.15.
At the end of one day a clearinghouse member is long 100 contracts, and the settlement price is $50,000 per contract. The original margin is $2,000 per contract. On the following day the member becomes responsible for clearing an additional 20 long contracts, entered into at a price of $51,000 per contract. The settlement price at the end of this day is $50,200. How much does the member have to add to its margin account with the exchange clearinghouse?

The clearinghouse member is required to provide $20 \times 2,000 = $40,000 as initial margin for the new contracts. There is a gain of $(50,200 - 50,000) \times 100 = $20,000$ on the existing contracts. There is also a loss of $(51,000 - 50,200) \times 20 = $16,000$ on the new contracts. The member must therefore add

$$40,000 - 20,000 + 16,000 = $36,000$$

to the margin account.

Problem 2.16.
On July 1, 2010, a Japanese company enters into a forward contract to buy $1 million with yen on January 1, 2011. On September 1, 2010, it enters into a forward contract to sell $1 million on January 1, 2011. Describe the profit or loss the company will make in dollars as a function of the forward exchange rates on July 1, 2010 and September 1, 2010.

Suppose $F_1$ and $F_2$ are the forward exchange rates for the contracts entered into July 1, 2010 and September 1, 2010, and $S$ is the spot rate on January 1, 2011. (All exchange rates are measured as yen per dollar). The payoff from the first contract is $(S - F_1)$ million yen and the payoff from the second contract is $(F_2 - S)$ million yen. The total payoff is therefore

$$(S - F_1) + (F_2 - S) = (F_2 - F_1)$$

million yen.

Problem 2.23.
Suppose that on October 24, 2010, you take a short position in an April 2011 live-cattle futures contract. You close out your position on January 21, 2011. The futures price (per pound) is 91.20 cents when you enter into the contract, 88.30 cents when you close out your position, and 88.80 cents at the end of December 2010. One contract is for the delivery of 40,000 pounds of cattle. What is your total profit? How is it taxed if you are (a) a hedger and (b) a speculator? Assume that you have a December 31 year end.

The total profit is

$$40,000 \times (0.9120 - 0.8830) = $1,160$$

If you are a hedger this is all taxed in 2011. If you are a speculator

$$40,000 \times (0.9120 - 0.8880) = $960$$

is taxed in 2010 and

$$40,000 \times (0.8880 - 0.8830) = $200$$

is taxed in 2011.
Problem 3.12.
Suppose that in Example 3.4 the company decides to use a hedge ratio of 0.8. How does the decision affect the way in which the hedge is implemented and the result?

If the hedge ratio is 0.8, the company takes a long position in 16 NYM December oil futures contracts on June 8 when the futures price is $68.00. It closes out its position on November 10. The spot price and futures price at this time are $75.00 and $72. The gain on the futures position is

\[(72 – 68.00) \times 16,000 = 64,000\]

The effective cost of the oil is therefore

\[20,000 \times 75 – 64,000 = 1,436,000\]

or $71.80 per barrel. (This compares with $71.00 per barrel when the company is fully hedged.)

Problem 3.16.
The standard deviation of monthly changes in the spot price of live cattle is (in cents per pound) 1.2. The standard deviation of monthly changes in the futures price of live cattle for the closest contract is 1.4. The correlation between the futures price changes and the spot price changes is 0.7. It is now October 15. A beef producer is committed to purchasing 200,000 pounds of live cattle on November 15. The producer wants to use the December live-cattle futures contracts to hedge its risk. Each contract is for the delivery of 40,000 pounds of cattle. What strategy should the beef producer follow?

The optimal hedge ratio is

\[\frac{0.7 \times 1.2}{1.4} = 0.6\]

The beef producer requires a long position in \[200000 \times 0.6 = 120,000\] lbs of cattle. The beef producer should therefore take a long position in 3 December contracts closing out the position on November 15.

Problem 3.18.
On July 1, an investor holds 50,000 shares of a certain stock. The market price is $30 per share. The investor is interested in hedging against movements in the market over the next month and decides to use the September Mini S&P 500 futures contract. The index is currently 1,500 and one contract is for delivery of $50 times the index. The beta of the stock is 1.3. What strategy should the investor follow? Under what circumstances will it be profitable?

A short position in

\[1.3 \times \frac{50,000 \times 30}{50 \times 1,500} = 26\]

contracts is required. It will be profitable if the stock outperforms the market in the sense that its return is greater than that predicted by the capital asset pricing model.
Problem 4.10.
A deposit account pays 12% per annum with continuous compounding, but interest is actually paid quarterly. How much interest will be paid each quarter on a $10,000 deposit?

The equivalent rate of interest with quarterly compounding is \( R \) where

\[
e^{0.12} = \left(1 + \frac{R}{4}\right)^4
\]

or

\[
R = 4(e^{0.03} - 1) = 0.1218
\]

The amount of interest paid each quarter is therefore:

\[
10,000 \times \frac{0.1218}{4} = 304.55
\]

or $304.55.

Suppose that zero interest rates with continuous compounding are as follows:

<table>
<thead>
<tr>
<th>Maturity (years)</th>
<th>Rate (% per annum)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.0</td>
</tr>
<tr>
<td>2</td>
<td>3.0</td>
</tr>
<tr>
<td>3</td>
<td>3.7</td>
</tr>
<tr>
<td>4</td>
<td>4.2</td>
</tr>
<tr>
<td>5</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Calculate forward interest rates for the second, third, fourth, and fifth years.

The forward rates with continuous compounding are as follows: to
Year 2: 4.0%
Year 3: 5.1%
Year 4: 5.7%
Year 5: 5.7%

Problem 5.9.
A one-year long forward contract on a non-dividend-paying stock is entered into when the stock price is $40 and the risk-free rate of interest is 10% per annum with continuous compounding.

a) What are the forward price and the initial value of the forward contract?

b) Six months later, the price of the stock is $45 and the risk-free interest rate is still 10%. What are the forward price and the value of the forward contract?

a) The forward price, \( F_0 \), is given by equation (5.1) as:

\[
F_0 = 40e^{0.1\times1} = 44.21
\]

or $44.21. The initial value of the forward contract is zero.
b) The delivery price $K$ in the contract is $44.21. The value of the contract, $f$, after six months is given by equation (5.5) as:

$$f = 45 - 44.21e^{-0.1\times0.5} = 2.95$$

i.e., it is $2.95. The forward price is:

$$45e^{0.1\times0.5} = 47.31$$

or $47.31.

Problem 5.10.
The risk-free rate of interest is 7% per annum with continuous compounding, and the dividend yield on a stock index is 3.2% per annum. The current value of the index is 150. What is the six-month futures price?

Using equation (5.3) the six month futures price is

$$150e^{(0.07-0.032)\times0.5} = 152.88$$

or $152.88.

Problem 5.14.
The two-month interest rates in Switzerland and the United States are 2% and 5% per annum, respectively, with continuous compounding. The spot price of the Swiss franc is $0.8000. The futures price for a contract deliverable in two months is $0.8100. What arbitrage opportunities does this create?

The theoretical futures price is

$$0.8000e^{(0.05-0.02)\times2/12} = 0.8040$$

The actual futures price is too high. This suggests that an arbitrageur should buy Swiss francs and short Swiss francs futures.

Problem 5.15.
The current price of silver is $15 per ounce. The storage costs are $0.24 per ounce per year payable quarterly in advance. Assuming that interest rates are 10% per annum for all maturities, calculate the futures price of silver for delivery in nine months.

The present value of the storage costs for nine months are

$$0.06 + 0.06e^{-0.1\times0.25} + 0.06e^{-0.1\times0.5} = 0.176$$

or $0.176. The futures price is from equation (5.11) given by $F_0$ where

$$F_0 = (15.000 + 0.176)e^{0.1\times0.75} = 16.36$$

i.e., it is $16.36 per ounce.
Problem 6.8.
The price of a 90-day Treasury bill is quoted as 10.00. What continuously compounded return (on an actual/365 basis) does an investor earn on the Treasury bill for the 90-day period?

The cash price of the Treasury bill is

\[
100 - \frac{90}{360} \times 10 = \$97.50
\]

The annualized continuously compounded return is

\[
\frac{365}{90} \ln \left(1 + \frac{2.5}{97.5}\right) = 10.27\%
\]

Problem 6.9.
It is May 5, 2010. The quoted price of a government bond with a 12% coupon that matures on July 27, 2014, is 110-17. What is the cash price?

The number of days between January 27, 2010 and May 5, 2010 is 98. The number of days between January 27, 2010 and July 27, 2010 is 181. The accrued interest is therefore

\[
6 \times \frac{98}{181} = 3.2486
\]

The quoted price is 110.5312. The cash price is therefore

\[110.5312 + 3.2486 = 113.7798\]
or $113.78.

Problem 6.10.
Suppose that the Treasury bond futures price is 101-12. Which of the following four bonds is cheapest to deliver?

<table>
<thead>
<tr>
<th>Bond</th>
<th>Price</th>
<th>Conversion Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>125-05</td>
<td>1.2131</td>
</tr>
<tr>
<td>2</td>
<td>142-15</td>
<td>1.3792</td>
</tr>
<tr>
<td>3</td>
<td>115-31</td>
<td>1.1149</td>
</tr>
<tr>
<td>4</td>
<td>144-02</td>
<td>1.4026</td>
</tr>
</tbody>
</table>

The cheapest-to-deliver bond is the one for which

\[
\text{Quoted Price} - \text{Futures Price} \times \text{Conversion Factor}
\]

is least. Calculating this factor for each of the 4 bonds we get

Bond 1: 125.15625 - 101.375 \times 1.2131 = 2.178
Bond 2: 142.46875 - 101.375 \times 1.3792 = 2.652
Bond 3: 115.96875 - 101.375 \times 1.1149 = 2.946
Bond 4: 144.06250 - 101.375 \times 1.4026 = 1.874

Bond 4 is therefore the cheapest to deliver.
**Problem 7.9.**

Companies X and Y have been offered the following rates per annum on a $5 million 10-year investment:

<table>
<thead>
<tr>
<th></th>
<th>Fixed Rate</th>
<th>Floating Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Company X</td>
<td>8.0%</td>
<td>LIBOR</td>
</tr>
<tr>
<td>Company Y</td>
<td>8.8%</td>
<td>LIBOR</td>
</tr>
</tbody>
</table>

Company X requires a fixed-rate investment; company Y requires a floating-rate investment. Design a swap that will net a bank, acting as intermediary, 0.2% per annum and will appear equally attractive to X and Y.

The spread between the interest rates offered to X and Y is 0.8% per annum on fixed rate investments and 0.0% per annum on floating rate investments. This means that the total apparent benefit to all parties from the swap is 0.8% per annum. Of this 0.2% per annum will go to the bank. This leaves 0.3% per annum for each of X and Y. In other words, company X should be able to get a fixed-rate return of 8.3% per annum while company Y should be able to get a floating-rate return LIBOR + 0.3% per annum. The required swap is shown in Figure S7.1. The bank earns 0.2%, company X earns 8.3%, and company Y earns LIBOR + 0.3%.

![Figure S7.1 Swap for Problem 7.9](image)

**Problem 7.10.**

A financial institution has entered into an interest rate swap with company X. Under the terms of the swap, it receives 10% per annum and pays six-month LIBOR on a principal of $10 million for five years. Payments are made every six months. Suppose that company X defaults on the sixth payment date (end of year 3) when the interest rate (with semiannual compounding) is 8% per annum for all maturities. What is the loss to the financial institution? Assume that six-month LIBOR was 9% per annum halfway through year 3.

At the end of year 3 the financial institution was due to receive $500,000 (= 0.5×10% of $10 million) and pay $450,000 (= 0.5×9% of $10 million). The immediate loss is therefore $50,000. To value the remaining swap we assume that forward rates are realized. All forward rates are 8% per annum. The remaining cash flows are therefore valued on the assumption that the floating payment is 0.5×0.08×10,000,000 = $400,000 and the net payment that would be received is $500,000 − $400,000 = $100,000. The total cost of default is therefore the cost of foregoing the following cash flows:
Discounting these cash flows to year 3 at 4% per six months we obtain the cost of the default as $413,000.

**Problem 7.11.**

A financial institution has entered into a 10-year currency swap with company Y. Under the terms of the swap, the financial institution receives interest at 3% per annum in Swiss francs and pays interest at 8% per annum in U.S. dollars. Interest payments are exchanged once a year. The principal amounts are 7 million dollars and 10 million francs. Suppose that company Y declares bankruptcy at the end of year 6, when the exchange rate is $0.80 per franc. What is the cost to the financial institution? Assume that, at the end of year 6, the interest rate is 3% per annum in Swiss francs and 8% per annum in U.S. dollars for all maturities. All interest rates are quoted with annual compounding.

When interest rates are compounded annually

$$F_0 = S_0 \left( \frac{1 + r}{1 + r_f} \right)^T$$

where $F_0$ is the $T$-year forward rate, $S_0$ is the spot rate, $r$ is the domestic risk-free rate, and $r_f$ is the foreign risk-free rate. As $r = 0.08$ and $r_f = 0.03$, the spot and forward exchange rates at the end of year 6 are

**Spot:** 0.8000

1 year forward: 0.8388

2 year forward: 0.8796

3 year forward: 0.9223

4 year forward: 0.967

The value of the swap at the time of the default can be calculated on the assumption that forward rates are realized. The cash flows lost as a result of the default are therefore as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>Dollar Paid</th>
<th>CHF Received</th>
<th>Forward Rate</th>
<th>Dollar Equiv of CHF Received</th>
<th>Cash Flow Lost</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>560,000</td>
<td>300,000</td>
<td>0.8000</td>
<td>240,000</td>
<td>-320,000</td>
</tr>
<tr>
<td>7</td>
<td>560,000</td>
<td>300,000</td>
<td>0.8388</td>
<td>251,600</td>
<td>-308,400</td>
</tr>
<tr>
<td>8</td>
<td>560,000</td>
<td>300,000</td>
<td>0.8796</td>
<td>263,900</td>
<td>-296,100</td>
</tr>
<tr>
<td>9</td>
<td>560,000</td>
<td>300,000</td>
<td>0.9223</td>
<td>276,700</td>
<td>-283,300</td>
</tr>
<tr>
<td>10</td>
<td>7,560,000</td>
<td>10,300,000</td>
<td>0.9670</td>
<td>9,960,100</td>
<td>2,400,100</td>
</tr>
</tbody>
</table>

Discounting the numbers in the final column to the end of year 6 at 8% per annum, the cost of the default is $679,800.
Note that, if this were the only contract entered into by company Y, it would make no sense for the company to default at the end of year six as the exchange of payments at that time has a positive value to company Y. In practice, company Y is likely to be defaulting and declaring bankruptcy for reasons unrelated to this particular contract and payments on the contract are likely to stop when bankruptcy is declared.