Chapter 11

Bond Valuation: Part I

- Bond pricing
- Time path of bond prices
- Bond yields
- Term structure of interest rates

Bond Pricing - 1

Several Assumptions:

To simplify the analysis, we make the following assumptions.

1. The coupon payments are made every six months.

2. The next coupon payment for the bond is received exactly six months from now.

3. The coupon interest is fixed for the term of the bond.

Therefore, the price of the bond is the sum of

1) The PV of the semi-annual coupon payments
2) The PV of the par value
In general, the price of bond can be computed as:

\[
P = \frac{c}{(1+i)^1} + \frac{c}{(1+i)^2} + \frac{c}{(1+i)^3} + \cdots + \frac{c}{(1+i)^n} + \frac{M}{(1+i)^n}
\]

where:
- \(c\) = semi-annual coupon payment
- \(i\) = semi-annual yield
- \(n\) = number of semi-annual periods
- \(M\) = maturity value (or face value)

The first \(n\) terms can be computed as an ordinary annuity: \(c \left[ \frac{1 - \frac{1}{(1+i)^n}}{i} \right] \), therefore \(P\) can be expressed as:

\[
P = c \left[ \frac{1 - \frac{1}{(1+i)^n}}{i} \right] + \frac{M}{(1+i)^n}
\]

Alternatively, we can use the financial calculator to find the price of the bond. We will use several examples to illustrate.

**Example 1:** Compute the price of a 9% coupon bond with 20 years to maturity and a par value of $1,000 if the required yield is 12%.

The cash flows are as follows:
1) 40 semi-annual coupon payments of $45.
2) $1,000 20 years from now.

Mathematically, price can be calculated as:

\[
P = \frac{45}{6\%} \left[ 1 - \frac{1}{(1+6\%)^{40}} \right] + \frac{1000}{(1+6\%)^{40}} = 677.08 + 97.22 = 774.30
\]

Alternatively, using the financial calculator, we can find the price with the following input:

<table>
<thead>
<tr>
<th>PMT</th>
<th>$45</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>40</td>
</tr>
<tr>
<td>FV</td>
<td>$1,000</td>
</tr>
<tr>
<td>INT</td>
<td>6%</td>
</tr>
</tbody>
</table>

The bond price can be computed by pressing “Compute” and enter “PV”. You will arrive at the same answer of $774.30.
**Example 2:** Compute the price of a 9% coupon bond with 20 years to maturity and a par value of $1,000 if the required yield is 7%.

The cash flows are unchanged, but the semi-annual yield is now 3.5%.

Mathematically, price can be calculated as:

\[
P = \frac{45}{3.5\%} \left[ 1 - \frac{1}{(1 + 3.5\%)^{40}} \right] + \frac{1000}{(1 + 3.5\%)^{40}} = \$1,213.55
\]

Alternatively, using the financial calculator, we can find the price with the following input:

- PMT: $45
- N: 40
- FV: $1,000
- INT: 3.5%

Compute PV:

\[PV = \$1,213.55\]

**Example 3:** Compute the price of a 9% coupon bond with 20 years to maturity and a par value of $1,000 if the required yield is 9%.

The cash flows are unchanged, but the semi-annual yield is now 4.5%.

Mathematically, price can be calculated as:

\[
P = \frac{45}{4.5\%} \left[ 1 - \frac{1}{(1 + 4.5\%)^{40}} \right] + \frac{1000}{(1 + 4.5\%)^{40}} = \$1,000\]

Alternatively, using the financial calculator, we can find the price with the following input:

- PMT: $45
- N: 40
- FV: $1,000
- INT: 4.5%

Compute PV:

\[PV = \$1,000\]
Relationship between required yield and price:

The price of a bond changes in the opposite direction to the change in the required yield. This can be seen by comparing the price of the 20-year 9% coupon bond that we priced in the previous three examples. If we calculate the price based on other yields, we get the following table:

<table>
<thead>
<tr>
<th>Yield to maturity</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>$2,800.00</td>
</tr>
<tr>
<td>1%</td>
<td>$2,446.89</td>
</tr>
<tr>
<td>2%</td>
<td>$2,149.21</td>
</tr>
<tr>
<td>3%</td>
<td>$1,897.48</td>
</tr>
<tr>
<td>4%</td>
<td>$1,683.89</td>
</tr>
<tr>
<td>5%</td>
<td>$1,502.06</td>
</tr>
<tr>
<td>6%</td>
<td>$1,346.72</td>
</tr>
<tr>
<td>7%</td>
<td>$1,213.55</td>
</tr>
<tr>
<td>8%</td>
<td>$1,098.96</td>
</tr>
<tr>
<td>9%</td>
<td>$1,000.00</td>
</tr>
<tr>
<td>10%</td>
<td>$914.20</td>
</tr>
<tr>
<td>11%</td>
<td>$839.54</td>
</tr>
<tr>
<td>12%</td>
<td>$774.31</td>
</tr>
<tr>
<td>13%</td>
<td>$717.09</td>
</tr>
<tr>
<td>14%</td>
<td>$666.71</td>
</tr>
<tr>
<td>15%</td>
<td>$622.17</td>
</tr>
<tr>
<td>16%</td>
<td>$582.64</td>
</tr>
</tbody>
</table>

If we graph the price/yield relationship for this 20 year 9% coupon bond, we will find it has the "bowed" shape.
Bond Pricing - 8

*Relationship among coupon rate, required yield and price:*

For a straight bond, the coupon rate and term to maturity are fixed. Consequently, as yield in the marketplace changes, the only variable that can change to compensate for the new yield is the price of the bond.

Generally, coupon rate of a bond at issue is set to approximately the prevailing yield in the market. The price of the bond will then approximately equal to its par value. When a bond sells below its par value, it is said to be selling at a *discount*. On the contrary, when the price is higher than the par value, it is said to be selling at a *premium*.

The following relation holds:

- Coupon rate < required yield $\Leftrightarrow$ price < par (discount)
- Coupon rate = required yield $\Leftrightarrow$ price = par
- Coupon rate > required yield $\Leftrightarrow$ price > par (premium)

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Bond Pricing - 9

**Zero-coupon bond:**

There is no coupon payment in a zero-coupon bond. Instead, the investor realizes interest by the amount of the difference between the face value and the purchase price.

The formula for the price of a zero-coupon bond that matures in $N$ years from now is:

$$ P = \frac{M}{(1 + i)^n} $$

Where

- $i =$ semi-annual yield
- $n =$ number of semi-annual periods = $2 \times N$
- $M =$ maturity value
**Zero-coupon bond:**

*Example 4:* Compute the price of a 10-year zero-coupon bond with a face value of $1,000 and required yield of 8.6%.

\[
M = \$1,000 \quad i = 0.043 \quad n = 2 \times 10 = 20
\]

\[
P = \frac{1,000}{(1.043)^{20}} = \$430.83
\]

With financial calculator:

- N 20
- FV $1,000
- INT 4.3%
- Compute PV: PV = $430.83.

*Example 5:* Compute the price of a 7-year zero-coupon bond with a maturity value of $100,000 and required yield of 9.8%.

\[
M = \$100,000 \quad i = 0.049 \quad n = 2 \times 7 = 14
\]

\[
P = \frac{100,000}{(1.049)^{14}} = \$51,185.06
\]

With financial calculator:

- N 14
- FV $100,000
- INT 4.9%
- Compute PV: PV = $51,185.06

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**Time Path of Bond Prices - 1**

The following table shows the relation between the bond price and time to maturity for a 9% coupon bond with a face value of $1,000, assuming the yield to maturity stays the same over the life of the bond:

<table>
<thead>
<tr>
<th>Years remaining to maturity</th>
<th>20</th>
<th>15</th>
<th>10</th>
<th>5</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Bond (YTM=12%)</td>
<td>$774.30</td>
<td>$793.53</td>
<td>$827.95</td>
<td>$889.60</td>
<td>$1,000</td>
</tr>
<tr>
<td>Premium Bond (YTM=6%)</td>
<td>$1346.72</td>
<td>$1294.01</td>
<td>$1223.16</td>
<td>$1127.95</td>
<td>$1,000</td>
</tr>
</tbody>
</table>

Assuming the required yield does not change, we have the following conclusion:

*For a bond selling at a discount, the bond price increases until it reaches par value at maturity;*

*For a bond selling at a premium, the bond price decreases until it reaches par value.*

*For a bond selling at par, the bond price will not change.*
Time Path of Bond Prices - 2

The following graph shows the relation between the bond price and time to maturity of the bond mentioned above.

Bond Prices Over Time

<table>
<thead>
<tr>
<th>Time remaining to maturity</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>$1,400.00</td>
</tr>
<tr>
<td>20</td>
<td>$1,300.00</td>
</tr>
<tr>
<td>18</td>
<td>$1,200.00</td>
</tr>
<tr>
<td>16</td>
<td>$1,100.00</td>
</tr>
<tr>
<td>14</td>
<td>$1,000.00</td>
</tr>
<tr>
<td>12</td>
<td>$900.00</td>
</tr>
<tr>
<td>10</td>
<td>$800.00</td>
</tr>
<tr>
<td>8</td>
<td>$700.00</td>
</tr>
<tr>
<td>6</td>
<td>$600.00</td>
</tr>
<tr>
<td>4</td>
<td>$500.00</td>
</tr>
<tr>
<td>2</td>
<td>$400.00</td>
</tr>
<tr>
<td>0</td>
<td>$300.00</td>
</tr>
</tbody>
</table>

Bond Yield - 1

**Current Yield:**

Current yield relates the annual coupon interest to the market price.

\[
\text{Current yield} = \frac{\text{Annual dollar coupon interest}}{\text{Price}}
\]

**Example:** What is the current yield for an 18-year, 6% coupon bond selling for $700.89?

Annual dollar coupon interest = $1,000 x 0.06 = $60.

Currently yield = 60/700.89 = 8.56%

Question: What is the problem with this measure?

1. Capital gain or loss is not considered.
2. Time value of money is ignored.
**Bond Yield - 2**

*Yield to Maturity:*

Yield to maturity of a bond is the internal rate of return the investment is generating. That is, the discount rate that will make the present value of the cash flows equal to the bond price.

We will assume that coupons are paid semi-annually. Denote the semi-annual yield as \( y \), then \( y \) must satisfy the following:

\[
P = \sum_{t=1}^{n} \frac{CF_t}{(1 + y)^t} + \frac{M}{(1 + y)^n}
\]

Note:
1. \( n \) is number of semi-annual periods.
2. \( y \) is semi-annual rate of return.
3. Market convention is to use \( 2y \) as the yield to maturity.
4. The yield to maturity computed this way is called the *bond-equivalent yield*.

**Bond Yield - 3**

*Yield to Maturity:*

The yield to maturity can be measured by trial and error.

**Example 1:** Suppose a 2-year bond makes annual payments of 8% and is priced at 102.5. What is the yield-to-maturity?

<table>
<thead>
<tr>
<th>cash flows</th>
<th>PV at 8%</th>
<th>PV at 7%</th>
<th>PV at 6%</th>
<th>PV at 6.625%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 8</td>
<td>7.41</td>
<td>7.48</td>
<td>7.55</td>
<td>7.50</td>
</tr>
<tr>
<td>2 108</td>
<td>92.59</td>
<td>94.33</td>
<td>96.12</td>
<td>95</td>
</tr>
<tr>
<td>Total</td>
<td>100.00</td>
<td>101.81</td>
<td>103.67</td>
<td><strong>102.50</strong></td>
</tr>
</tbody>
</table>

Therefore, the yield to maturity is 6.625%.

We can use EXCEL function IRR to calculate the yield to maturity. internal rate of return = IRR (range);

To calculate the YTM of the above example, we set the range to be [-102.50, 8, 108].
Then the YTM is: IRR(range) = 6.625%.

If the cash flows are semi-annual, the IRR function will give us the semi-annual yield. By convention, annual yield will be \( 2 \times \text{IRR} \text{(range)} \).
Bond Yield - 4

Yield to Maturity:

Example 2: Compute the yield to maturity of an 18-year 6% coupon bond selling at $700.89.

Two sources of cash flows:
1. 36 coupon payments of $30 every six months.
2. $1,000 36 six-month periods from now.

Using IRR function, the range is [-700.89, 30, 30, ..., 1030]

Therefore, the semi-annual yield is: \( y = \text{IRR (range)} = 4.75\% \),

\( \text{YTM} = 2y = 9.5\% \).

Using the financial calculator, we can find YTM with the following input:

\[
\begin{array}{c|c}
 N & 36 \\
\hline
\text{PMT} & \$30 \\
\text{PV} & -$700.89 \\
\text{FV} & \$1,000 \\
\end{array}
\]

The semi-annual yield can be computed by pressing “Compute” and enter “INT”. You will get 4.75%.

Therefore, the yield-to-maturity is \( 2y = 9.5\% \).

Bond Yield - 5

Example 3: Compute the yield to maturity of a 10-year 8% coupon bond selling at $1,100.

Two sources of cash flows:
1. 20 coupon payments of $40 every six months.
2. $1,000 10 years from now.

Using IRR function, the range is [-1100, 40, 40, ..., 1040]

Therefore, the semi-annual yield is: \( y = \text{IRR (range)} = 3.31\% \),

\( \text{YTM} = 2y = 6.62\% \).

Using the financial calculator, we can find YTM with the following input:

\[
\begin{array}{c|c}
 N & 20 \\
\hline
\text{PMT} & \$40 \\
\text{PV} & -$1100 \\
\text{FV} & \$1,000 \\
\end{array}
\]

Compute INT: 3.31%.

Therefore, the yield-to-maturity is \( 2y = 6.62\% \).
**Yield to Maturity of Zero-Coupon Bond:**

A zero-coupon bond is characterized by a single cash flow resulting from the investment. Consequently, we can compute the yield to maturity directly.

*Example 4:* Compute the yield to maturity of a 15-year zero-coupon bond selling for $274.78 with a face value of $1,000.

Suppose the semiannual yield is \( y \), we have:

\[
\begin{align*}
274.78 \times (1+y)^{30} &= 1,000 \\
(1+y)^{30} &= 1,000/274.78 \\
1+y &= (1,000/274.78)^{1/30} \\
y &= 0.044
\end{align*}
\]

Therefore, yield to maturity is: \( 2y = 8.8\% \).

That is,

\[
y = \left( \frac{\text{Future value per dollar invested}}{\text{maturity value}} \right)^{1/n} - 1
\]

where

\[
\begin{array}{c|c}
\text{Future value per dollar} & \text{maturity value} \\
\hline
\text{invested} & \text{Price}
\end{array}
\]

*Example 5:* A 3-year Treasury STRIP with a face value of $1,000 is currently selling at $837.48. Calculate the bond-equivalent yield and effective annual yield.

1) Bond-equivalent yield = \( 2y \) (\( y \) is the semi-annual yield)

(Ans: 6\%).

*Bond-equivalent yield is the same as yield-to-maturity.*

2) Effective annual yield = \( (1+y)^2 - 1 \)

(Ans: 6.09\%).

*Effective annual yield takes into account semi-annual compounding.*
**Yield to Call:**

For a callable bond, a commonly quoted yield measure is *yield to call*. It is the interest rate that will make the present value of the cash flows equal to the price of the bond if the bond is held to the first call date.

Mathematically,

$$P = \frac{c}{(1 + y)^1} + \frac{c}{(1 + y)^2} + \cdots + \frac{c}{(1 + y)^{n^*}} + \frac{M^*}{(1 + y)^{n^*}}$$

where:
- $M^* = $ call price
- $n^* = $ number of periods until first call date

**Example 6:** Suppose an 18-year 11% coupon bond with face value of $1,000 is selling for $1,168.97. If the first call date is 13 years from now and that the call price is $1,055. Compute the yield to call. If the bond is called in 13 years, the two sources of cash flows are:
1. 26 coupon payments of $55 every six months.
2. $1,055 due in 13 years.

Using financial calculator, we input the following:

| N  | 26 |
| PMT | $55 |
| PV | -$1,168.97 |
| FV | $1,055 |

Compute the interest rate to get $y$: $y=4.5\%$.
Yield to call = $2y = 9\%$.

This is different from yield to maturity.

**Question:** What is the yield to maturity of this bond?
Relation between current yield, yield-to-maturity, coupon rate

For premium bond:

*Coupon rate > current yield > yield to maturity*

*Exercise:* Verify the relationship using a 10-year, 8% coupon bond with 6% yield to maturity. (Coupons are paid semi-annually)

For discount bond:

*Coupon rate < current yield < yield to maturity*

*Exercise:* Verify the relationship using a 10-year, 5% coupon bond with 9% yield to maturity. (Coupons are paid semi-annually)

Yield curve: The graphical relationship between the yield to maturity and term to maturity. It is also known as the term structure of interest rates.

The following is the Treasury yield curve as of Oct. 28, 2005.
Two types of yield curve:

- Downward-sloping (inverted) curve
- Upward-sloping curve

The slope of yield curve is affected by:

- Inflation expectations
- Liquidity preferences of investors
- Supply and demand

There are three theories on the shape of the yield curve:

- Expectations hypothesis
- Liquidity preference theory
- Market segmentation theory

**Expectations Hypothesis:** The theory that yields to maturity is determined solely by expectations of future short-term interest rates.

- Shape of yield curve is based upon investor expectations of future behavior of interest rates.
- If expecting higher inflation, investors demand higher interest rates on longer maturities to compensate for risk.
- Increasing inflation expectations will result in upward-sloping yield curve.
- Decreasing inflation expectations will result in downward-sloping yield curve.
Liquidity Preference Theory: The theory that investors demand a risk premium on long-term bonds.

Liquidity premium is the extra compensation investors demand for holding longer term bonds, as longer term bond faces more interest rate risk. It is a type of risk premium.

- Shape of yield curve is based upon the length of term, or maturity, of bonds.
- If investors’ money is tied up for longer periods of time, they have less liquidity and demand higher interest rates to compensate for real or perceived risks.
- Investors won’t tie their money up for longer periods unless paid more to do so.

Market Segmentation Theory: Shape of yield curve is based upon the supply and demand for funds.

- The supply and demand changes based upon the maturity levels: short-term vs. long-term.
- If more borrowers (demand) want to borrow long-term than investors want to invest (supply) long-term, then the interest rates (price) for long-term funds will go up.
- If fewer borrowers (demand) want to borrow long-term than investors want to invest (supply) long-term, then the interest rates (price) for long-term funds will go down.

Interpreting yield curve:

- Upward-sloping yield curves result from:
  - Higher inflation expectations
  - Lender preference for shorter-maturity loans
  - Greater supply of shorter-term loans

- Flat or downward-sloping yield curves result from:
  - Lower inflation expectations
  - Lender preference for longer-maturity loans
  - Greater supply of longer-term loans