1.) The operator $L^2$ is given by the following expression:

$$L^2 = -\frac{\hbar^2}{2\pi} \left[ \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \right]$$

(15) a.) Show that the function

$$\psi = \sin^2 \theta \cos \theta e^{2i\phi}$$

is an eigenfunction of this operator.

(3) b.) What is the eigenvalue corresponding to this eigenfunction? ______

(6) c.) The eigenfunction $\psi$ given in part a.) is an angular part of the hydrogen atom wavefunction. Assign the quantum numbers $\ell$ and $m$ that correspond to this wavefunction. Hint: $L_z = -i\hbar \frac{\partial}{\partial \phi}$
(12) 2.) Prove that the average value of the momentum for a harmonic oscillator in quantum mechanics equals zero. Hint: consider the even, odd properties of the wavefunction.

3.) The radial probability distribution function is given by the expression $4\pi r^2 R'(r)$. The unnormalized radial part of the wavefunction of the hydrogen atom is given by the expression:

$$ R(r) = (2 - \frac{r}{a_0}) e^{-\frac{r}{2a_0}} $$

(14) a.) Find the value(s) of $r$ at which the function $4\pi r^2 R'(r)$ assumes maximum value(s).

(4) b.) Find the value of $r$ at which $R(r)$ has a node.
(15) 4.) The first excited electronic states of He₂ are formed by exciting an electron from the antibonding \( 1\sigma \) molecular orbital to the bonding \( 2\sigma_3 \) orbital. Give the term symbol(s) corresponding to this excited configuration. Give also the bond order in this excited configuration. Give the term symbol of the ground state and indicate which emission line(s) from the excited to the ground state is an allowed transition.

(9) 5.) Give the point group symmetry of
   a.) anthracene ___________________________
   b.) \( \text{H}_2\text{O}_2 \) (the 2 HOO planes are at \( 90^\circ \) with respect to one another) ________________
   c.) an egg ________________

(20) 6.) The Stokes lines of the rotational Raman spectrum are given by the expression \( \nu = 4B_0 (J+3/2) \). For the \( \text{H}_2 \) molecule the following rotational lines are observed: 355, 588, 815, and 1033 \( \text{cm}^{-1} \). Find \( B_0 \) and \( r_0 \) for the \( \text{H}_2 \) molecule. \( h=6.626\times10^{-14} \text{ J sec} \), \( c=2.998\times10^8 \text{ m/sec} \), \( m_n=1.01 \text{ amu} \).
(12) 7.) A radical containing two inequivalent protons with splitting constants 2.0 mT and 2.6 mT gives a spectrum centered on 332.5 mT. At what fields do the hyperfine lines lie?

(15) 8.) A molecule exists in singlet and triplet forms, with the singlet having the higher energy by $4.11 \times 10^{-21}$ J per molecule. The singlet level is nondegenerate and the triplet level is 3-fold degenerate. Ignoring higher electronic levels, calculate the electronic partition function. Find the ratio of the populations of the singlet to triplet levels at 298 K.
(10) 9.) The most intense line in the electronic transition X-->A of a diatomic molecule is the 0-->0 transition, but for another electronic transition X--B the most intense vibrational band is the 0-->5 band. What can you say about the bond lengths in the A and B electronic states versus the bond length in the ground electronic state?

(15) 10.) The partition function of a molecule is a product of translational, rotational, vibrational, and electronic partition functions. Only the translational partition function depends on the volume. The translational partition function is given by the expression:

\[ q_t = \left( \frac{2\pi m kT}{h^2} \right)^{3/2} V \]

Using the expression for the Helmholtz free energy 
\[ A = -N kT \ln q \] . Derive the ideal gas equation from these relationships. Hint: Remember the Thermodynamic equation 
\[ dA = -SdT - PdV \]