I and Q Components in Communications Signals and Single Sideband

SHARLENE KATZ
DAVID SCHWARTZ
JAMES FLYNN
OVERVIEW

- Description of I and Q signal representation
- Advantages of using I and Q components
- Using I and Q to demodulate signals
- I and Q signal processing in the USRP
- Single Sideband (SSB)
- Processing I and Q components of a SSB signal in the USRP
# Standard Representation of Communications Signals

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### AM
- Modulation: Amplitude Modulation
- Frequency Domain: $X_{AM}(f) = A \cos(2\pi f - 2\pi f_c t)$

### DSB
- Frequency Domain: Two sidebands

### FM
- Frequency Domain: Carrier and sidebands
I and Q are the In-phase and Quadrature components of a signal.

Complete description of a signal is:

\[ x(t) = I(t) + jQ(t) \]

\( x(t) \) can therefore be represented as a vector with magnitude and phase angle.

Phase angle is not absolute, but relates to some arbitrary reference.
Overview of I and Q Representation

• In Digital Signal Processing (DSP), ultimate reference is local sampling clock.

• DSP relies heavily on I and Q signals for processing. Use of I and Q allows for processing of signals near DC or zero frequency.
  o If we use “real” signals (cosine) to shift a modulated signal to baseband we get sum and difference frequencies
  o If we use a “complex” sinusoid to shift a modulated signal to baseband we ONLY get the sum
  o This avoids problems with images
Overview of I and Q Representation

- Nyquist frequency is twice highest frequency, not twice bandwidth of signal.

  For example: common frequency used in analog signal processing is 455 kHz. To sample in digital processing, requires 910 kS/s. But if the signal bandwidth is only 10 kHz. With I & Q, sampling requires only 20 kS/s.
Overview of I and Q Representation

- I and Q allows discerning of positive and negative frequencies.
  - If: \[ H(f) = a + jb \]
  - Then: \[ H(-f) = a - jb \]
Overview of I and Q Representation

Representing familiar characteristics of a signal with I and Q:

- **Amplitude:** \( A(t) = \sqrt{I^2(t) + Q^2(t)} \)
- **Phase:** \( \phi(t) = \tan^{-1}\left(\frac{Q(t)}{I(t)}\right) \)
- **Frequency:**

\[
f(t) = \frac{\partial \phi(t)}{\partial t} = \frac{I(t)\frac{\partial Q(t)}{\partial t} - Q(t)\frac{\partial I(t)}{\partial t}}{I^2(t) + Q^2(t)}
\]
DEMODULATION

- **AM:** \( x(t) = \sqrt{i^2(t) + q^2(t)} \)

- **SSB:** \( x(t) = i(t) \)

- **FM:** \[ x(t) = \left( \frac{1}{\Delta t} \right) \tan^{-1} \left[ \frac{i(t)q(t-1) + q(t)i(t-1)}{i(t)i(t-1) - q(t)q(t-1)} \right] \]

- **PM:** \[ x(t) = \tan^{-1} \left[ \frac{q(t)}{i(t)} \right] \]
Overview of I and Q Representation

- The traditional FM equation:

\[ x_{FM}(t) = \cos(\omega_c t + k \int x_m(t) dt) \]

- The analytic equation:

\[ x_{FM}(t) = I(t) \cos(\omega_c t) + jQ(t) \sin(\omega_c t) \]

- Modulation and Demodulation methods are different when I and Q representation is used
USRP DAUGHTER BOARD

AMP

\[ \cos \omega_c t \]

\[ \sin \omega_c t \]

LPF \rightarrow ADC \rightarrow I

LPF \rightarrow ADC \rightarrow Q

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FPGA

\[
\begin{align*}
\cos \omega_f t_n & \quad \text{n = sample number} \\
\sin \omega_f t_n & \\
\text{complex multiply} & \quad \text{decimate} \\
I & \quad \Rightarrow \quad I \quad \text{To USB and PC} \\
Q & \quad \Rightarrow \quad Q
\end{align*}
\]
Complex Multiply

\[(A + j B) \times (C + j D) = AC - BD + j(BC + AD)\]
Sideband Modulation

- Where’s the intelligence?
  - A signal carries useful information only when it changes.
  - Change of **ANY** carrier parameter produces sidebands.
  - The intelligence or information is in the sidebands.
- Why not just send the sidebands or just a sideband?
AM Review

- **AM review:**
  - Carrier is modulated by varying amplitude linearly proportional to intelligence (baseband) signal amplitude.

- **Block Diagram**

\[
x(t) \rightarrow m \rightarrow x \rightarrow + \rightarrow x_{AM}(t) = A_c [1 + mx(t)]\cos \omega_c t
\]
AM: Time Domain

- AM in the Time Domain

Unmodulated carrier

100% modulated carrier
AM: Frequency Domain

- AM in the Frequency Domain

![Graph showing carrier and sidebands](image-url)
Double Sideband Modulation (DSB)

- Let’s just transmit the sidebands

\[ x(t) \rightarrow m \rightarrow x \rightarrow + \rightarrow x_{DSB}(t) = A_c \cdot m \cdot x(t) \cdot \cos\omega_c t \]
DSB: Time Domain

- Double Sideband in the Time Domain
Double Sideband in the Frequency Domain

carrier was here

lower sideband
upper sideband
Example of a DSB Signal
DSB Spectrum

- Note: the upper and lower sidebands are the same
- Do we need both of them?

carrier was here

![Graph showing DSB Spectrum with upper and lower sidebands marked]

- lower sideband
- upper sideband
A sideband signal is obtained by adding a sideband filter to capture the upper or lower sideband.
Example of a USB Signal
Comparison of DSB and SSB

- Power: SSB requires half of the power of DSB
- Bandwidth: SSB requires half of the bandwidth of DSB
- Complexity: SSB modulators/demodulators are more complex
SSB Example

- Start with arbitrary waveform in baseband:
SSB Example

- Modulate as Upper Sideband Signal:
SSB Example

\[ \cos \omega_c t \]

\[ \sin \omega_c t \]
SSB Example

\[ \cos(\omega_c t) \]

\[ \sin(\omega_c t) \]

LPF

ADC

AMP
SSB Example

\[ I = \cos(\omega_c t) \]
\[ Q = \sin(\omega_c t) \]
SSB Example

\[ \text{AMP} \rightarrow \cos \omega_c t \rightarrow \text{LPF} \]

\[ \text{AMP} \rightarrow \sin \omega_c t \rightarrow \text{LPF} \]
SSB Example

\[ I = \cos \omega_c t \]

\[ Q = \sin \omega_c t \]
SSB Example

\[
\begin{align*}
\cos \omega_f t_n &\quad \text{n = sample number} \\
\sin \omega_f t_n &\quad \text{decimate} \\
\end{align*}
\]

To USB and PC
SSB Example

\[ I = \cos \omega_f t_n \]

\[ Q = \sin \omega_f t_n \]

decimate

To USB and PC

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SSB Example

cos \omega f t_n

sin \omega f t_n

n = sample number

decimate

decimate

To USB and PC

I

Q

complex multiply

I

Q
SSB Example

\[ \text{complex multiply} \quad \text{decimate} \]

\[ \cos \omega_f t_n \quad \text{n = sample number} \quad \text{decimate} \]

\[ \sin \omega_f t_n \]

\[ I \quad Q \]

To USB and PC

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Example above assumed no other signals on the band and perfectly synchronized oscillators

Need to isolate (filter) the signal of interest and deal with oscillators slightly out of sync

GRC tutorial demonstrates Weaver’s Method of demodulating SSB that solves these problems