Problem Set #1
Plane Euclidean Geometry

1. (Triangle Geometry) In triangle $ABC$, $AB=AC$. Point $P$ strictly between $A$ and $B$ such that $AP=PC=CB$. Find $\angle A$.

2. (Triangle Geometry) Triangle $ABC$ has a right angle at $C$, $AC=3$ and $BC=4$. Triangle $ABD$ has a right angle at $A$ and $AD=12$. Points $C$ and $D$ are in opposite sides of $\overline{AB}$. The line through $D$ parallel to $\overline{AC}$ meets $\overline{CB}$ extended at $E$. Find $\frac{DE}{DB}$.

3. (Circle Geometry) Triangle $ABC$ is inscribed in a circle, and $\angle B = \angle C = 4\angle A$. Also $B$ and $C$ are adjacent vertices of a regular polygon of $n$ sides in this circle. Find $n$.

4. (Circle Geometry) An acute isosceles triangle, $ABC$, is inscribed in a circle. Through $B$ and $C$, tangents to the circle are drawn, meeting at point $D$. If $\angle ABC = \angle ACB = 2\angle D$, find $\angle A$ (in radians).

5. (Coordinate Geometry) In the $xy$-plane, consider the L-shaped region bounded by horizontal and vertical segments with vertices $(0,0)$, $(0,3)$, $(3,3)$, $(3,1)$, $(5,1)$, and $(5,0)$. Find the slope of the line passing through the origin that divides the area of this region exactly in half.

6. (Coordinate Geometry) If $ABCD$ is a $2\times2$ square, $E$ is the midpoint of $\overline{AB}$, $F$ is the midpoint of $\overline{BC}$, $\overline{AF}$ and $\overline{DE}$ intersect at $I$, and $\overline{BD}$ and $\overline{AF}$ intersect at $H$, find the area of quadrilateral $BEIH$.

7. (Triangle Inequality) A triangle with integral sides has perimeter 8. Find its area.

8. (Areas) The length of rectangle $ABCD$ is 5 inches and its width is 3 inches. Diagonal $AC$ is divided into three equal segments by points $E$ and $F$. Find the area of triangle $BEF$.

9. (Areas) An 8 by $2\sqrt{2}$ rectangle has the same center as a circle of radius 2. Find the area of the region common to both the rectangle and the circle.

10. (Law of Cosines) In a triangle with sides of length $a$, $b$, and $c$, 
    
    \[(a + b + c)(a + b - c) = 3ab .\]

    Find the measure of the opposite angle to the side of length $c$. 

Required Topics

Angles, triangles, and circle theorems (e.g. triangle inequality, Pythagorean Theorem, similar triangles, inscribed and central angles in a circle). Understanding of perimeters and areas. Coordinate geometry in the plane (e.g. distance formula, equation of a line). Trigonometry (e.g. trigonometric identities, law of sines, and law of cosines).

References & Resources

   
   [http://www.studyworksonline.com/cda/explorations/main/0,,NAV2-21,00.html](http://www.studyworksonline.com/cda/explorations/main/0,,NAV2-21,00.html)

   You can also go to [http://www.studyworksonline.com](http://www.studyworksonline.com), click on “Explorations” (upper menu) and then on “Interactive Geometry” (left menu).