Review for Final Exam
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Mechanical Engineering 483
Alternative Energy Engineering II
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Final Exam
• Monday, May 10, 3–5 pm
• Open book and notes
  – No books other than course text
  – No homework solutions or in-class exercise solutions
• Will be problems similar to those on homework and in-class exercise
• More credit for correct approach than for details of algebra or arithmetic
• No questions on material since second midterm

What is energy?
• Energy and power (energy/time) units
  – Energy units: joules (J), kilowatt-hours (kWh), British thermal units (Btu)
    • 1 Btu = 1055.056 J
  – Power units: watts (W), Btu/hr
    • 1 W = 1 J/s = 3.412 Btu/hr
  – Fuel equivalencies: 1 ft³ natural gas ≈ 1000 Btu; 1 bbl crude = 5.8 MMBtu; 1 Mtoe oil = 41.868 \times 10^{15} J = 0.0387 quads
  – World energy use (2006) was 466 quads

Energy Costs
• Home costs (San Fernando Valley 2008)
  – Electricity: $0.115/kWh = $32/GJ
  – Increase from $0.11/kWh to $0.12/kWh
  – Natural gas: $1.07/therm = $11/GJ
  • One therm = 10^9 Btu is approximately the energy in 100 standard cubic feet of natural gas
  • Range was $0.69 to $1.22 per therm
  – Gasoline at $3.00 per gallon (including taxes) costs $26/GJ
  • Assumes energy content of gasoline is 5.204 MMBtu per (42 gallon) barrel
  • $100/bbl oil costs $6.20/GJ (5.80 MMBtu/bbl)
  – Energy cost without California gasoline taxes ($0.585/gallon) is $21/GJ

Resources vs. Reserves
<table>
<thead>
<tr>
<th></th>
<th>Known</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economical to Recover</td>
<td>Reserves</td>
<td>Resources</td>
</tr>
<tr>
<td>Not economical to recover</td>
<td>Resources</td>
<td>Resources</td>
</tr>
</tbody>
</table>

Resource Probabilities

ME 483 – Alternative Energy Engineering II
Hubbert Peak

- Analysis due to M. King Hubbert
- Main publications in 1949 and 1956
- Correctly predicted peak in US oil production in early 1970s
- Not so accurate in other predictions
- Some recent applications show world oil production peak in next ten years
- Many other studies show later peak

Basic Combustion Analysis

- General fuel formula: \( C_{x}H_{y}S_{z}O_{w}N_{v} \)
- \( x, y, z, w, \) and \( v \) from ultimate analysis or analysis of gas mixtures
- Ultimate analyses:
  - \( x = \frac{wt\%C}{12.0107}, y = \frac{wt\%H}{1.00794}, \\
  z = \frac{wt\%S}{32.065}, \ w = \frac{wt\%O}{16.0004}, \\
  v = \frac{wt\%N}{14.0067}, \ m_{\text{fuel}} = 100 \)
  - \( M_{\text{fuel}} = 12.0107x + 1.00794y + 32.065z + 15.9994w + 14.0067v = m_{\text{fuel}}(1 - \%MM) \)
- For mixture of compounds \( (ω_k = \text{mole fraction}) \)
  \[
  x = \sum_{\text{species}} ω_k x_k, \ y = \sum_{\text{species}} ω_k y_k, \ M_{\text{fuel}} = \sum_{\text{species}} ω_k M_k
  \]

Combustion Air

- \( A = x + \frac{y}{4} + z - \frac{w}{2} = \) stoichiometric moles \( O_2/\text{mole fuel} \)
- Need input data on Actual \( O_2/\text{Stoichiometric} \ O_2 = \text{Relative air/fuel ratio} = \lambda \)
- Air/fuel ratio \( = \frac{m_{\text{air}}}{m_{\text{fuel}}} = 138.28\lambda A/m_{\text{fuel}} \)
- \( C_{x}H_{y}S_{z}O_{w}N_{v} + \lambda A(O_2 + 3.77N_2) \rightarrow xCO_2 + \frac{y}{2}H_2O + zSO_2 + (\lambda - 1)AO_2 + 3.77\lambda A + \frac{v}{2}N_2 \)

Exhaust Oxygen and \( \lambda \)

- Can relate these two quantities with fuel properties
- Can compute theoretical \( \%O_2 \) for given \( \lambda \)
- Dry exhaust has water removed to protect chemical analyzers
- \[
  \%O_2_{\text{dry}} = \frac{(\lambda - 1)A}{x + 4.77\lambda A - A + z + \frac{v}{2}}
  \]
- \[
  \lambda = \frac{\%O_2_{\text{dry}}}{100} \left( x + \frac{4.77\lambda A - A + z + \frac{v}{2}}{\%O_2_{\text{dry}}} \right)
  \]

Emission Rates

- Often stated as pollutant mass per unit heat input from fuel
- Equation used: \( E_i = \rho_{i,d} F_d \frac{20.9}{20.9 - \%O_2_{d,f}} \)
- Compute \( \rho_{i,d} = y_i,d MP_{\text{std}}/R_{u,T_{\text{std}}} \)
- \( F_d \) is dry exhaust volume/heat input
  - Use default values or compute by equation
    - Feb 3 notes have values of \( K's \) and default \( F_d's \)
    - \[
      F_d = \frac{K(K',%C + K_7%H + K_8%O + K_9%S + K_9%N)}{Q}
    \]
Other Equations

- Pollutant mass per unit heat input
  \[
  \frac{m_{\text{CO}_2}}{Q_{\text{fuel}}} = \frac{3.6642 \text{ wt}\%}{100} \quad \frac{m_{\text{SO}_2}}{Q_{\text{fuel}}} = \frac{1.9979 \text{ wt}\%}{100}
  \]

- Combustion Efficiency (definitions on next slide)
  \[
  \eta_{\text{comb}} = \frac{P_{\text{max}}}{Q_{\text{fuel}}} = 1 - \left(1 + \frac{\text{Air}}{\text{Fuel}}\right) \int_{T_a}^{T_c} \left(\frac{dT}{T} - \frac{\Delta h_{\text{CO}_2}}{M_{\text{fuel}}Q_{\text{fuel}}}\right)
  \]

Combustion Efficiency

- Air/fuel is the air to fuel (mass) ratio
- \(C_{p,\text{air}} = 0.24 \text{ Btu/lbm} \cdot R = 1.005 \text{ kJ/kg} \cdot \text{K}\)
- \(f = \text{molar exhaust ratio CO/(CO + CO}_2\))
- \(x = \text{carbon atoms in fuel formula, C}_x\text{H}_y\ldots\)
- \(Q_{\text{c}} = \text{heat of combustion (Btu/lbm or kJ/kg)}\)
  - Use lower heating value for water vapor (usual case)
  - \(\Delta h_{\text{CO}_2} = 282,990 \text{ kJ/kgmol} = 121,665 \text{ Btu/lbmol}\)
- \(M_{\text{fuel}} = \text{combustible fuel molar mass lbm/lbmol or kg/kmol}\)

Energy Economics

- Look at balance between initial cost and ongoing costs
  - Uses interest rate to consider time value of money
- Key formula relates equivalence between initial cost, \(P\) (present value), and ongoing payment stream, \(A\) (annual cost)
  \[
  \frac{A}{P} = \frac{i}{1-(1+i)^{-n}} \quad \frac{P}{A} = \frac{1-(1+i)^{-n}}{i}
  \]

Using the A/P formula

- Formula applies to any time period so long as \(i\) is interest rate per time period
- E.g., for monthly costs with \(i = 6\% / \text{yr} = 0.5\% / \text{month for N months}\)
  \[
  \frac{A}{P} = \frac{0.005}{1-(1+0.005)^{-N}}
  \]
- Need trial-and-error solution (or financial calculator) to find \(i\), given \(n\) and \(A/P\)
- Can find \(n\) for given \(i\) and \(A/P\)
  \[
  N = \frac{\ln\left(1 - \frac{P}{A}\right)}{\ln(1+i)}
  \]

Energy Storage Measures

- Energy per unit mass (kJ/kg; Btu/lbm)
- Energy per unit volume (kJ/m³; Btu/ft³)
- Rate of delivery of energy to and from storage (kW/kg; Btu/hr-lbm)
- Efficiency (energy out/energy in)
- Life cycles – how many times can the storage device be used
  - Particularly important for batteries

Compare

- Batteries versus other motive power
  http://www.nap.edu/books/0309092612/html/40.html
Renewable/Alternative

- Alternative or renewable resources
  - Solar energy
  - Wind energy
  - Ocean energy (tides, waves and temperature gradients)
  - Geothermal energy
  - Hydropower especially small hydro
  - Biomass fuels
  - Conservation as an alternative resource
    - Reduced usage and improved efficiencies including vehicle fuel economy

US Electric Net Summer Capacity (EIA Data)


Power Generation Costs

Wind Power and Betz Limit

- Power in incoming air = \( m = m V^2/2 = (\rho VA)V^2/2 = \rho AV^2/2 = P_0 \)
  - Air density, \( \rho \approx 1.2 \text{ kg/m}^3 \)
  - \( A \) = swept area of rotor = \( \pi(D_{\text{rotor}})^2/4 \)
  - \( V \) = wind velocity

- \( c_p = \text{power coefficient} = \text{turbine power divided by power in wind} \)
  - Alternative: \( (\text{generator power}) / (\text{wind power}) \)

- Betz Limit: Maximum theoretical \( c_p = 16/27 \approx 0.593 \)
Effect of V^3 Dependence

- Site Wind Distribution
- Energy calculations assume Betz c_p and a 100 m rotor diameter

Rayleigh Distribution

- At least three variations are used
  
  \[ V_{mp} = \beta = \frac{c}{\sqrt{2}} \quad f(V) = \frac{Ve^{-V^2/2\beta^2}}{\beta^2} \quad 0 \leq V < \infty \]
  
  \[ 2\beta^2 = c^2 \quad f(V) = \frac{2Ve^{-V^2/c^2}}{c^2} \quad 0 \leq V < \infty \]
  
  \[ \bar{V} = \beta \sqrt{\frac{\pi}{2}} = \frac{c}{2\sqrt{\pi}} \quad f(V) = \frac{\pi Ve^{-\pi V^2/4\bar{V}^2}}{2\bar{V}} \quad 0 \leq V < \infty \]

Weibull Distribution

- A two-parameter distribution with shape parameter, k, and scale parameter, c
- Rayleigh distribution is Weibull distribution with k = 2
- Mean = c\Gamma(1 + k^{-1})
- Variance = c^2[\Gamma(1 + 2k^{-1}) - \Gamma^2(1 + k^{-1})]
  
  \[ f(V) = \frac{k}{c} \left( \frac{V}{c} \right)^{k-1} e^{-\left( \frac{V}{c} \right)^k} \quad 0 \leq V < \infty \]

Gamma Function

\[ \Gamma(x+1) = x\Gamma(x) \quad \Gamma(1) = \Gamma(2) = 1 \quad \Gamma\left( \frac{1}{2} \right) = \sqrt{\pi} \]
Wind Power

- Instantaneous wind power: \( P_0 = \rho V^3 A/2 \)
- Total or average wind power:
  \[
  \bar{P} = \frac{\rho A V^3}{2} - \frac{\rho A}{2} \int_0^\infty V^3 f(V) dV
  \]
- Total or average turbine power:
  \[
  \bar{P}_{\text{total}} = c_p \bar{P} = c_p \rho A V^3 / 2
  \]

\[
\bar{V} = c^3 \Gamma \left( \frac{3}{k} + 1 \right)
\]

Wind Power Distribution

- Wind power between \( V_1 \) and \( V_2 \)
  - Weibull (Set \( k = 2 \) for Rayleigh)
  \[
  \bar{P} = \frac{\rho A c^3 (V_2/c)^3}{2} \int_{(V_1/c)^3}^{(V_2/c)^3} y^2 e^{-y} dy
  \]
  - Found by numerical integration with results in tables

Wind Turbine Operation

- No operation until wind velocity reaches a minimum called the cut-in velocity
- Then operate at full turbine output power until turbine output is greater than generator can accept
- Limit turbine output power to full generator power at high wind speeds
- No operation above maximum velocity called cut-out velocity

Frequency and Power Distributions

- Generator uses turbine power between \( V_{\text{cut-in}} \) and rated (maximum power) velocity, \( V_{\text{Pmax}} = [2P_{\text{max}}/(c_p A)]^{1/3} \)
  - Power coefficient \( c_p \) = generator power divided by wind power
- Between \( V_{\text{Pmax}} \) and \( V_{\text{cut-out}} \) operate at maximum power
  \[
  \bar{P}_{\text{operation}} = \int_{V_{\text{cut-in}}}^{V_{\text{Pmax}}} c_p \rho A V^3 f(V) dV + \int_{V_{\text{Pmax}}}^{V_{\text{cut-out}}} \bar{P}_{\text{max}} f(V) dV
  \]
Average Operating Power II

- Using power fraction table
  \[ \frac{V_{\text{pmax}} e^{\Delta V^3 \int f(V)dV}}{V_{\text{cut-in}}} = \left( f_p(V_{\text{cut-in}}) - f_p(V_{\text{max}}) e^{\Delta V^3 \int f(V)dV} \right) \]

- Using cumulative distribution
  \[ \int_{V_{\text{max}}}^{V_{\text{pmax}}} f(V)dV = \frac{P_{\text{max}}}{P_{\text{cut-in}}} \left[ \frac{1 - e^{-\Delta V^3 f(V)}}{1 - e^{-\Delta V^3 f(\text{max})}} \right] \]

+ \[ P_{\text{operation}} = \left( f_p(V_{\text{cut-in}}) - f_p(V_{\text{max}}) e^{\Delta V^3 f(\text{max})} \right) + \frac{P_{\text{max}}}{f_p(\text{cut-in})} \]

Electromagnetic Radiation

- Radiation heat transfer by electromagnetic radiation
  - Part of much larger spectrum
  - Thermal radiation transfers heat without contact
    - Use of fire or electric resistance heating are best examples
    - Thermal radiation lies in infrared and visible part of spectrum (with some in ultraviolet)

Black-body Radiation Spectrum

- Basic black body equation: \( E_b = \sigma T^4 \)
  - \( E_b \) is total black-body radiation energy flux
    - \( W/m^2 \) or \( \text{Btu/hr·ft}^2 \); \( \sigma \) is the Stefan-Boltzmann constant
- \( E_{b\lambda} \) is spectral radiation
  - Units are \( W/(m^2·\mu m) \)
  - \( E_{b\lambda} d\lambda \) is fraction of black body radiation in range \( d\lambda \) about wavelength \( \lambda \)
- Maximum occurs at \( \lambda T = 2897.8 \mu m·K \)
  - \( T \) increase shifts peak shift to lower \( \lambda \)

Partial Black-body Power

- Black body radiation between \( \lambda = 0 \) and \( \lambda = \lambda_1 \) is \( E_{b,0:0} = \int_0^{\lambda_1} E_{b\lambda} d\lambda \)

Fraction of total radiation \( (\sigma T^4) \) between \( \lambda = 0 \) and any given \( \lambda \) is \( f_{\lambda} \)

\[ f_{\lambda,0:0} = \frac{1}{\sigma T^4} \int_0^{\lambda_1} E_{b\lambda} d\lambda \]
Emissivity

- Emissivity = ratio of actual radiated power to that of black body
  - Diffuse surface – emissivity does not depend on direction
  - Gray surface – emissivity does not depend on wavelength
  - Gray, diffuse surface – emissivity is the same and does not depend on direction or wavelength
  - Simplest surface to handle and often used in radiation calculations

Properties

- When radiation, G, hits a surface a fraction ρG is reflected; another fraction, αG is absorbed, a third fraction τG is transmitted
- Energy balance: ρ + α + τ = 1

Kirchhoff’s Law

- Absorptivity equals emissivity (at the same temperature) αλ = ελ
- True only for values in a given direction and wavelength
- Assuming total hemispherical values of α and ε are the same simplifies radiation heat transfer calculations, but is not always a good assumption

Effect of Temperature

- Emissivity, ε, depends on surface temperature
- Absorptivity, α, depends on source temperature (e.g. Tsun ≈ 5800 K)
- For surfaces exposed to solar radiation
  - high α and low ε will keep surface warm
  - low α and high ε will keep surface cool
  - Does not violate Kirchoff’s law since source and surface temperatures differ

<table>
<thead>
<tr>
<th>Surface</th>
<th>αs</th>
<th>ε</th>
<th>Surface</th>
<th>αs</th>
<th>ε</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>0.09</td>
<td>0.63</td>
<td>Black nickel oxide</td>
<td>0.92</td>
<td>0.08</td>
</tr>
<tr>
<td>Polished</td>
<td>0.14</td>
<td>0.84</td>
<td>Black chrome</td>
<td>0.87</td>
<td>0.09</td>
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<tr>
<td>Anodized</td>
<td>0.15</td>
<td>0.84</td>
<td>Concrete</td>
<td>0.60</td>
<td>0.88</td>
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<tr>
<td>Foil</td>
<td>0.15</td>
<td>0.84</td>
<td>White marble</td>
<td>0.46</td>
<td>0.99</td>
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<tr>
<td>Copper</td>
<td>0.18</td>
<td>0.84</td>
<td>Red brick</td>
<td>0.63</td>
<td>0.93</td>
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<tr>
<td>Polished</td>
<td>0.18</td>
<td>0.84</td>
<td>Asphalt</td>
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<td>Tarnished</td>
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<td>0.75</td>
<td>Black paint</td>
<td>0.97</td>
<td>0.97</td>
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<tr>
<td>Stainless steel</td>
<td>0.37</td>
<td>0.60</td>
<td>White paint</td>
<td>0.14</td>
<td>0.93</td>
</tr>
<tr>
<td>Polished</td>
<td>0.37</td>
<td>0.60</td>
<td>Snow</td>
<td>0.28</td>
<td>0.97</td>
</tr>
<tr>
<td>Dull</td>
<td>0.50</td>
<td>0.21</td>
<td>Human skin (Caucasian)</td>
<td>0.62</td>
<td>0.97</td>
</tr>
</tbody>
</table>

From Çengel, Heat and Mass Transfer
Computing the Sun Path

- Input data: Latitude, $L$, date, hour $h$
- Find declination from serial date, $n$
  \[
  \delta = \left(23.45^\circ\right) \sin \left(\frac{360}{365} \left(284 + n\right) \frac{\pi}{180}\right) \quad (\delta \text{ in degrees})
  \]
- Two angles: altitude ($\alpha$) and azimuth ($\phi$)
  - $\sin(\alpha) = \sin(L) \sin(\delta) + \cos(L) \cos(\delta) \cos(h)$
  - $\sin(\alpha_s) = \sin(\phi) = \cos(\delta) \sin(h) / \cos(\alpha)$
  - Sun path is plot of $\alpha$ vs. $\phi = \alpha_s$ for one day
  - Plot is symmetric about solar noon
  - Typically plot data for 21st of month

Solar Irradiation by Month in Los Angeles (LAX)

Optimum Fixed Collector Tilt

Solar Time = Standard Time +
Equation of Time +
$\left(4 \text{ min/h}^2\right) \times$
(Standard Longitude - Local Longitude)
Standard Time = DST - 1 hour

Notes:
- All fixed collectors are facing south
- $L = 33.93^\circ$ for Los Angeles, USA
- Fixed, tilt = 0
- Fixed, tilt = $L$
- Fixed, tilt = $L+15$
- Fixed, tilt = 90
- 1-axis, track, EW horizontal
- 1-axis, track, NS horizontal
- 1-axis, track, tilt = $L$
- 1-axis, tilt = $L+15$
- 2-axis, track

Notes for solar irradiation:
- All collectors facing south
- Showing data for 1961-1990 NREL data
- Showing data for different collectors

Optimum collector tilt is $35^\circ$
Solar Insolation by collector orientation and month at 40°N latitude.

Active Indirect Solar Water Heating

Flat Plate Collector

Passive Direct Solar Water Heating
Solar Collector Analysis

- Three analysis steps for solar energy to heat fluid (Hottel-Whillier-Bliss equation)
  - Solar energy into plate flows across plate to location of tubes at some line on plate
  - At same line heat flow into collector fluid from plate is determined
  - Integrate heat flow into fluid from inlet to exit to get total useful heat transfer to fluid

\[ \dot{Q}_u = F_R A_c \left[ H_a - U_c (T_{f,in} - T_u) \right] \]

\[ \dot{Q}_u = m_f (T_{f,out} - T_{f,in}) \]

Loss Through Top

- Analyze set of series thermal resistances with common Q_{top}
  - Heat transfer between absorber plate and lower glass plate shown below

\[ Q_{top} = \left( h_{g2-g1} + h_{p-g2} \right) A \left( T_{p} - T_{g2} \right) - \frac{T_{p} - T_{g2}}{R_{g2-g1}} \]

\[ h_{p-g2} = \frac{A \sigma (T_{p}^4 + T_{g2}^4)(T_{p}^4 + T_{g2}^4)}{1 + \frac{1}{\varepsilon_{p}} - \frac{1}{\varepsilon_{g2}} - 1} \]

Remaining Top Loss Path

- Between glass plates
  \[ Q_{top} = \left( h_{g2-g1} + h_{p-g2} \right) A \left( T_{g2} - T_{g1} \right) = \frac{T_{g2} - T_{g1}}{R_{g2-g1}} \]

- Top plate to ambient
  \[ Q_{top} = \left( h_{g2-g1} + h_{g2-g1} \right) A \left( T_{g2} - T_a \right) = \frac{T_{g2} - T_a}{R_{g2-g1}} \]

Total Loss

- \( Q_{sides} = U_{side}' A_{side} (T_p - T_a) \)
  - Can estimate \( U_{side}' = 0.5 \text{ W/m}^2\text{K} \)
  - Use \( U_{side}' A_{side} = U_{side}' A_{side} \) for common area
  - \( Q_{sides} = U_{side}' A_{side} (T_p - T_a) = (T_p - T_a)/A_{side} \)

- Total is sum of individual losses

- \( Q_{loss} = U_{top} A_{top} (T_p - T_a) = (T_p - T_a)/R_c \)

- Overall conductance and resistance

\[ U_c = U_{top} + U_{bottom} + U_{sides} \]

\[ \frac{1}{R_c} = \frac{1}{R_{top}} + \frac{1}{R_{bottom}} + \frac{1}{R_{sides}} \]

Approximate \( U_{top} \) Equation

\[ U_{top} = \frac{1}{A} \left( \frac{T_p - T_a}{N + B} \right) + \frac{1}{R_c} \]

\[ A' = 250 \left[ 1 - 0.0044 (s - 90) \right] \]

\[ B = (1 - 0.04 h_w + 0.0005 h_w^2)(1 + 0.091N) \]

\[ h_w = \text{heat transfer coefficient from top to ambient} \]

Equation uses SI units: \( U \) and \( h \) in W/m²·K, \( T \) in K, \( \sigma = 5.670 \times 10^{-8} \text{ W/m}^2\text{K}^4 \), \( \varepsilon \) is same for all glass covers
Absorber Plate Analysis

- Define \( m^2 = \frac{U_c}{tk_{plate}} \)
- Effectiveness factor, \( F = \frac{\tanh(mL)}{mL} \)
- Total (useful) heat transfer per unit length of tube

\[
q_{out} = (2LF + D)[H_a - U_c(T_f - T_a)] = q'_u
\]

Factors, \( F' \) and \( F_R \)

- Collector efficiency factor, \( F' \)
  \[
  F' = \frac{1}{\frac{1}{U_c} + \left[\frac{1}{(2L + D)U_c} + \frac{1}{C_B} + \frac{1}{(h_c + \pi D)}\right]} 
  \]
- Heat removal factor, \( F_R \)
  \[
  F_R = \frac{U_cA_cF'_u}{mc_p} \left[ 1 - e^{-F'_a \frac{mc_p}{A_c}} \right] \quad a = \frac{U_cA_cF_u}{mc_p} 
  \]

Collector Efficiency, \( \eta_c = \frac{Q_u}{A_cH_i} \)

- Start with Hottel-Whillier-Bliss Equation
  \[
  Q_u = F_RA_c[H_a - U_c(T_f - T_a)] 
  \]
- Replace \( H_a \) by \( H_i \tau_a \)
  \[
  Q_u = F_RA_c[H_i \tau_a - U_c(T_f - T_a)] 
  \]
- Substitute into efficiency equation
  \[
  \eta_c = \frac{Q_u}{A_cH_i} = \frac{F_RA_c[H_i \tau_a - U_c(T_f - T_a)]}{A_cH_i} = \frac{F_RU_c(T_f - T_a)}{H_i} 
  \]

Absorber Plate Analysis II

- Heat flow into fluid at any point

\[
q'_u = \frac{1}{U_c} \left[ \frac{H_a - U_c(T_f - T_a)}{2LF + D} \right] + \frac{1}{C_B} + \frac{1}{h_c + \pi D} \]

Summary of Results

- \( Q_u = \) useful heat transfer to working fluid
  \[
  m^2 = \frac{U_c}{k_{tp} + \frac{1}{U_c} + \left[\frac{1}{(2L + D)U_c} + \frac{1}{C_B} + \frac{1}{(h_c + \pi D)}\right]} 
  \]
- \( F' = \frac{1}{\frac{1}{U_c} + \left[\frac{1}{(2L + D)U_c} + \frac{1}{C_B} + \frac{1}{(h_c + \pi D)}\right]} \)
- \( H_a = H_i(\tau_a) \)
  \[
  F_R = \frac{U_cA_cF_u}{mc_p} \left[ 1 - e^{-F'_a \frac{mc_p}{A_c}} \right] 
  \]
- \( Q_u = AF_R[H_a - U_c(T_f - T_a)] \)
  \[
  T_{f,out} = T_{f,in} + \frac{Q_u}{mc_p} 
  \]

Solar Collector Efficiency Tests
Sample Rating Sheet

Sample Rating Sheet II

**Sample Rating Sheet**

<table>
<thead>
<tr>
<th>COLLECTOR TYPE</th>
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**Sample Rating Sheet II**

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<td>Model: ECO-10R</td>
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**f-chart Method**

- Predicts fraction of demand over a time period (usually monthly) than can be supplied by solar
- Two empirical parameters, $X$ and $Y$
  - $X$ is ratio of reference collector loss to total heating load
  - $Y$ is ratio of absorbed solar energy to total heating load

**Computing X (dimensionless)**

$$X = A_c \left( \frac{F_R U_c}{F_R} \right) \left( \frac{\Delta T_{in}}{T_H - T_{in}} \right)$$

- $A_c = \text{collector area (m}^2\text{)}$
- $F_R U_c (W/m^2\cdot K)$ from slope of collector test data
- $F_R/F_R$ computed or assumed = 0.97
- Usual averaging period, $\Delta t = 1$ month, converted to seconds
- $D = \text{heating demand for averaging period (J)}$
- $T_H = 100^\circ C; T_{in}$ from NREL data

**Computing Y (dimensionless)**

$$Y = A_c \left( \frac{F_R (\tau \alpha)}{F_R (\tau_n \alpha_n)} \right) \left( \frac{H_{total}}{D} \right)$$

- $F_R (\tau \alpha)_n$ from intercept of collector test
- $F_R/F_R$ computed or assumed = 0.97
- Ratio $\tau \alpha / (\tau_n \alpha_n)$ = 0.94 (October – March), = 0.90 (April – September) or computed
- $H_{total}$ is available from NREL data for $\Delta t$ = 1 month (convert to J/m$^2$)
- $D$ is heating demand J

**f Equations**

- For water heating: $f = 1.029 Y - 0.065 X - 0.245 Y^2 + 0.0018 X^2 + 0.0215 Y^3$
  - Adjustments required
    - Adjust $X$ for hot water supply only and storage capacity different from standard
    - Adjust $Y$ for load heat exchanger capacity
- For air heating: $f = 1.040 Y - 0.065 X - 0.159 Y^2 + 0.00187 X^2 - 0.0095 Y^3$
  - Solar collectors heating air have no heat exchanger so $F_R = F_R$
Adjustments

• Adjust X for storage capacity, M, in L/m²
  \[ X' = X \left( \frac{75}{M} \right)^{1/4} \]
• Adjustment for water heating only
  – See f-chart notes for details
• Adjust Y for load heat exchanger factor, Z:
  \[ Y' = Y \left( 0.39 + 0.65e^{-0.139/Z} \right) \]
  – \( \varepsilon_c \) = heat exchanger effectiveness

\[
Z = \varepsilon_c \left( \frac{\dot{m}c_p}{\varepsilon_m} \right) / (UA)_L
\]

NREL 1961-1990 LAX Average

SOLAR RADIATION FOR FLAT-PLATE COLLECTORS FACING SOUTH AT A FIXED-TILT (kWh/m² day): Percentage Uncertainty = 9

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