Solar Collector Analysis and Design
Larry Caretto
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Outline
- Midterm exam solutions
- Types of solar collectors
- Review Heat Transfer Basics
- Overview of solar collector analysis
- Analysis of losses from solar collectors
- Transfer of net heat gain to fluid tubes
- Increase in temperature of fluid
- Design equation for solar collectors

Midterm Exam
- Gas turbine using landfill gas with 85% CH₄ (Qᵣ = 802,802 kJ/kmol) 15% CO₂, 5% N₂
  - Pᵣ, comp,in = 100 kPa; Tᵣ, comp,in = 290 K; ηᵣ, comp = 87%;
  - DPᵣ, combustor = 30 kPa; Tᵣ, combustor = 1450 K; Pᵣ, out,turb = 110 kPa; ηᵣ, turb = 90%; P = 50 MW, ηᵣ, generator = 94%
- Find: air and fuel mass flows and outlet %O₂
- Start by finding heating value of landfill gas

Midterm Problem One II
- Compute compressor and turbine
  - \( \frac{W_{c}}{K} = 2.600 \text{ kPa} \) \( \frac{T_{c}}{T_{T}} = 112 \text{ kPa} \) \( \eta_{s,c} = 87\% \)
  - \( \frac{D_{P,combustor}}{K} = 30 \text{ kPa} \) \( \frac{T_{in,turbine}}{K} = 1450 \text{ K} \) \( \frac{P_{out,turb}}{K} = 110 \text{ kPa} \)
  - \( \eta_{s,turb} = 90\% \)
- Find: \( K_{N} \) \( P = 50 \text{ MW} \)
- \( \eta_{generator} = 94\% \)

Midterm Problem One III
- Get fuel/air ratio from combustor
  - \( \frac{w_{fuel}}{w_{air}} = \frac{1.148}{0.158} \frac{kJ}{K} \)
  - \( \frac{30,827 \text{ kJ}}{kg} \) \( \frac{1.148}{0.158} \frac{kJ}{K} \)

Midterm Problem One IV
- Find: %O₂ in exhaust
  - Need to find fuel formula components, A and \( \lambda \)
  - Get fuel formula as average over all components
  - \( \%O₂ = \frac{100(\lambda - 1)A}{x + \lambda A B_d - A + z + v} \)
Midterm Problem Two

- Given: Landfill-gas gas turbine produces 50 MW electricity at 80% capacity factor; maintenance cost = $0.007/kWh; sales price = $0.06/kWh
- Find: Cost to return 9% for 23 years
- 50 MW and 80% capacity factor gives (50,000 kW) (80%) (7866 h/yr) = 3.506X10^7 kWh/yr
- Sales minus maintenance = $0.06/kWh – $0.07/kWh = $0.053/kWh
- Annual income = (3.506x10^8 kWh/yr)($0.053/kWh) = $1.858x10^7/yr

\[ P_{\text{max}}(0.299) = (2.5 \text{ MW})(0.299) = 0.748 \text{ MW} \]

Midterm Problem Three

(a) show: Relation for mean cubed velocity in one band between \( V = a \) and \( V = b \)

\[ \overline{V^3}_{\text{band}} = \frac{b}{b-a} \int_{b-a}^{b} V^3 dV = \frac{b^4 - a^4}{4(b-a)} = \frac{b^2 + a^2}{4} \]

(b) Find: contribution of wind speeds between rated and cut-out speeds to the average operating power
- This is \( P_{\text{max}} \), times fraction of speeds in this range

Midterm Problem Three IV

(c) Find: contribution of wind speeds between cut-in speed and rated wind speeds

\[ P(V_{\text{cut-in}} < V < V_{\text{rated}}) = \frac{\text{Frac}}{k=\text{Start}} \int_{k=\text{Start}}^{k=\text{End}} \frac{c_k p_k A_k^3}{2} \]

- Start term has cut-in speed as lower bound
- \[ \int_{k=\text{Start}}^{k=\text{End}} \frac{c_k p_k A_k^3}{2} \] / \[ \text{Frac} \] = 93.775 kW

- End term has rated speed as upper bound
- \[ \int_{k=\text{Start}}^{k=\text{End}} \frac{c_k p_k A_k^3}{2} \] / \[ \text{Frac} \] = 19,128 kW
Midterm Problem Three V
• (d) Find: capacity factor if sum in part (c) = 0.3007 MW
• Capacity factor = average operating power divided by maximum power
• Average operating power = sum of part (b) and part (c) components = 0.7480 MW + 0.3007 MW = 1.0487 MW
• Capacity factor = (1.0487 MW) / (2.5 MW) = 41.9%

Active Indirect Solar Water Heating

Passive Direct Solar Water Heating

Liquid Flat Plate Collector

Review Fourier’s Law
• Basic law for heat conduction
• Actually a vector equation \( \hat{q} = -k \nabla T \)
• \( k \) is thermal conductivity
  – Units of \( k \) are W/m·K or Btu/hr·ft·°R
• For one dimensional heat transfer, \( \dot{q}_x \)
  = \( -kdT/dx \); integration (constant \( q_x \)) gives
  \[ \dot{q} = k \frac{(T_1 - T_2)}{L} \quad \text{or} \quad \dot{Q} = qA \frac{kA(T_1 - T_2)}{L} \]
**Review Convection Basics**

\[ \dot{Q}_{\text{conv}} = h A_f (T_f - T_s) \]

Velocity variation of air

\[ V \]

Air flow

Temperature variation of air

\[ T \]

\[ T_f \]

\[ T_s \]

- \( h \) = heat transfer coefficient (W/m²·K) or Btu/hr·ft²·°F
- \( h \) is found from empirical or theoretical equation

**Review Convection Types**

- **Free (natural) convection** comes from buoyancy, **forced convection** has a driven flow
- Flows contained in pipes and ducts are internal flows; egg pictures show external flow
- Other considerations are laminar vs. turbulent flow and convection during boiling or condensation

**Review Thermal Resistance**

- Heat flow analogous to current
- Temperature difference analogous to potential difference
- Both follow Ohm’s law with appropriate resistance term
- Current: \( I = \frac{(V_1 - V_2)}{R} \)
- Heat Transfer: \( Q = \frac{(T_1 - T_2)}{R_{\text{thermal}}} \)

**Review Thermal Resistance II**

- Conduction: \( \dot{Q} = \frac{L}{kA}(T_1 - T_2) \) \( \Rightarrow \dot{Q} = \frac{T_1 - T_2}{R_{\text{cond}}} \) \( \Rightarrow R_{\text{cond}} = \frac{L}{kA} \)
- Convection: \( \dot{Q} = hA(T_f - T_s) \) \( \Rightarrow \dot{Q} = \frac{T_f - T_s}{R_{\text{conv}}} \) \( \Rightarrow R_{\text{conv}} = \frac{1}{hA} \)
- Radiation: \( R_{\text{rad}} = \frac{1}{A_f\varepsilon T_1^3} \)

**Review Thermal Circuits**

- Electric current flow

\[ \dot{Q} = \frac{T_1 - T_2}{R} \]

\[ V_1 \]

\[ R_e \]

\[ V_2 \]

\[ R_e \]
Parallel Resistances ($T_{∞} = T_{sun}$)

$$\frac{1}{R_{rad}} = \frac{1}{R_{conv}} + \frac{1}{R_{total}}$$

Define total heat transfer coefficient, $h_{total}$

$$h_{total} = \frac{1}{A_{r}R_{total}} = h_{conv} + h_{rad}$$

Review Combined Modes

$$\dot{q} = k(T_{1} - T_{2})$$

$$\dot{q} = h(T_{2} - T_{∞})$$

Review Combined Modes II

$$\dot{Q} = \frac{T_{a1} - T_{a2}}{R_{total} + \frac{1}{h_{a1}} + \frac{1}{h_{a2}}}$$

Series Resistance Formula

$$\dot{Q} = \frac{T_{a1} - T_{a2}}{h_{a1} + \frac{1}{k} + h_{a2}}$$

Basic Solar Collector Analysis

- Overall heat balance
  - Incoming solar radiation
  - Heat loss from collector to environment
  - Useful energy gain = Incoming Solar Radiation − Environmental Heat Loss

- Environmental heat loss proportional to
  $\Delta T = T_{collector} - T_{ambient}$
  Applications that require high collector temperatures will have more heat loss

Useful Heat Transfer

- Heat is added to a collector fluid
  - Typically collector fluid is water or water and anti-freeze solution
  - Air is also used as collector fluid for home heating

- Energy added from simple first law for open system with constant pressure heat addition
  $$\dot{Q}_{u} = m_{c} c_{p} (T_{f, out} - T_{f, in})$$
**Solar to Useful Energy**

- Solar transmission through glass covers provides absorbed radiation, $H_a$
- Consider three losses
  - Conduction through bottom of solar collector box
  - Conduction through edge of box
  - Loss through top
    - Convection between absorber plate and glass covers with conduction through glass
    - Convection from top glass cover to ambient

**Absorbed radiation minus losses different for different areas of collector**

- Fluid temperature increases from inlet to outlet
- What is fluid temperature increase from absorbed radiation minus losses?


**Loss Through Top**

- In steady state the following heat rates will be the same
  - Between absorber plate and bottom glass
  - From bottom glass to top glass
    - Consider two-plate collector
    - From top glass to ambient
    - Look at exchange between absorber plate at temperature $T_P$ and bottom glass at temperature $T_{g2}$
    - Have convection plus radiation

**Loss Through Top II**

$$Q_{ap} = h_{_{g2-g1}}A_g(T_{g2} - T_{g1}) + \frac{A}{1 + \frac{1}{\varepsilon_g} - 1}$$

- Equation for convection plus radiation between two parallel plates
  - $A_g$ = Collector Area
  - $h_{_{g2-g1}}$ = convection coefficient for air gap
  - $\varepsilon_g$, $\varepsilon_{g2}$ = emissivities of plate and glass
  - $T_P$, $T_{g2}$ = temperatures of plate and glass

**Loss Through Top III**

$$\frac{A}{1 + \frac{1}{\varepsilon_g} - 1} = \frac{A}{1 + \frac{1}{\varepsilon_{g2}} - 1}$$

- Define radiation heat transfer coefficient, $h_{_{g2-g1}}$, as shown in dashed red square
  - Get equation for heat transfer and thermal resistance, $R_{_{g2-g1}}$
  - $Q_{ap} = (h_{_{g2-g1}} + h_{_{g2-g1}})A_g(T_{g2} - T_{g1}) = \frac{T_{g2} - T_{g1}}{R_{_{g2-g1}}}$

**Loss Through Top IV**

- Have similar equation for heat transfer between top and bottom glass plate
  - $Q_{ap} = (h_{_{g2-g1}} + h_{_{g2-g1}})A_g(T_{g2} - T_{g1}) = \frac{T_{g2} - T_{g1}}{R_{_{g2-g1}}}$
  - $h_{_{g2-g1}} = \frac{A}{1 + \frac{1}{\varepsilon_g} - 1}$
  - $T_{g1}$, $\varepsilon_{g1}$ = Temperature, Emissivity of top glass
  - $h_{_{g2-g1}}$ = convection coefficient for $g_1$ to $g_2$
Loss Through Top V

- Similar equation for heat transfer between top glass plate and ambient
  \[ Q_{top} = (h_{g1-a} + h_{g2-a}) A (T_{top} - T_a) = \frac{T_{top} - T_a}{R_{g2-g1}} \]
  \[ h_{g1-a} = \frac{A \sigma (T_{g1}^4 + T_{g2}^4) (T_{g1} + T_{g2})}{\varepsilon_{g1} + \varepsilon_{g2} - 1} \]
  - Radiation h different here because \( \Delta T \) for radiation uses \( T_{sky} \), not \( T_a \)
  - \( h_{g1-a} \) = convection coefficient for \( g_1 \) to ambient

Loss Through Top/Bottom

- Combine three resistances in series to get \( R_{top} = R_{g2-g1} + R_{g1-a} + R_{top} \)
  \[ Q_{top} = (T_{top} - T_a) / R_{top} = U_{top} A_c (T_{top} - T_a) \]
- Loss through bottom is conduction through insulation \((k_{ins}, \Delta x_{ins})\) in series with convection to ambient with \( h_{b-a} \)
  \[ Q_{bottom} = \frac{T_b - T_a}{R_{ins}} + \frac{k_{ins}}{\Delta x_{ins}} + \frac{1}{h_{b-a} A_c} \]

Total Loss

- \( Q_{sides} = U'_{side} A_{side} (T_{P} - T_a) \)
  - Can estimate \( U'_{side} \) = 0.5 W/m²·K
  - Use \( U_{side} A_{c} = U'_{side} A_{side} \) for common area
- \( Q_{loss} = U_{c} A_c (T_p - T_a) = (T_p - T_a) / R_{c} \)
- Overall conductance and resistance
  \[ U_c = U_{top} + U_{bottom} + U_{sides} \]
  \[ \frac{1}{R_c} = \frac{1}{R_{top}} + \frac{1}{R_{bottom}} + \frac{1}{R_{sides}} \]

Approximate \( U_{c} \) Equation

- \( U_c = \frac{1}{A'} \left( \frac{A' (T_p - T_a)}{T_p} \right)^{0.1} + \frac{1}{h_a} \left( \sigma (T_p + T_a) \right)^{0.1} \)
  \[ h_a = \left( 1 - 0.091N \right) \left( 1 - 0.04h_a + 0.0005h_a^2 \right) \]
  \[ N = \text{number of glass covers} \]
  \[ A' = 250(1 - 0.0044(s - 90)) \]
  \[ s = \text{tilt angle (degrees)} \]
  \[ B = (1 - 0.04h_a + 0.0005h_a^2)(1 + 0.091N) \]
  \[ \sigma = 5.670 \times 10^{-8} \text{ W/m}^2\cdot\text{K}^4 \]
  \( \varepsilon_{g} \) is same for all glass covers

Absorber Plate Analysis

- Small plate thickness, \( t \), gives uniform \( T \) in \( z \) direction
- Main heat transfer is in \( x \) direction with smaller heat transfer in \( y \) direction

Absorber Plate Analysis II

- Absorber plate is like a fin with \( dT/dx = 0 \) at midpoint between tubes \( (x = 0) \) and \( T = T_b \) at tube bond point \( (x = \pm L) \)
- Energy balance over \( \delta x \) for unit thickness in \( y \) direction has \( h_x \delta x \) solar input and loss to ambient = \( U_x \delta x (T - T_a) \)
Absorber Plate Analysis III

- Energy balance in x direction conducts net heat (solar input – loss)

$$H_x \frac{d}{dx}(U(T-T_a)) - tk \frac{dT}{dx} = 0$$

Absorber Plate Analysis IV

- Boundary conditions are $T = T_b$ at $x = L$ and $dT/dx = 0$ at $x = 0$

Define $\theta = T - T_a - \frac{H_a}{U_c}$ so $d\theta/\text{dx} = \frac{\partial \theta}{\partial x}$.

$$\frac{d^2 \theta}{dx^2} = \frac{U_c}{tk} \left( T - T_e \right)$$

Absorber Plate Analysis V

- For $d\theta/\text{dx} = 0$ at $x = 0$

$$0 = \frac{d\theta}{\text{dx}} = Am \cosh m0 + Bm \sinh m0 = Am \Rightarrow A = 0$$

- For $\theta = T_b - T_a - \frac{H_a}{U_c}$ at $x = L$

$$T_b - T_a - \frac{H_a}{U_c} = B \cosh mL$$

$$B = \cosh mL$$

$$\theta = B \cosh mx = \cosh mL$$

Absorber Plate Analysis VI

- Heat flow from plate to tube $= -tk dT/\text{dx}\big|_{x=L}$

$$\frac{dT}{\text{dx}} \big|_{x=L} = \frac{U_c}{tk} \left( T_b - T_e \right)$$

$$q = \frac{-U_c}{tk} \left( T_b - T_e \right) \tanh mL$$

$$q = \frac{U_c}{tk} \left( T_b - T_e \right) \tanh mL - \frac{1}{m} \left[ H_a - U_c (T_b - T_a) \right] \tanh mL$$

Absorber Plate Analysis VII

- Account for heat flows into tube from two sides and define $F = \tanh(mL)/(mL)$

$$q_{\text{in}} = \frac{2}{mL} \left[ H_a - U_c(T_b - T_a) \right] \tanh mL - 2LF \left[ H_a - U_c(T_b - T_a) \right]$$

Absorber Tube Analysis

- Relationship between $q_u$ (useful heat per unit length), $q_u = \frac{1}{C_B} + \frac{h_{cl} \pi D_i}{C_B}$

$$\frac{1}{C_B} + \frac{h_{cl} \pi D_i}{C_B}$$

- $C_B = \text{bond conductance} > 35 \text{ W/m·K}$

- $C_B = k_B w_B/t_B$ where $k_B$, $w_B$, and $t_B$ are bond thermal conductivity, width and thickness

- $h_{cl} = \text{Heat transfer coefficient inside tube}$

- $D_i = \text{inside tube diameter}$
Absorber Tube Analysis II

- Can eliminate bond temperature between two equations for $q''_u$

$$q_u = \frac{T_a - T_f}{\frac{1}{C_a} + \frac{1}{h_{ja}A_d}} \implies T_a = T_f + q_u \left( \frac{1}{C_a} + \frac{1}{h_{ja}A_d} \right)$$

$$q_u = (2LF + D) \left[ H_s - U_c (T_f - T_a) \right]$$

Absorber Tube Analysis III

- Divide by $U_c (2LF + D)$ and solve for $q'_u$

$$q'_u = \frac{1}{U_c \left( 2LF + D \right)} \left( \frac{1}{C_a} + \frac{1}{h_{ja}A_d} \right) \left[ H_a - U_c (T_f - T_a) \right]$$

Absorber Tube Analysis IV

- $F' = \text{Collector efficiency factor}$
- $w = \text{Distance between tube centerlines}$

$$F' = \frac{1}{U_c \left( 2LF + D \right)} \left[ \frac{1}{C_a} + \frac{1}{h_{ja}A_d} \right]$$

- Now find fluid temperature increase from inlet to exit

Absorber Tube Analysis V

- Energy balance over differential $\delta y$ in one of $n$ tubes where added heat is $q''_u \delta y$ and $m$ is total mass flow rate

$$\frac{m}{n} c_p \frac{dT_f}{dy} = q'_u = wF' \left[ H_s - U_c (T_f - T_a) \right]$$

Absorber Tube Analysis VI

- Rearrange to separate variables and integrate

$$\int_{T_{fi}}^{T_{fo}} \frac{dT_f}{T_f - T_a - H_s/U_c} = -\frac{mF'U_c}{c_p} \int_0^H \frac{dy}{\delta y}$$

$\ln \left( \frac{T_{fo}}{T_{fi}} \right) - H_s/U_c \left( T_{fo} - T_{fi} \right) = -\frac{mF'U_c}{c_p} \delta y$

- $nwl = (\text{number of tubes})(\text{distance between tubes})(\text{length of tube}) = \text{collector area} = A_c$

Absorber Tube Analysis VII

- Introduce $F_R = (\text{Actual heat transfer}) / (\text{Heat transfer is entire plate is at } T_{fin})$

$$F_R = \frac{m c_p \left( T_{fin} - T_{fin} \right)}{A \left( H_a - U_c (T_{fin} - T_a) \right)}$$

$$F_R = \frac{m c_p \left( T_{fin} - T_{fin} - H_s/U_c \right)}{U_c A \left( H_a - U_c (T_{fin} - T_a + T_s) \right)}$$

$$F_R = \frac{m c_p \left( T_{fin} - T_{fin} - H_s/U_c \right)}{U_c A \left( T_{fin} - T_s - H_s/U_c \right)}$$
Absorber Tube Analysis VIII

\[ F_r = \frac{mc_p}{U_c A} \left[ -\left( \frac{T_{f,inf} - T_a - H_a / U_c}{T_{f,in} - T_a - H_a / U_c} \right) \right] + mc_p \left[ 1 - e^{-\frac{U_{inf}}{mc_p}} \right] \]

- Last step uses previous result
- Definition: \( F_r = \frac{mc_p}{U_c [T_{f,in} - T_a]} - \frac{mc_p}{U_c [T_{f,out} - T_a]} \)
- Result called Hottel-Whillier-Bliss Equation

\[ \dot{Q}_a = F_r A_c \left[ H_a - U_c (T_{f,in} - T_a) \right] \]

Summary of Results

\[ F_r = \frac{1}{U_c (2L + D) + \frac{1}{C_b} + \frac{1}{\pi D} h_i} \]

\[ F_r = \frac{mc_p}{U_c A} \left[ 1 - e^{-\frac{U_{inf}}{mc_p}} \right] \]

\[ Q_a = cP \left( H_a - U_c (T_{f,in} - T_a) \right) \]

Transmission and Absorption

- Radiation entering top glass cover can be transmitted, absorbed and reflected
- Amount transmitted to second glass cover can also be transmitted, absorbed, and reflected
- Want overall proportion of incident radiation that is absorbed by collector
- This is given by \( H_a = H_i \tau_a \), where \( \tau_a \) is the mean of the transmissivity times the absorptivity for the total process

Solar Input

- \( H_a \) used previously is solar energy absorbed by the collector
- \( H_i \) is the solar radiation incident on the collector
- Two sources of solar radiation
  - Direct radiation from the sun
  - Diffuse radiation from atmosphere and ground reflection
- Direction affects amounts transmitted and absorbed

Collector Efficiency

- Instantaneous collector efficiency, \( \eta_c \)
  \[ \eta_c = \frac{\dot{Q}_a}{H_i} \]

- Average collector efficiency over a time period, \( \tau_c \)
  \[ \bar{\eta_c} = \frac{\int_0^\tau \dot{Q}_a \, dt}{A \int_0^\tau H_i \, dt} \]

Solar Efficiency Testing

- Start with Hottel-Whillier-Bliss Equation
  \[ \dot{Q}_a = F_r A_c \left[ H_a - U_c (T_{f,in} - T_a) \right] \]

- Replace \( H_a \) by \( H_i \tau_a \)
  \[ \dot{Q}_a = F_r A_c \left[ H_i \tau_a - U_c (T_{f,in} - T_a) \right] \]

- Substitute into efficiency equation
  \[ \eta_c = \frac{\dot{Q}_a}{A \frac{H_i}{H_i} \left( \int_0^\tau \dot{Q}_a \, dt \right)} = \frac{F_r A_c \left[ H_i \tau_a - U_c (T_{f,in} - T_a) \right]}{A \frac{H_i}{H_i} \left( \int_0^\tau \dot{Q}_a \, dt \right)} \]
Solar Efficiency Testing II

- Last equation shows how to determine collector parameters in testing
  \[ \eta_c = \frac{Q}{A H_i} = F_R \tau \eta c \left( \frac{T_{f, in} - T_a}{H_i} \right) \]
  - Measure and plot collector efficiency, \( \eta_c \), as a function of \( (T_{f, in} - T_a)/H_i \)
  - Measure \( Q = m c_p (T_{f, out} - T_{f, in}) \)
  - Intercept is \( F_R \tau \eta c \)
  - Slope is \(-F_R U_c\)

Solar Efficiency Test Results

<table>
<thead>
<tr>
<th>Type of Collector</th>
<th>Intercept = ( F_R \tau \eta c )</th>
<th>Value at x = 0.1</th>
<th>Slope = (-F_R U_c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-cover, black</td>
<td>0.77</td>
<td>0.096</td>
<td>-6.75</td>
</tr>
<tr>
<td>1-cover, selective</td>
<td>0.77</td>
<td>0.23</td>
<td>-5.4</td>
</tr>
<tr>
<td>2-cover, black</td>
<td>0.74</td>
<td>0.30</td>
<td>-4.4</td>
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<tr>
<td>2-cover, selective</td>
<td>0.74</td>
<td>0.41</td>
<td>-3.3</td>
</tr>
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One Final Word

- Efficiency tests are usually done with the collector normal to the sun's rays
- This measures a particular \( \tau \alpha \) product, called the normal \( \tau \alpha \) product, \((\tau \alpha)_n\)
- For actual collector operation the \( \tau \alpha \) product can vary over the year with the position of the sun
- Adjustments can be made to account for this variation to the \( \tau \alpha \) product