Pipe Flow

Larry Caretto
Mechanical Engineering 390
Fluid Mechanics
April 8 and 15, 2008

Quiz Seven Results

- 32 students
- 25 maximum possible
- Average (mean) = 17.3
- Standard deviation = 3.58
- Median = 17
- Grade distribution
  8 12 13 13 14 15 15 15 15 15 15 15 15 15 16 16 17 17 18 19 19 20
  20 20 20 20 22 22 22 22 22 22 22

Quiz Eight Solution

- Find prototype velocity for \( V_m = 160 \) mph in 70°F air for various scales and prototype pressures such that \( Ma < 0.3 \)
- Reynolds number similarity with ideal gas law, \( \rho = P/RT \) and \( \mu = \text{function of } T \) only

\[
\text{Re}_m = \frac{\rho_m V_m}{\mu_m} = \text{Re} = \frac{\rho V}{\mu} \quad \Rightarrow \quad \frac{V}{V_m} = \frac{\rho_m \mu_m}{\rho \mu_m} \quad \Rightarrow \quad V = \frac{V_m \rho_m \mu_m}{\rho \mu_m}
\]

\[
\frac{V}{V_m} = \frac{\rho_m \mu_m}{\rho \mu_m} = \frac{P_m}{P_T} \frac{RT_m}{RT_m} \frac{\ell_m}{\ell_m} \frac{\mu_m}{\mu_m} = \frac{\ell_m}{\ell} \frac{P_m}{P} \quad \Rightarrow \quad V = \frac{V_m \ell_m}{\ell} \frac{P_m}{P}
\]

Quiz Eight Solution II

- Have \( \ell_m/\ell = 1/3 \) and \( 1/8 \) with \( p_m/p = 1 \) and 5

- For \( V_m = 160 \) mph, we have \( V = 53.3, 20, 267, \) and \( 100 \) mph
- For \( T = 70°F, \ c = 1128 \) ft/s = 769 mph
  (Table B.3, p. 762, 30 mph = 44 ft/s
- For \( Ma = 0.3, V = (0.3)(769) = 231 \) mph
- All speeds except 267 mph speed okay

Quiz Eight Solution III

- Need equal drag coefficients

\[
C_{D_m} = \frac{F_{D_m}}{rac{1}{2} \rho_m V^2} = C_D = \frac{F_D}{\frac{1}{2} \rho V^2} \quad \Rightarrow \quad F_{D_m} = F_D \frac{\rho_m V^2}{\rho V^2}
\]

\[
= F_D \frac{V_m}{\rho_m} \frac{V_m}{\rho} = F_D \frac{\rho V}{\rho} \frac{V_m}{\rho_m} = F_D \frac{\rho V}{\rho} \frac{V}{V_m} = F_D \frac{\rho V}{\rho} \frac{V}{V_m} = \frac{\rho V}{\rho} F_D
\]

- When \( p_m = p \), drag forces are the same
- When \( p_m = 5p \) prototype drag force is 5 times model drag force

Quiz Seven Comments

- Energy equation has specific terms for inlet (i) and outlet (o) in various forms

\[
\frac{P_i}{\rho} + \frac{V_i^2}{2g} + z_i = \frac{P_o}{\rho} + \frac{V_o^2}{2g} + z_o + h_i - h_L \quad \text{head}
\]

\[
\frac{P_i}{\rho} + \frac{V_i^2}{2g} + gz_i = \frac{P_o}{\rho} + \frac{V_o^2}{2g} + gz_o + W_{shaft} - \text{loss} \quad \text{energy}
\]

\[
\dot{m} \left( \frac{P_i}{\rho} + \frac{V_i^2}{2g} + gz_i \right) = \dot{m} \left( \frac{P_o}{\rho} + \frac{V_o^2}{2g} + gz_o \right) + W_{shaft} - P_{Loss} \quad \text{power}
\]
Quiz Seven Comments II

- Work outputs are negative
- Vacuum pressures are negative
- Still have problems with units
  - Always convert pressures to lb/ft²
  - Conversion factor: 70.726 psf/in Hg found as γ_Hg = 847 lb/ft³ divided by 12 in/ft
- Note conversions for head
  \[ \dot{W}_{shaft} = \dot{m}w_{shaft} = \dot{m}gh = \gamma Qh \]
  \[ P_{Loss} = \dot{m}(loss) = \dot{m}gh_{L} = \gamma Qh_{L} \]

Outline

- Laminar and turbulent flows
- Developing and fully-developed flows
- Laminar and turbulent velocity profiles: effects on momentum and energy
- Calculating head losses in pipes
  - Major losses from pipe only
  - Minor losses from fittings, valves, etc.
- Noncircular ducts

Piping System

- System consists of
  - Straight pipes
  - Joints and valves
  - Inlets and outlets
  - Work input/output

What We Want to Do

- Determine losses from friction forces in straight pipes and joints/valves
  - Will be expressed as head loss or “pressure drop” − \( h_L = \Delta P/\gamma \)
  - Losses in straight pipes are called “major” losses
  - Losses in fittings, joints, valves, etc. are called “minor” losses
  - Minor losses may be greater than major losses in some cases

Pipe Cross Section

- Most pipes have circular cross section to provide stress resistance
- Main exception is air conditioning ducts
- Consider round pipes first then extend analysis to non-circular cross sections
  - Extension based on using same equations as for circular pipe by defining hydraulic diameter = 4 (area) / (perimeter), which is \( D \) for circular cross sections

The Pipes are Full

- Consider only flows where the fluid completely fills the pipe
- Partially filled pipes are considered under open-channel flow

Driving force is pressure

Driving force is gravity
Pipe flow April 8 and 15, 2008

Laminar vs. Turbulent Flow

- Laminar flows have smooth layers of fluid
- Turbulent flows have fluctuations

Laminar vs. Turbulent Flow II

- Most flows of engineering interest are turbulent
  - Analysis relies mainly on experimentation guided by dimensional analysis
  - Even advanced computer models, called computational fluid dynamics (CFD) rely on “turbulence models” that have large degree of empiricism
- Can get some (very limited) analytical results for laminar flows

Laminar vs. Turbulent Flow III

- Condition of flow as laminar or turbulent depends on Reynolds number
- For pipe flows
  - \( \text{Re} = \frac{\rho V D}{\mu} < 2100 \) is laminar
  - \( \text{Re} = \frac{\rho V D}{\mu} > 4000 \) is turbulent
  - \( 2100 < \text{Re} < 4000 \) is transition flow
- Other flow geometries have different characteristics in \( \text{Re} = \frac{\rho V L_c}{\mu} \) and different values of \( \text{Re} \) for laminar and turbulent flow limits

Flow Development

- Entrance regions and bends create changing flow patterns with different head losses
- Once flow is “fully developed” the head loss is proportional to the distance
- Entrance pressure drop is complex
  - Complete entrance region treated under minor losses
  - Will not treat partial entrance region here

Developing Flows II

- Entrance regions and bends create changing flow patterns with different head losses
- Once flow is “fully developed” the head loss is proportional to the distance
- Entrance pressure drop is complex
  - Complete entrance region treated under minor losses
  - Will not treat partial entrance region here
Developing Flows III

- After development region, pressure drop (head loss) is proportional to pipe length
- Equations for entrance region length, $\ell_e$
  - Laminar flow: $\frac{\ell_e}{D} = 0.06 \text{Re}$
  - Turbulent flow: $\frac{\ell_e}{D} = 4.4 \text{Re}^{1/6}$
  - Turbulent flow rule of thumb $\ell_e \approx 10D$

Fluid Element in Pipe Flow

- Look at arbitrary element, with length $\ell$, and radius $r$, in fully developed flow
- What are forces on this element?

Fully Developed Flow

\[ \nabla F_x = \rho \frac{d^2 \mathbf{u}}{dt^2} \quad \text{No change in momentum} \]

\[ \sum F_x = \rho \frac{d}{dt} \left( \rho \mathbf{u} \right) = 0 \]

\[ \Delta p = -\frac{2 \epsilon }{r} \quad \text{Pressure drop is due to viscous stresses} \]

Extend Relation to Wall

- Have $\Delta p = 2 \tau \ell / r$ for any $r$: $0 < r < R = D/2$
- For wall $r = R = D/2$ and $\tau = \tau_w = \text{wall shear stress}$: $\Delta p = 2 \tau_w \ell / R = 4 \tau_w \ell / D$

Fully Developed Laminar Flow

- Can get exact equation for pressure drop
  \[ \Delta p = \frac{128 \mu Q}{\pi D^4} \]
- Laminar velocity profile
  \[ u = u_c \left[ 1 - \left( \frac{r}{R} \right)^2 \right] \]

Fully Developed Laminar Flow II

- Laminar shear stress profile found from
  \[ \tau = \mu \frac{du}{dr} \]
  \[ \tau = \frac{2 \mu c}{R} \frac{2r}{D^2} \]
Pipe flow April 8 and 15, 2008

ME 390 – Fluid Mechanics 5

Fully Developed Laminar Flow III

- What is centerline velocity, \( u_c \)?

\[ Q = V A = \pi R^2 \int u dA = \int R \left[ u_c - \left( \frac{r}{R} \right)^2 \right] 2\pi r dr \]

\[ Q = 2\pi u_c \int_0^R r dr - \frac{R^3}{R^2} \int_0^R dr = 2\pi u_c \left( \frac{R^2}{2} - \frac{R^4}{4R^2} \right) = 2\pi u_c \frac{R^2}{4} \]

\[ Q = \pi u_c \frac{R^2}{2} \Rightarrow u_c = \frac{2Q}{\pi R^2} = \frac{2V}{\pi R} = \frac{2\pi R^2}{\pi R^2} = 2V \]

Centerline \( u_c \) is twice the mean velocity.

Effect of Velocity Profile

- Momentum and kinetic energy flow for mean velocity, \( V \)
  - Flow_{Momentum} = \( \dot{m}V = \rho V A V = \rho V^2 (\pi R^2) \)
  - Flow_{KE} = \( \frac{V^2}{2} = \rho V A \frac{V^2}{2} = \rho V^3 (\pi R^2) / 2 \)

- Accurate representation uses profile

\[ \text{Flow}_{Momentum} = \int_0^R \rho u dA = \int_0^R \left( \int u_c \left( 1 - \left( \frac{r}{R} \right)^2 \right)^2 2\pi r dr \right) \frac{4}{3} \rho V^2 A \]

\[ \text{Flow}_{KE} = \int_0^R \frac{u^2}{2} = \int_0^R \left( \int u_c \left( 1 - \left( \frac{r}{R} \right)^2 \right)^3 2\pi r dr \right) = 2\rho A V^3 / 2 \]

Turbulent Flow

- For laminar and turbulent flows, the velocity at the wall is zero
  - This is called the no-slip condition
  - Momentum is maximum in the center of the flow and zero at the wall
    - Laminar flows: momentum transport from wall to center is by viscosity, \( \tau = \mu du/dr \)
    - Turbulent flows: random fluctuations exchange eddies of high momentum from the center with low momentum flow from near-wall regions

Moments Exchange

Velocities at one point as a function of time

\[ u(t) = \text{instantaneous velocity} \]

\[ u' = \text{velocity fluctuation} = u - \bar{u} \]

Turbulence Regions/Profiles

- Very thin viscous sublayer next to wall
  - 0.13% of \( R = 3 \) in for \( H_2O \) at \( \dot{V} = 5 \text{ ft/s} \)

- Flat velocity profile in center of flow
Pipe flow

Profile

Turbulent velocity profiles with \( n \) as a function of Reynolds number

\[
\frac{\bar{u}}{V_c} = \left( 1 - \frac{r}{R} \right)^{1/n}
\]

Pipe Roughness

- Effect of rough walls on pressure drop may depend on surface roughness of pipe
- Typical roughness values for different materials expressed as roughness length, \( \epsilon \), with units of feet or meters
- Only turbulent flows depend on roughness length, laminar flows do not

<table>
<thead>
<tr>
<th>TABLE 8.1</th>
<th>Equivalent Roughness for New Pipes [From Moody (Ref. 7) and Colebrook (Ref. 8)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pipe</td>
<td>Equivalent Roughness, ( \epsilon ) (Feet)</td>
</tr>
<tr>
<td>Riveted steel</td>
<td>0.003–0.03</td>
</tr>
<tr>
<td>Concrete</td>
<td>0.001–0.01</td>
</tr>
<tr>
<td>Wood stave</td>
<td>0.0006–0.003</td>
</tr>
<tr>
<td>Cast iron</td>
<td>0.00085</td>
</tr>
<tr>
<td>Galvanized iron</td>
<td>0.0005</td>
</tr>
<tr>
<td>Commercial steel</td>
<td>0.0005</td>
</tr>
<tr>
<td>or wrought iron</td>
<td>0.00015</td>
</tr>
<tr>
<td>Drawn tubing</td>
<td>0.000005</td>
</tr>
<tr>
<td>Plastic, glass</td>
<td>0.0 (smooth)</td>
</tr>
</tbody>
</table>

Use this table (p 433 of text) to find \( \epsilon \)

Energy Equation

- Energy equation between inlet (i) and outlet (o)
  \[
  \frac{P_i}{\gamma} + \frac{V_i^2}{2g} + z_i = \frac{P_o}{\gamma} + \frac{V_o^2}{2g} + z_o + h_i - h_e
  \]
- Previous applications allowed us to compute the head loss from all other data in this equation
- Call this the measured head loss
  - We can compute it but do not know its cause
**Pressure Drop/Head Loss**
- We now seek a design calculation for $h_L$
- Use level pipe ($z_1 = z_2$) with constant area ($V_1 = V_2$) and no shaft head ($h_s = 0$)

$$\frac{P_2 - P_1}{\rho g} = \frac{V_2^2}{2g} + z_2 - z_1 + h_L$$

$$h_L = \frac{P_2 - P_1}{\rho g} - \frac{V_2^2}{2g}$$

**Pressure Drop/Head Loss II**
- Calculated $\Delta p$ for $z_1 = z_2$, $V_1 = V_2$, and $h_s = 0$ gives $h_L$ for more general flows
- Will later define friction factor, $f$, such that

$$f = \frac{\Delta p}{\frac{1}{2} \rho V^2} = \frac{\Delta p}{\frac{1}{2} \rho V^2}$$

Will use this to define head loss

$$h_L = \frac{\Delta p}{\rho g} = \frac{f}{2} \frac{1}{\pi} \frac{V^2}{D^2} = f \frac{1}{\pi} \frac{V^2}{D^2}$$

**Head Loss in Pipes**
- Dimensional analysis shows that dimensionless pressure drop, $\Delta p/\rho V^2$, is a function of Reynolds number, $\rho V D/\mu$, the $t/D$ ratio and relative roughness, $\epsilon/D$
- Expressed in terms of friction factor, $f$

$$f = \frac{\Delta p}{\frac{1}{2} \rho V^2} = \frac{f}{\rho V^2}$$

• What is the form of $f(Re, \epsilon/D)$?

**Moody Diagram Equations**
- Colebrook equation (turbulent)

$$\frac{1}{\sqrt{f}} = -2.0 \log_{10} \left( \frac{\epsilon/D}{3.7} + \frac{2.51}{Re \sqrt{f}} \right)$$

- Haaland equation (turbulent)

$$\frac{1}{\sqrt{f}} = -1.8 \log_{10} \left( \frac{\epsilon/D}{3.7} \right)$$

- Laminar

$$f = \frac{64}{\pi D^4} \frac{Q}{A} = \frac{256 \mu Q}{\pi D^4} = \frac{64 \mu Q}{\pi D^4} = \frac{64 \mu Q}{\pi D^4}$$

**Wholly Turbulent Flows**
- Large Reynolds numbers: $f$ independent of Re depends only on $\epsilon/D$

$$\Delta p = \frac{f}{D} \frac{Q^2}{2} \frac{1}{\pi D^2} = \frac{Q^2}{4 \pi D^2} \Rightarrow V^2 = \frac{16 Q^2}{\pi D^2}$$

$$\Delta p = \frac{f}{D} \frac{Q^2}{2} \frac{1}{\pi D^2} \frac{1}{\pi D^2} = \frac{8 f}{\pi D^2} \frac{Q^2}{2} \frac{1}{\pi D^2} \frac{1}{\pi D^2} = \frac{8 f}{\pi D^2} \frac{Q^2}{\pi D^2}$$

- Pressure drop varies as $D^{-5}$
  - Similar to $D^{-4}$ dependence in laminar flow
### Pressure Drop Problems

- Find the pressure drop given fluid data, pipe dimensions, $\varepsilon$, and flow (volume flow, mass flow, or velocity)
  - Get $A = \pi D^2/4$
  - Get $V = Q/A$ or $V = \rho \nu A$ if not given $V$
  - Find $\rho$ and $\mu$ for fluid at given $T$ and $P$
  - Compute $Re = \rho V D/\mu$ and $d/D$
  - Find $f$ from diagram or equation
    - Laminar $f = 64/Re$; Colebrook for turbulent
  - Compute $\Delta p = f (L/D) \rho V^2/2$