Unit Four – First Law for Ideal Gases
Mechanical Engineering 370
Thermodynamics
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September 16, 2010

Outline
• Quiz three solution
• Unit four – first law for ideal gases
  – Heat capacities, $c_v$ and $c_p$ are properties
  – For ideal gases $du = c_v dT$ and $dh = c_p dT$, regardless of path
  – For ideal gases $u = u(T)$ only and $h = u + Pv = u + RT = h(T)$ only
  – Solving first law problems with ideal gases

Unit Four Goals
• As a result of studying this unit you should be able to
  – describe the path for a process and determine the work with greater confidence than you had after completing unit three
  – understand the heat capacities $C_x$ (e.g. $C_p$ and $C_v$) as $dQ = C_x \, dT$ in a “constant-x” process
  – use the property relations for ideal gases $du = c_v \, dT$ and $dh = c_p \, dT$ for any process

Unit Four Goals Continued
– find changes in internal energy and enthalpy for an ideal gas where the heat capacity is constant or a function of temperature.
– use ideal gas tables to find changes in internal energy and enthalpy where the heat capacities are functions of temperature
– find internal energy changes for ideal gases as $\Delta h = \Delta u + R \Delta T$
– convert results from a per-unit-mole basis to a per-unit-mass basis and vice versa

Why Use Ideal Gas?
• Simple equations
• Real gas behavior close to ideal gas for low pressure (or high temperature)
• Want $P$ low compared to $P_c$ or $T$ high compared to $T_c$
  – Have seen that ideal gas gives good results for $P$-$v$-$T$ data when $P < \sim 0.2 P_c$ or $T > \sim 2 T_c$
  – Will also see that this region gives good results for internal energy change, $\Delta u$
Heat Capacity

- Heat capacity, $C_v = m c_v$, gives heat transfer $dQ = C_v dT$ in constant $x$ process
- Specific heat capacity, $c_p = C_p / m$ is a property
- Use $c_v$ and $c_p$ for constant volume and pressure, respectively
- For liquids and solids $c_v$ and $c_p$ are essentially the same
  - $dq = c_v dT$ at constant volume
  - $dq = c_p dT$ at constant pressure

Ideal Gases

- For ideal gases, $du = c_v dT$, $dh = c_p dT$ regardless of path
- Note these differences
  - Path dependent heat transfer
    - $dq = c_v dT$ for constant volume only
    - $dq = c_p dT$ for constant pressure only
  - Path independent properties $du$ and $dh$
  - $\Delta u = \Delta U / m = \int c_v dT$, $\Delta h = \Delta H / m = \int c_p dT$
  - $h = u + Pv = u + RT$ for ideal gases
  - $dh = du + RdT \Rightarrow c_p dT = c_v dT + RdT$

Compare/Contrast Tables

- For any property relations we have:
  - First law: $Q = \Delta U + W$
  - Work: $W = \int_{path} PdV$
  - So what is different?
    - With property tables we use tables to find $P$-$\nu$-$T$ relationships and internal energy
    - Ideal gases use $Pv = RT$ and $du = c_v dT$
      - Use data sources for $c_v$ as average value, $c_v$ as a function of temperature and integrate
      - Find $u(T)$ in ideal gas tables

Example Calculation

- Given: 10 kg of H$_2$O at 100 kPa, 200$^\circ$C expanded to 400$^\circ$C at constant pressure
- Find: Heat Transfer
  - using H$_2$O tables (for comparison)
  - using ideal gas with constant heat capacity
  - using ideal gas with variable heat capacity
- First Law: $Q = \Delta U + W = m(u_2 - u_1) + W$
- Path: $W = \int_{path} PdV = \int_{P=1}^{P=2} (V_2 - V_1)$ for constant pressure, $P_{1-2} = P_1 = P_2$
  - $u_2 - u_1 = \int c_v dT$ for ideal gas
Using H₂O Tables

- At T₁ = 200°C and P₁ = 100 kPa, v₁ = 2.1724 m³/kg and u₁ = 2658.2 kJ/kg
- At T₂ = 400°C and P₂ = P₁ = 100 kPa, v₂ = 3.1027 m³/kg and u₂ = 2968.3 kJ/kg
- W = P₁(v₂ - v₁) = P₁ m(v₂ - v₁) = (10 kg)(100 kPa)(3.1027 - 2.1724) m³/kg = 931 kPa·m³ = 931 kJ
- Q = m(u₂ - u₁) + W = (10 kg)(2968.3 - 2658.2) kJ/kg + 931 kJ = 4,029 kJ

Ideal Gas Calculations

- Q = ΔU + W = m(u₂ - u₁) + ∫PdV
- Q = m(u₂ - u₁) + mP∫dv
- PV = mRT
- We use PV = mRT to determine mass and specific volume from P and T
- The work calculation does not depend on assumptions about cᵥ (or cₚ = cᵥ + R)
- Find R = 0.4615 kJ/kg·K = 0.4615 kPa·m³/kg·K in Table A-1, page 908

Work – Ideal Gas Assumption

- At T₁ = 200°C and P₁ = 100 kPa, v₁ = RT₁/P₁ = (.4615 kJ/kg·K)(473.15 K)/(100 kPa) = 2.1836 m³/kg
- At T₂ = 400°C and P₂ = P₁ = 100 kPa, v₂ = RT₂/P₂ = (.4615 kJ/kg·K)(673.15 K)/(100 kPa) = 3.1066 m³/kg
- W = P₁(v₂ - v₁) = P₁ m(v₂ - v₁) = (10 kg)(100 kPa)(3.1066 - 2.1836) m³/kg = 923 kPa·m³ = 923 kJ

Ideal Gas Internal Energy

- u₂ - u₁ = [cᵥ(T)dT] = [cₚ(T)dT] - RdT
- Possible calculations for cᵥ (or cₚ)
  - Assume constant (easiest) Δu = cᵥΔT
  - Integrate equation for cᵥ or cₚ as a function of temperature (Table A-2(c), p 911)
  - Use ideal gas tables giving u(T) and h(T) (Tables A-17 to A-26, pp 934-947)
  - Last two give molar properties (except air)
    - Divide by molar mass, M, to get per-unit-mass property values

Table A-2

<table>
<thead>
<tr>
<th>Gas</th>
<th>Formula</th>
<th>R</th>
<th>cᵥ</th>
<th>cₚ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>—</td>
<td>0.2870</td>
<td>0.105</td>
<td>0.718</td>
</tr>
<tr>
<td>Argon</td>
<td>Ar</td>
<td>0.0861</td>
<td>0.500</td>
<td>0.312</td>
</tr>
<tr>
<td>Bismuth</td>
<td>Bi₂H₆</td>
<td>0.1433</td>
<td>1.7164</td>
<td>1.3734</td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>CO₂</td>
<td>0.1889</td>
<td>0.850</td>
<td>0.657</td>
</tr>
<tr>
<td>Carbon monoxide</td>
<td>CO</td>
<td>0.2968</td>
<td>1.040</td>
<td>0.741</td>
</tr>
<tr>
<td>Ethane</td>
<td>C₂H₆</td>
<td>0.2765</td>
<td>1.7602</td>
<td>1.4877</td>
</tr>
<tr>
<td>Ethylene</td>
<td>C₂H₄</td>
<td>0.2964</td>
<td>1.5462</td>
<td>1.2518</td>
</tr>
<tr>
<td>Helium</td>
<td>He</td>
<td>0.2079</td>
<td>5.1906</td>
<td>3.1106</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>H₂</td>
<td>4.1240</td>
<td>14.307</td>
<td>10.183</td>
</tr>
<tr>
<td>Methane</td>
<td>CH₄</td>
<td>0.5182</td>
<td>2.2597</td>
<td>1.7354</td>
</tr>
<tr>
<td>Neon</td>
<td>Ne</td>
<td>0.6119</td>
<td>1.0509</td>
<td>0.6179</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>N₂</td>
<td>0.2968</td>
<td>1.039</td>
<td>0.743</td>
</tr>
<tr>
<td>Oxygen</td>
<td>O₂</td>
<td>0.2588</td>
<td>0.908</td>
<td>0.658</td>
</tr>
<tr>
<td>Propane</td>
<td>C₃H₈</td>
<td>0.1885</td>
<td>1.6794</td>
<td>1.4999</td>
</tr>
<tr>
<td>Sulfur</td>
<td>S</td>
<td>0.4815</td>
<td>1.8723</td>
<td>1.4108</td>
</tr>
</tbody>
</table>
First law for ideal gases

**Constant $c_v$ Ideal Gas**

- Get $c_v = 1.4108 \, \text{kJ/kg} \cdot \text{K}$ at 300 K (26.85°C) for water from Table A-2(a), p. 909 (no data for water in Table A-2(b), page 910).
- $\Delta U = m \Delta u = m [c_v(T_d) \Delta T = m c_v(T_2 - T_1) = m c_v \Delta T,$ if $c_v$ is constant
- Here, $\Delta U = m c_v(T_2 - T_1) = (10 \, \text{kg}) (1.4108 \, \text{kJ/kg} \cdot \text{K})(673.15 \, \text{K} - 473.15 \, \text{K}) = 2,822 \, \text{kJ}$
- $Q = \Delta U + W = 2,822 \, \text{kJ} + 923 \, \text{kJ} = 3,745 \, \text{kJ},$ a 7% error compared to actual properties

**Ideal Gas Tables for Water**

- $T$ is in kelvins
- Need to interpolate for values at $T = 200^\circ\text{C} = 473.15 \, \text{K}$
- $u(200^\circ\text{C}) = 11,953 \, \text{kJ/kmol}$

**Ideal Gas Tables**

- Find molar $u(T)$ for H$_2$O in Table A-23 on page 946
- Have to interpolate to find the values of $u_1 = u(473.15 \, \text{K}) = 11,953 \, \text{kJ/kmol}$ and $u_2 = u(673.15 \, \text{K}) = 17,490 \, \text{kJ/kmol}$
- $\Delta U = (10 \, \text{kg})(17,490 \, \text{kJ/kmol} - 11,953 \, \text{kJ/kmol}) / (18.015 \, \text{kg/kmol}) = 3,074 \, \text{kJ}$
- $Q = \Delta U + W = 3,074 \, \text{kJ} + 923 \, \text{kJ}$
- $Q = 3,997 \, \text{kJ}$
Ideal gases: du = c_v dT

- This does not depend on path
- We just computed the Q = ΔU + W for a constant pressure path of an ideal gas
- We used ΔU = mΔu = m[c_v dT] to compute the internal energy change
- We would use the same equation regardless of the path between state 1 and state 2

Assuming c_v Constant

- Assumption of constant heat capacity introduces about a 7% error for this problem
- Accounting for temperature variation of heat capacity reduces error to <= 0.8%
- Next page shows figure 4-24, page 177
  - Constant heat capacity assumption is best for noble gases (e.g., argon, neon) and reasonable for diatomic molecules at ambient temperatures
  - Assumption worsens as the temperature range increases

Variation

- Monatomic gases have constant c_p
- Diatomic gases have moderate variation with T
- Triatomic gases have larger variation with T
- What is c_v?

In-class exercise

- 2 kg of air initially at 100 kPa and 300 K heated to 1200 K. Find the heat transfer for each of the following processes:
  - Constant volume
  - Constant pressure
  - A straight line path to a pressure of 250 kPa
- Use air tables to find the change in internal energy
- How will path affect ΔU?

In-class Exercise Solution

- Path will not affect ΔU
  - For an ideal gas, u is a function of temperature only. Since ΔT is the same, ΔU = mΔu will be the same.
  - From air tables, Table A-17, pp 934-935
    - u(300 K) = 214.07 kJ/kg
    - u(1200 K) = 933.33 kJ/kg
    - mΔu = (2 kg)(719.24 kJ/kg) = 1438.48 kJ
  - Q = W + 1438.48 kJ

Exercise Solution II

- Look at constant volume path first
- For constant volume path, W = 0 so Q = ΔU = 1438.48 kJ
Exercise Solution III

- For constant pressure, \( V_2 = V_1 \left( \frac{T_2}{T_1} \right) = 6.888 \text{ m}^3 \)
- \( W = P_1(V_2 - V_1) = (100 \text{ kPa})(6.888 \text{ m}^3 - 1.722 \text{ m}^3) = 516.6 \text{ kJ} \)
- For constant pressure path, \( Q = \Delta U + W = 1438.48 \text{ kJ} + 516.66 \text{ kJ} = 1955.14 \text{ kJ} \)

Exercise Solution IV

- For straight line path, \( V_2 = V_1 \left( \frac{P_1 T_2}{P_2 T_1} \right) = 2.7552 \text{ m}^3 \)
- \( W = \frac{(P_1 + P_2)(V_2 - V_1)}{2} = (100 \text{ kPa} + 250 \text{ kPa})(2.7552 \text{ m}^3 - 1.722 \text{ m}^3) = 180.81 \text{ kJ} \)
- For this straight-line path, \( Q = \Delta U + W = 1438.48 \text{ kJ} + 180.81 \text{ kJ} = 1619.29 \text{ kJ} \)

In-Class Exercise

- Air, initially at 100 psia, 1000 R and 1 ft³ undergoes an expansion following the polytropic path \( PV^{1.3} = \text{constant} \) to a final temperature of 500 R. Find the heat transfer.
- For a polytropic path the work is given by the following equation
  \[
  W = \frac{P_2 V_2 - P_1 V_1}{1 - n}
  \]

In Class Exercise II

- Given data:
  - Initial state: \( T_1 = 1000 \text{ R}, P_1 = 100 \text{ psia}, V_1 = 1 \text{ ft}^3 \)
  - Path: \( PV^{1.3} = \text{constant} \)
  - \( P_1 V_1^{1.3} = P_2 V_2^{1.3} \)
  - Final state: \( T_2 = 500 \text{ R} \)
  - Substance: Air
- Property relations: ideal gas
  - \( PV = mRT \) and \( du = c_vdT \)
  - Air tables give integral of \( c_vdT \)

In Class Exercise III

- Equations and data:
  - First Law: \( Q = \Delta U + W = m(u_2 - u_1) + W \)
  - \( W = \int \text{path} \text{PdV} \)
  - \( W = (P_1 V_2 - P_1 V_1) / (1 - n) \) for polytropic path, \( PV^n = \text{constant} \)
  - \( PV = mRT \)
  - \( du = c_vdT \) (or use air tables for \( u \), A-17E, pp 982–983 for engineering units)
  - For air, \( R = 0.3704 \text{ psia} \cdot \text{ft}^3/\text{lb}_m \cdot \text{R} = 0.06855 \text{ Btu/} \text{lb}_m \cdot \text{R} \) (Table A-1E, page 958)

In Class Exercise IV

- From air tables, \( u_1 = u(1000 \text{ R}) = 172.43 \text{ Btu/lb}_m \) and \( u_2 = u(500 \text{ R}) = 85.20 \text{ Btu/lb}_m \)
  \[
  m = \left( \frac{100 \text{ psia}}{1 \text{ ft}^3} \right) \frac{0.3704 \text{ psia} \cdot \text{ft}^3}{(1000 \text{ R})} = 0.2700 \text{ lb}_w
  \]
  \[
  W = \frac{m V_1^{1.3}}{1 - n} = \frac{m V_2^{1.3} - m V_1^{1.3}}{1 - n} = \frac{m R(T_2 - T_1)}{1 - n}
  \]
  \[
  = \frac{0.2700 \text{ lb}_w \cdot (0.06855 \text{ Btu/} \text{lb}_m \cdot \text{R})(500 \text{ R} - 1000 \text{ R})}{1 - 1.3} = 30.845 \text{ Btu}
  \]
In Class Exercise V

- Q = ΔU + W = m(u_2 - u_1) + W

\[
Q = (0.2700 \text{ lb}) \left( \frac{85.20 \text{ Btu}}{\text{lb}_n} - \frac{172.43 \text{ Btu}}{\text{lb}_n} \right) + 30.845 \text{ Btu}
\]

\[ Q = 7.29 \text{ Btu} \]

- Assuming constant c_v = 0.174 Btu/lbm, R at mean temperature of 750 R = 290.3°F from Table A-2E(b), page 960 gives Q = 7.36 Btu

Northridge