Outline

- Quiz 12 solution
- Review for final
  - Property tables and ideal gases
  - First law for closed and open (steady and unsteady) systems
  - Entropy and maximum work calculations
  - Isentropic efficiencies
  - Cycle calculations (Rankine, refrigeration, air standard) with mass flow rate ratios

Review, but, first a word about units

- Units and dimensions
- SI units and engineering units
- Extensive, intensive and specific
  - \( E \) is extensive, e.g., \( V \), \( U \), \( H \), \( S \), \( Q \), \( W \)
  - \( T \) and \( P \) are intensive
  - \( e = E/m \) is specific (e.g. \( \text{kJ/kg} \), \( \text{Btu/lbm} \))
- Unit conversions (\( \text{kPa} \cdot \text{m}^3 = \text{kJ} \)) (\( \text{m}^2/\text{s}^2 = \text{J/kg} \)) (\( \text{lb} \cdot \text{ft}^3 \), \( \text{Btu} \cdot \text{lbm} \cdot \text{s}^2/\text{ft}^2 \))
  - Factors on equation sheet

Property Data and Relations I

- Find properties from tables
  - Given \( T \) and \( P \), \( T < T_{\text{sat}}(P) \) or \( P > P_{\text{sat}}(T) \) is liquid; \( T > T_{\text{sat}}(P) \) or \( P < P_{\text{sat}}(T) \) is gas
  - Liquid at \( P \), \( T \) approximately saturated liquid at given \( T \)
  - When given \( P \) or \( T \) and \( e \) where \( e \) may be \( v \), \( u \), \( h \), \( s \), compare \( e \) to saturation properties
    - \( e < e_{\text{l}}(P \text{ or } T) \) is liquid; \( e > e_{\text{g}}(P \text{ or } T) \) is gas
    - otherwise compute \( x = (e - e_{\text{l}})/(e_{\text{g}} - e_{\text{l}}) \)
  - Forget all this if you find the state point

Property Data and Relations II

- Ideal gas equations and properties
  - \( PV = RT \), \( du = c_v dT \), \( dh = c_p dT \), \( ds = c_v dT/T \) + \( Rdv/v = c_p dT/T - RdP/P \)
  - \( e = u + RT \)
  - \( u, h, c_v \) and \( c_p \) = \( f(T) \) only \( (k = c_p/c_v) \)
  - Pick constant heat capacity at average \( T \)
  - Handle variable heat capacities by equations or use ideal gas tables for \( u(T) \), \( h(T) \) and \( s(T) \)
  - Isentropic relations for constant and variable heat capacities, e.g \( P_i V_i^2 = P_2 V_2^2 \), \( P_i/P_2 = P_i(T_2)/P_i(T_1) \), \( V_2/V_1 = v_i(T_2)/v_i(T_1) \)

Basic First Law Terms

- Energy terms (per unit mass): internal, \( u \), kinetic, \( \mathbf{u}^2/2 \), and potential, \( gz \)
- Heat, \( Q \) (or \( q \)), is energy in transit due only to a temperature difference
- Work, \( W \), is action of force over distance
  - \( W = \int_{\text{path}} PdV \) or \( w = \int_{\text{path}} Pdv \)
- Heat added to a system is positive, heat removed from a system is negative
- Work done by a system is positive, work done on a system is negative
Energy Balances
- System energy change = Heat added to system – work done by system + Energy from inflows – Energy outflows
- Usually in kJ (or Btu), but open systems can use power (kW or Btu/hr)
- Can use $q = \dot{Q}/\dot{m}$ and $w = \dot{W}/\dot{m}$ or equivalent rates: $q = \dot{Q}/\dot{m}$, $w = \dot{W}/\dot{m}$
- Flowing stream terms include flow work to give $h = u + Pv$

Closed Systems
- $Q = \Delta U + \dot{W} = m(u_{\text{final}} - u_{\text{initial}}) + \int P \text{d}V$
- Integral is area under path
  - Path equation gives $P(V)$ for process
  - Integrate equation or find area
  - Watch sign
- Internal energy depends on state
  - Tables, may have to use $u = h - Pv$
  - Ideal gases: $du = c_v \text{d}T$ or $u(T)$ in ideal-gas tables

Work as Area Under Path
- This works if the path has a simple shape
- Here we have a path with three components
- $W = W_{1-2} + W_{2-3} + W_{3-4}$
- $W = (P_1 + P_2)(V_2 - V_1)/2 + 0 + P_{3-4}(V_4 - V_3)$
- $W$ is zero if $V$ is constant and is negative when volume decreases

Formal Integration of Path
- Analytical path equation examples
  - Isothermal ideal gas: $P = RT/v$
  - Polytropic process: $P v^n = \text{const} (n \neq k)$
  - Arbitrary: $P = P_1 + a(V - V_1)^2 + ...$
- Evaluate $\int P \text{d}V$ from $V_1$ to $V_2$
- Use $P(V) \text{d}V$ for work in kJ (or Btu) or use $P(v) \text{d}v$ for kJ/kg (or Btu/lbm)
- Path equation includes initial and final states and can be solved for these

Open Systems/Assumptions
- General energy and mass balances
  - Steady flow: $\frac{\text{d}m_{\text{in}}}{\text{dt}} = \frac{\text{d}m_{\text{out}}}{\text{dt}}$
  - One inlet and one outlet
  - Negligible kinetic and potential energies

Steady-Flow Systems
- $\dot{Q} = \dot{W} + \sum m_i \left( h_i + \frac{v_i^2}{2} + g z_i \right) - \sum m'_i \left( h'_i + \frac{v_i^2}{2} + g z_i \right)$
- Mass balance
  - First law for $\Delta KE = \Delta PE = 0$
  - For $\Delta KE = \Delta PE = 0$, one inlet and one outlet
    - $\dot{Q} = \dot{W} + \dot{m}(h_{\text{in}} - h_o)$
    - $q = \dot{w}_e + h_{\text{in}} - h_o$
Unsteady Flow Equations

\[
\begin{align*}
    \left[ m_i \left( u + \frac{v_i^2}{2} + gz_i \right) - m_i \left( u + \frac{v_i^2}{2} + gz_i \right) \right]_{\text{system}} & = Q - W_u \\
    - \sum m_i \left( h_i + \frac{v_i^2}{2} + gz_i \right) + \sum m_i \left( h_i + \frac{v_i^2}{2} + gz_i \right) & = m_i \left( h_{\text{out}} - h_{\text{in}} \right) \\
    m_i - m_i \text{,inlet} & = \sum m_i - \sum m_i
\end{align*}
\]

First Law Comparisons

- Closed system: \( Q = m(u_{\text{final}} - u_{\text{initial}}) + W \)
- Steady open system, \( \Delta KE = \Delta PE = 0 \), one inlet and one outlet
  \[ \dot{Q} = m(h_{\text{out}} - h_{\text{in}}) + W \]
- Transient open system, \( \Delta KE = \Delta PE = 0 \), one inlet and one outlet
  \( m_{\text{final}} = m_{\text{initial}} + m_{\text{in}} - m_{\text{out}} \)
  \( Q = m_{\text{final}} u_{\text{final}} - m_{\text{initial}} u_{\text{initial}} + W_u + m_{\text{out}} h_{\text{out}} - m_{\text{in}} h_{\text{in}} \)

Which First Law?

- If the problem mentions the following terms use the first law indicated
  
  Flows, inlet, outlet? | Initial and final states? | Use this first law
  No | Yes | Closed System
  Yes | No | Steady open system
  Yes | Yes | Transient open system

The Second Law

- There exists an extensive thermodynamic property called the entropy, \( S \), defined as follows:
  \( dS = \frac{dU + PdV}{T} \)
- For any process \( dS \geq 0 \)
- For an isolated system \( dS \geq 0 \)
- \( T \) must be absolute temperature

Entropy as a Property

- Dimensions of entropy are energy divided by temperature (kJ/K or Btu/R for S, kJ/kg-K or Btu/lbm-R for \( s = S/m \))
- If we know the state we can find the entropy (tables or ideal gas relations)
- If we know the entropy, we can use it to find the state (tables or ideal gases)
- Use in tables similar to specific volume

Cycles with \( |Q_H| = |Q_L| + |W| \)

- Engine cycle converts heat to work
- Refrigeration cycle transfers heat from low to high temperature
  \[ \eta_{\text{ref}} = \frac{|Q_L|}{|Q_H|} \]
  \[ \text{cop} = \frac{|Q_H|}{|W|} \]
Cycle Parameters

- Engine cycle efficiency \( \eta = \frac{W}{Q_H} \)
- Refrigeration cycle COP (coefficient of performance): \( \text{COP} = \frac{|Q_L|}{W} \)
- General definitions, valid for any cycle
- Engine efficiency always less than one
- COP can be greater than one

From Blobs to Real Cycles

- Analyze each device or step in the cycle
- Can use per-unit-mass quantities or rate quantities
- Complex cycles have different mass flow rates
- In an engine cycle:
  - Add all heat transfers that are positive to get the total \( Q_H \) \( |Q_H| = Q_H \)
  - Add all heat transfers that are negative to get the total \( Q_L \) \( |Q_L| = -Q_L \)
  - The work \( W \) is the algebraic sum of all the work terms in the cycle \( |W| = W \)
  - Engine cycle efficiency \( \eta = \frac{|W|}{|Q_H|} \)

From Blobs to Real Cycles II

- In a refrigeration cycle:
  - Add all heat transfers that are positive to get the total \( Q_L \) \( |Q_L| = Q_L \)
  - Add all heat transfers that are negative to get the total \( Q_H \) \( |Q_H| = -Q_H \)
  - The work \( W \) is the algebraic sum of all the work terms in the cycle \( |W| = W \)
- Coefficient of performance \( = \frac{|Q_L|}{|W|} \)
  - \( |Q_H| = |Q_L| + |W| \) for all cycles

Reversible Process

- Idealization (the \( \geq \) sign), cannot do better than a reversible process
- Internal reversibility \( dS = dQ/T \)
  - For a reversible process, \( Q = \int TdS \), \( \Delta S = \int dQ/T = \int mc_v dT/T \)
  - Reversible isothermal process: \( \Delta S = Q/T \)
- External reversibility \( dS_{\text{isolated system}} = 0 \)
- Maximum work in a reversible process:
  - Minimum work input for work input device
  - For adiabatic process \( \Delta S = 0 \) for maximum

Ideal Gas Entropy

- For constant heat capacity
  \[ s_2 - s_1 = c_v \ln \left( \frac{T_2}{T_1} \right) + R \ln \left( \frac{v_2}{v_1} \right) = c_p \ln \left( \frac{T_2}{T_1} \right) - R \ln \left( \frac{P_2}{P_1} \right) \]
- For constant heat capacity with \( s_2 = s_1 \)
  - \( T_2 / T_1 = (P_2 / P_1)^{(k-1)/k} \Rightarrow T/P^{(k-1)/k} = \text{constant} \)
  - \( v_2 / v_1 = (v_1 / v_2)^{(k-1)} \Rightarrow v_k^{(k-1)} = \text{constant} \)
  - \( P_2 / P_1 = (v_1 / v_2)^k \Rightarrow \text{constant} \)

Ideal Gas Entropy II

- For variable heat capacity
  \[ s_2 - s_1 = c_p \frac{dT}{T} - R \ln \left( \frac{P_2}{P_1} \right) = s^o(T_2) - s^o(T_1) - R \ln \left( \frac{P_2}{P_1} \right) \]
- For variable heat capacity with \( s_2 = s_1 \)
  - \( P(T_2) = P(T_1) (P_2 / P_1) \)
  - \( v(T_2) = v(T_1) (v_2 / v_1) \)
  \[ s^o(T_2) = s^o(T_1) + R \ln \left( \frac{P_2}{P_1} \right) \]
Isentropic Efficiencies

<table>
<thead>
<tr>
<th>Work output: $w = \eta_s w_s$</th>
<th>Work input: $w = w_s/\eta_s$</th>
</tr>
</thead>
</table>

Isentropic Efficiency Problems

- Find ideal work from given inlet state and one outlet state property: $\Delta s = 0$
  - e.g., $w_s = h_{in} - h_{out,s}$
- Actual work $= \eta_s w_s$ for work output or $w_s/\eta_s$ for work input
- Actual outlet state: $h_{out} = h_{in} - w$
  - Use sign convention for work in $h_{in} - w$
- Note that $h_{out}$ is different from $h_{out,s}$

Control Volume Entropy

- General (use $= \text{part of } \geq$ for reversible)
  \[
  \frac{dS}{dt} + \sum_{\text{outlet}} m_i s_i - \sum_{\text{inlet}} m_s s_s \geq \frac{Q_s}{T_s}.
  \]
- Reversible adiabatic process
  \[
  \frac{dS}{dt} = \sum_{\text{outlet}} m_i s_i - \sum_{\text{inlet}} m_s s_s
  \]
- Steady reversible adiabatic process
  \[
  \sum_{\text{outlet}} m_i s_i = \sum_{\text{inlet}} m_s s_s
  \]
- Transient reversible adiabatic process
  \[
  \left[ m_i s_i - m_s s_s \right]_{i-1} = \sum_{\text{outlet}} m_i s_i - \sum_{\text{inlet}} m_s s_s
  \]

Cycle Idealizations

- Use these idealizations in lieu of data
  - No line losses (output state of one device is input to the next device)
  - Work devices are isentropic
  - Heat transfer and mixing devices have no work and $\Delta P = 0$
  - Exit from two-phase device is saturated
  - Air standard cycles assume air (an ideal gas) as working fluid with combustion modeled as heat transfer into fluid

Rankine Cycle

Compute Rankine cycle efficiency given only $T_3$, $P_3$ and $P_{\text{cond}}$

\[
\eta = \frac{(h_1 - h_4) - (h_1 - h_2)}{h_1 - h_2}
\]

\[
\begin{align*}
h_1 &= h(P_{\text{cond}}) \\
h_2 &= h_1 + v_1(P_3 - P_1) \\
h_3 &= h(T_3, P_3); s_3 = s(T_3, P_3) \\
h_4 &= h(P_{\text{cond}}); s_4 = s_3)
\end{align*}
\]

Modified Rankine Cycle

- Different mass flows rates in different parts
- Results depend on ratio of mass flows
- Get mass flow rate ratios from analysis of devices where all $h$ values are known
Mixing Heat and Mass Balance

\[ m_2 h_2 + m_3 h_3 = (1 - m_2) h_1 + m_2 h_2 \]

\[ \frac{m_2}{m_3} = \frac{h_2 - h_3}{h_1 - h_3} = \frac{m_2}{m_3} \]

\[ \eta = 1 - \frac{q_L}{q_H} = 1 - \frac{f(h_4 - h_1)}{(h_1 - h_2)} \]

Condenser Analysis

- Usually simpler than work analysis

\[ \dot{Q}_L = \dot{Q}_{\text{cond}} = m_b (h_1 - h_5) = m_b (h_5 - h_1) \]

\[ \dot{Q}_H = \dot{Q}_{\text{SG}} = m_a (h_5 - h_4) = m_a (h_4 - h_5) \]

\[ \eta = \frac{\dot{W}_{\text{net}}}{\dot{Q}_H} = \frac{\dot{Q}_H - \dot{Q}_L}{\dot{Q}_H} = 1 - \frac{m_b (h_5 - h_1)}{m_a (h_4 - h_5)} \]

Refrigeration Cycles

- \( P_{\text{evaporator}} = P_1 = P_4 = P_{\text{sat}}(T_4 = T_1) \)
- \( P_{\text{condenser}} = P_2 = P_3 = P_{\text{sat}}(T_3 < T_2) \)

State 1: \( h_1 = h_0(P_1) \)
State 2: \( h_2 = h(P_2) \)
State 3: \( h_3 = h_0(P_3) \)
State 4: \( h_4 = h_3 \)

cop = \frac{(h_1 - h_4)}{(h_2 - h_1)}

Air-Standard Cycle Analysis

- Use air properties as ideal gas with variable or constant heat capacity
- Model chemical energy release as heat addition (~1,200 Btu/lbm or 2,800 kJ/kg)
- Heat addition at constant pressure, volume or temperature
- Isentropic work
- Closed system except Brayton Cycle

Constant \( c_p \) Brayton Cycle

- Brayton Cycle
- Given: \( P_R, P_1, T_1, q_{in} \)
- Find: \( \eta \)
- \( P_2 = P_1/P_R \)
- Isentropic compression to \( P_2 \)
- \( T_2 = T_1(P_R)^{k-1}/k \)
- \( T_3 = T_2 + q_{in} / c_p \)

Brayton Example (const \( c_p \))

- Isentropic expansion from \( P_3 = P_2 \)
- to \( P_4 = P_1 \)
- \( T_4 = T_3(P_R)^{k-1}/k \)
- \( |q_L| = c_p |T_1 - T_4| \)
- \( \eta = 1 - |q_L| / |q_H| \)
- Can show that \( \eta = 1 - 1 / (PR)^{k-1}/k \) for constant \( c_p \)
Variable $c_p$ Brayton Cycle

- Brayton Cycle
- Given: $PR, P_1, T_1, q_H$
- Find: $\eta$
  - Get $h(T_1)$
  - Isentropic compression to $P_2$
  - $P_r(T_2) = P_r(T_1)(PR)$
  - Find $h(T_2)$ from $P_r(T_2)$
  - $h(T_3) = h(T_2) + q_H$

Brayton Cycle Example

- Isentropic expansion to $P_4 = P_1$
- $P_r(T_4) = P_r(T_3)/(PR)$
- $|q_L| = h(T_1) - h(T_4)$
- $\eta = 1 - |q_L|/|q_H|$
- $w_{net} = [h(T_3) - h(T_4)] - [h(T_2) - h(T_1)]$
- $\eta = 1 - w_{net}/|q_H|$

And, in conclusion

- Need to know property relations (tables and ideal gases) to work problems
- First law energy balances in a variety of systems (closed, steady and unsteady)
- Main application of second law is isentropic work and efficiencies
- Cycle analysis looks at groups of devices to get overall efficiency or coefficient of performance

The Final

- Tuesday, December 14, 10:15 am to 12:15 pm
- Similar to the homework, midterm and quiz problems
- Use a clean equation sheet (with no writing)
  - Can add equation $dh = Tds + vdP$
- More credit for showing how to get solution than for details of algebra/arithmetic
- Sample final exam questions online
  - Will post solutions on Thursday

The Final Continued

- Like four quiz problems
  - Cycles: air-standard, refrigeration, Rankine
  - At least one problem with different mass flow rates
  - Isentropic efficiencies in at least one problem
  - Properties for liquid, mixed and gas
  - At least one problem with open and one with closed systems
    - Low, but non-zero, probability of transient problem (may involve second law)