Part 8. Topics in Capital Budgeting

In part 7 we learned the basics of capital budgeting. However, we ignored some of the complications that can arise when evaluating projects. In this section we look at a few of those issues.

How Uncertainty Affects the Capital Budgeting Decision

Every project has uncertainty and so we need to determine how risk affects how we make decisions. Large corporations often use very sophisticated methods to incorporate risk into capital budgeting, but every businessperson needs to know a few basic techniques for evaluating uncertainty.

Break-Even Analysis

It is often useful to know at what point a project neither makes nor loses money – the break-even point. Often managers have a sense of whether the project could at least make that level. For example, imagine a project that has a fixed cost of $100,000 and earns on net $4 per unit sold. You expect to sell 35,000 units, which generates a profit of $40,000 (35,000*$4 - $100,000 = $40,000). The break even-point is 25,000 units (25,000*4-$100,000=0). You might be wrong about selling 35,000 units, but if you know that the project will sell at least 25,000 units then you know you won’t lose money, and would feel more confident about going ahead with the project.

Example: The upfront cost of a project is $100,000. Sales are for five years, starting next year. The company earns $0.50 per sale, and expects to sell 60,000 units per year. If the interest rate is 5%, the NPV is -100,000 +129,884 = 29,884. At what level of sales does the project break even in terms of NPV?

Set PV = -100,000 (so that NPV = 0) and find the amount of sales per year. The answer is sales in dollars = 23,097 so that sales in units = 46,194.

Break-even analysis is a good starting point, but it ignores some important information. It doesn’t tell us the probability of getting a specific result or how good or bad the results could be. It might be worth it to risk losing money if there is a chance of us getting a big reward. To answer these kinds of questions, we need to look at our calculations in more detail.

Sensitivity Analysis

Anytime we are making capital budgeting decisions we need to make assumptions about the project: how many units we sell, how long it will take to complete the project, what will be our cost of capital? We need to know how reliable our assumptions are and how the results for the project change if we are wrong about the assumptions. Sensitivity analysis is a way of measuring how sensitive the outcome is to our assumptions about the project.

Sensitivity analysis changes one of the assumptions, leaving the other assumptions the same, and determines how the NPV or IRR changes. Let’s go through a detailed example:

You are considering buying a machine that costs an extra $5,000 over the base model (in terms of purchase price) and offers cost savings of 10% over the next three years. Total cost of running
the machine is $20,000 per year, so savings would be $2,000 per year. The cost of capital is 8%. At this point we are making assumptions about four things: the extra cost of buying the machine, the cost savings percentage, the cost of running the machine and the cost of capital.

At the assumed values of the parameters, the present value of the cost savings is $5,154. The cost is $5,000, so the NPV is $154, and so we should do this project.

However, what if we are not sure about the amount of savings? This is where we can use sensitivity analysis. We change our assumption on the cost savings, leaving the other assumptions the same, and see what happens to the desirability of the project.

Say that the costs savings turns about to be only 7%. The cost savings is now $1,400. The present value of the three-year annuity is $3,608, the NPV is -$1,392. If this were the true cost savings, then we would not want to do this project. We would want to do this exercise for any value of cost savings that we thought likely.

A follow-up question might be ‘how much cost savings do we need to make this machine worthwhile?’ This is just a break-even question. We want to find the cost savings such that the NPV = 0. The cost savings are $1,940. If the cost of the machine is $20,000, then cost savings are 9.7%. In other words, the cost saving percentage has to be at least 9.7% for this to be a worthwhile project. This is pretty close to our original estimate of 10%, which tells us that there may be a substantial risk of losing money on this project if our estimate is wrong.

You can repeat this exercise by examining the sensitivity to our assumption about the cost of capital. For example, what would happen if the cost of capital turned out to be 10%?

A question that always comes up in scenario analysis is, which assumptions do you look at and what alternate values do you try? There are no absolute rules for this; it depends of the specifics of the project. You want to examine the sensitivity to assumptions that you are least sure about since there is a greater chance that you might be wrong. You also want to examine the assumptions that have the biggest effect on NPV, since if you are wrong it could dramatically change the outcome. When picking alternate values, try values that seem likely or plausible. You don’t need to be limited to just one value. Try several, both good and bad, to give you some idea about the range of outcomes.

Scenario Analysis

Often we are unsure about more than one of the assumptions. We would like to extend sensitivity analysis to include changing a number of variables at the same time. This is called scenario analysis. Again, which assumptions you change and how much to change them depends on the specifics of the problem. One approach is to see what happens at the extremes. The “best case” is when everything turns out better than expected. This is always good to know, but probably more important is the “worst case” when everything turns out worse than expected. You would like to know how much trouble the firm would be in if everything went bad.

When calculating best and worst cases, you compare the results with the “base case”, usually the most likely, or expected outcome. In the previous example, the base case assumption produced a NPV of $154. If we think that the worst case is cost savings of only 7%, a cost of capital of 10% and low usage so that energy costs are only $15,000, then the present value of the cost savings is 2,611, and so the NPV is -2,389. Clearly this is a bad outcome (we knew it would be from our previous calculations). How concerned the firm should be depends on the probability that this
happens and how it can handle the loss of money. In this case, the outcome might not be too bad, but in other case, firms that risk a lot of money in projects can find themselves in great difficulty, which is why it is always worthwhile to do this kind of analysis before making decisions.

Simulation

So far, we have not said much about the probabilities of different outcomes. Often we are not able to give exact probabilities, and can only say that something could happen, or alternatively, that it is not very likely. However, if we do have a good idea about the probabilities we can take advantage of computer technology to improve upon our scenario analysis. If we know what the probabilities of each of the outcomes are, and we knew how they are interrelated (for example, how likely is it to have both a low cost-savings percentage and low energy usage) we could come up with a distribution of NPVs through a process called simulation. This gives us the probability of each possible NPV that we could get. Once we had this, we could use a discount rate appropriate for the amount of uncertainty about the NPV.

How to Evaluate Projects when there is Risk

As we know, people do not like risk, and so the return to a project must be adjusted to reflect the cost of the risk. The basic principle is that riskier projects should have to meet higher hurdles then low-risk projects. How much higher depends on how much risk there is. There are basically two ways to adjust the returns to a project to make riskier projects face a higher hurdle: increasing the discount rate or reducing the expected payment.

*Increasing the Discount Rate*

We can think of the discount rate as consisting of a base risk-free rate and a premium added depending on the level of risk: the higher the risk, the higher the premium. Greater uncertainty will reduce the NPV of risky projects since future payments are discounted at a higher rate. Equivalently, it implies that projects must have a higher IRR to exceed the hurdle rate.

*Example:* A project promises to pay $150,000 two years in the future. If we knew that this payment was certain we would calculate the present value using an interest rate of 4%. However, this payment is uncertain, and so we decide that a 3% risk premium is appropriate, giving us an interest rate of 7%. The higher interest rate means that the NPV of the project will be less since future payments are discounted at a higher rate. If we are using the IRR criteria, the project now must reach the higher standard of 7% to be approved.

How do you determine the appropriate size of the risk premium? The basic idea is that we can look at investors and see how much extra return they demand in the market to hold securities with the same amount of risk.

*Convert to Risk-Adjusted Cash Flows.*

Instead of increasing the discount rate, we can instead reduce the values of the cash flows. For example, say that we will get $200,000 five years in the future, but the amount could be higher or lower. If we think that $180,000 *for certain* is the equivalent of the risky $200,000, we could then do the NPV calculation with $180,000 using the risk-free interest rate. There are ways of determining the appropriate risk-adjusted value if we know the amount of risk and how risk-
averse we are. In more casual settings, a similar correction is made by just being conservative in our estimates of future cash flows, or by taking a “haircut” from the cash flows (reducing the cash flows by a fixed percentage).

Back to the Project at Digital Solutions

If we had the detailed information on how the cash flows were estimated, we could perform sensitivity analysis to determine how sensitive the results are to key parameters (Ms. Garcia would certainly have done this). However, we can examine certain scenarios using the information we have been given. We repeat the NPV calculation below for four scenarios.

**Best Case**

In this case, we assume that everything turns out great. This means using our optimistic estimates: development costs are low and sales high. Given these assumptions, we calculate the NPV.

<table>
<thead>
<tr>
<th>Year</th>
<th>Payment</th>
<th>Present Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-250,000</td>
<td>-231,481</td>
</tr>
<tr>
<td>2</td>
<td>-200,000</td>
<td>-171,468</td>
</tr>
<tr>
<td>3</td>
<td>150,000</td>
<td>119,075</td>
</tr>
<tr>
<td>4</td>
<td>200,000</td>
<td>147,006</td>
</tr>
<tr>
<td>5</td>
<td>300,000</td>
<td>204,175</td>
</tr>
<tr>
<td>6</td>
<td>300,000</td>
<td>189,051</td>
</tr>
<tr>
<td>7</td>
<td>600,000</td>
<td>350,094</td>
</tr>
<tr>
<td><strong>NPV</strong></td>
<td></td>
<td><strong>606,452</strong></td>
</tr>
</tbody>
</table>

Not surprisingly, NPV is greater than zero. If the project was worth doing in the most likely scenario, it is certainly worth doing in the best-case scenario.

**High Development Costs**

In this scenario we assume that development costs prove to be higher than expected. We take the pessimistic estimate for development costs and the most likely estimate for sales.

<table>
<thead>
<tr>
<th>Year</th>
<th>Payment</th>
<th>Present Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-350,000</td>
<td>-324,074</td>
</tr>
<tr>
<td>2</td>
<td>-600,000</td>
<td>-514,403</td>
</tr>
<tr>
<td>3</td>
<td>100,000</td>
<td>79,383</td>
</tr>
<tr>
<td>4</td>
<td>200,000</td>
<td>147,006</td>
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<tr>
<td>5</td>
<td>300,000</td>
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<tr>
<td>7</td>
<td>600,000</td>
<td>350,094</td>
</tr>
<tr>
<td><strong>NPV</strong></td>
<td></td>
<td><strong>131,232</strong></td>
</tr>
</tbody>
</table>
NPV is still positive, though lower than in the most likely case. We should find it somewhat reassuring that even if development costs turn out to be higher than expected, the project would still be worth doing.

**Low Sales**

Here we assume that sales are lower than expected. We take the most likely estimate of development costs and the pessimistic estimate of sales.

<table>
<thead>
<tr>
<th>Year</th>
<th>Payment</th>
<th>Present Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-300,000</td>
<td>-277,778</td>
</tr>
<tr>
<td>2</td>
<td>-400,000</td>
<td>-342,936</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>50,000</td>
<td>36,751</td>
</tr>
<tr>
<td>5</td>
<td>50,000</td>
<td>34,029</td>
</tr>
<tr>
<td>6</td>
<td>50,000</td>
<td>31,508</td>
</tr>
<tr>
<td>7</td>
<td>600,000</td>
<td>350,094</td>
</tr>
</tbody>
</table>

NPV: \(-168,330\)

Here, the NPV is negative and so the project is not worth doing. Sales, not surprisingly, turn out to be very important in determining the profitability of the project. This presents a significant risk to the firm, and suggests that the company may want to see if it can do anything to reduce the uncertainty about the sales estimates.

**Worst Case**

In the worst case, sales turn out to be lower than expected and development costs are higher.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>1</td>
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<tr>
<td>2</td>
<td>-600,000</td>
<td>-514,403</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
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<tr>
<td>7</td>
<td>600,000</td>
<td>350,094</td>
</tr>
</tbody>
</table>

NPV: \(-386,094\)

The NPV is even more negative.

Notice, that by itself, this information doesn’t tell us whether we should do the project. For that, we would need to know the probability of each situation. If we do have some idea about the probabilities, then we can draw conclusions. For example, if we know that it is very likely that
we will make our most-likely sales forecast, then the project looks pretty good since even if development costs are high we will make money.

Even if we don’t know the probabilities, scenario analysis can still be quite useful. It is a good way to present information. It is likely that each of the senior managers of Digital Solutions have their own idea about the probability of each outcome, so Ms. Garcia can present the possibilities and let them decide how much importance to give to each situation. Furthermore, this analysis can be used to determine what additional information is needed.

Indeed, this would be the approach of Ms. Garcia’s report. She would show that this could be a profitable project for the company but that it will depend on the assumptions, particularly the amount of sales. If the company decides to proceed with the project they need to be aware of this risk, and should probably make further investigations into how much sales to expect.

Conclusion

All projects have uncertainty associated with them so it is important that you take the risks into account when making capital budgeting decisions. The most important thing is to be aware of the risks and to use techniques such as break-even analysis and scenario analysis to develop a sense of the possible outcomes. When making decisions, riskier projects need to meet a higher hurdle rate, both when using NPV or IRR. You need to be as specific as possible about the size of the risks facing the project; in larger projects, it is typical to use quantified measures of risk and market-based measures of the extra burden of that risk.

The Value of Having Flexibility

Up to this point, we have assumed that you make a one-time decision at the start whether or not to do a project, and that this decision never changes. In many situations, that is not the case. Often things can change or new information can arrive that may cause you to abandon the project or want to change the amount you produce. Being able to change your mind (and either avoid some of the costs of the project or get an extra benefit) is valuable and should be included in your calculation.

Flexibility comes in a number of different forms. You could have an abandonment option, so that if you later decide not to pursue the project you do not have to pay all the costs. Or you could have capital equipment that can be easily sold if you decide to abandon the project. Our you might build a factory larger than needed to give you the ability to expand production in the future.

Having flexibility is called having a real option. It gets its name because it is similar to a financial option. Some proposals offer more flexibility than others and this needs to be taken into account when determining the value of the project. However, how to do that is a topic for a more advanced course.
What to Do When Comparing Projects of Unequal Lengths

A special kind of project evaluation is when you are comparing two or more options that require repeated purchases, but last unequal lengths of time. While this sounds like an unusual case, it can happen in a number of circumstances. Imagine that you are evaluating two different kinds of light bulbs for your company. One kind of light bulb is cheap but only lasts for two years. The other light bulb lasts for four years, and uses less energy each year, but costs more initially. Which to choose?

Notice that this problem has a different structure than the previous projects we have looked at. In those examples there was a flow of cash out at the start and then positive cash flow later. In this situation, cash is flowing out at the start when you buy a light bulb, but also later when you pay for the electricity. And there doesn’t seem to be any cash flowing in! Does it matter? No, this is still a problem in discounted cash flow. We know we must have the light bulbs, and both types of light bulbs get the job done (although at different costs). We only need to concentrate on the cost part. The light bulb that has the lowest cost is the light bulb that provides the greatest NPV to the company.

Let’s add some numbers to the problem. The first light bulb (that lasts for two years) costs $1 per bulb and uses $0.40 of electricity per year. The second light bulb (that lasts for four years) costs $2 per bulb and uses $0.30 of electricity per year. For convenience, we will assume that the light bulbs are purchased now (year zero) but aren’t used until year 1 (nothing depends on this, it just means that we can use an ordinary annuity rather than an annuity due). We are also ignoring the costs of replacing the light bulbs, a significant benefit of the longer-lasting bulbs. The difficulty is that we cannot directly compare the two light bulbs. The first one is cheaper on the face of it, but we know they will have to be replaced sooner. There are two ways to compare projects when they have equal length.

Replacement Chain Method

The replacement chain method asks that you imagine replacing the light bulbs sequentially in both cases until both sequences have the same length. In our situation, two two-year light bulbs will last for four years, the same as one four-year light bulb. What we need to do is compare the cost of one four-year bulb with the cost of a sequence of two two-year bulbs. We can compare the two choices on a time line.

\[
\begin{array}{cccccc}
0 & 1 & 2 & 3 & 4 \\
1.0 & 0.4 & 1.4 & 0.4 & 0.4 & \text{Two Two-Year Light Bulbs} \\
2.0 & 0.3 & 0.3 & 0.3 & 0.3 & \text{One Four-Year Light Bulb} \\
\end{array}
\]

The four-year bulb is easy. The cost is $2 plus the present value of a four-year annuity of $0.30 per year, which equals $2.95 (assuming an interest rate of 10%). The two two-year bulbs cost $1 in year 0 and $1 in year 2 plus the present value of a four-year annuity of $0.40, which equals $3.10 (rounded). The four-year bulbs are slightly cheaper so should be chosen.
Equivalent Annual Annuity

Another approach is to calculate the equivalent annual annuity value of the project. This translates all the cost and benefits of the project into a dollar value per year. In the light bulb example, we are trying to find a regular sequence of payments that has the same cost (in present value) as the light bulb.

Four-Year Light Bulb

Find the value of X such that the present values of the two streams of payments are the same.

\[
\begin{array}{cccc}
$2.0 & $0.3 & $0.3 & $0.3 \\
0 & 1 & 2 & 3 \\
\end{array}
\quad \quad \quad \quad \quad
\begin{array}{cccc}
$X & $X & $X & $X \\
0 & 1 & 2 & 3 \\
\end{array}
\]

Two-Year Light Bulb

Find the value of Y such that the present values of the two streams of payments are the same.

\[
\begin{array}{ccc}
$1.0 & $0.4 & $0.4 \\
0 & 1 & 2 \\
\end{array}
\quad \quad \quad \quad \quad
\begin{array}{cc}
$Y & $Y \\
0 & 1 \\
\end{array}
\]

EAA values, since they are the same each year, can be compared directly. For our four-year light bulb, we first calculate the present value of the total cost, which equals $2.95 at a 10% discount rate. A four-year annuity that has a present value of $2.95 would have annual payments of $0.93. For the two-year bulb, the present value of the costs is $1.69. The annual payment for an equivalent two-year annuity is $0.97. The four-year bulbs are slightly cheaper so should be chosen.

While we have used the EAA approach in the context of a problem with only costs, it can be used in any capital budgeting problem. Simply find the annuity payment that produces the same present value as the NPV of the project.