The NBC television network, a subsidiary of the General Electric Company (GE), uses optimization-based sales systems to improve its revenues and productivity. GE’s corporate research and development center (CRD) developed these systems using operations research and management science techniques to improve NBC’s sales processes. These systems remove bottlenecks caused by manual development of sales plans, helping NBC to respond quickly to client requests with sales plans that meet their requirements. These systems also enable NBC to make the most profitable use of its limited inventory of valuable advertising slots by estimating demands for airtime by show and by week and to schedule commercials. Between 1996 and 2000, the systems increased revenues by over $200 million, improved sales-force productivity, reduced rework by over 80 percent, and improved customer satisfaction. They have become an integral and essential part of NBC’s sales process.

(Industries: communications, journalism. Programming: multiple criteria.)
process was laborious, taking several hours. Moreover, most plans required a great deal of rework because, owing to their complexity, they initially met neither management’s goals nor the customer’s requirements. Typically, planners generate over 300 plans for prime-time sales during the up-front market, and the sales force works 12 to 16 hours a day during that two- to three-week period.

We saw a need for a system that would generate sales plans quickly in a manner that made optimal use of the available inventory. The system would take user-entered inputs and automatically generate plans to meet all the customer requirements and make the best possible use of the inventory. In addition, it would track the business under negotiation, estimate its pressure on inventory, and generate reports to help management to dynamically adjust pricing based on market demand and sell-out percentages of shows by air date.

The Up-Front Market

The up-front sales process begins with the announcement of the program schedule (Figure 1). The major networks (ABC, CBS, Fox, NBC, UPN, and WB) announce their programming schedules in the middle of May for the broadcast year that starts in the following September. Immediately after announcing their program schedules, the networks finalize their ratings forecasts and estimate the market demand. The ratings forecasts are projections of the number of people in each of several demographic groups that are expected to watch each airing of the shows in the program schedule for the entire broadcast year. Advertisers are interested in reaching audiences in different demographic groups based on their product categories. For example, car companies are generally interested in advertising to adults 25 to 49 years old.

The ratings estimates are based on several factors, such as the strength of the show, historical ratings data for the time slot, the competing shows on other networks, and the performance of the adjacent shows. The total market demand depends largely on the strength of the economy, while a network’s share of the market is determined by the expected performance of its program schedule over the broadcast year.

After they finalize their ratings projections and market demand estimates, the networks set the rate cards. Rate cards contain base prices that networks wish to charge for a 30-second commercial on an airing of a particular show. Typically, each airing of a show has a different price. The show prices are normally very high during the sweeps periods in November, February, and May. They tend to be low during early January and in the summer when advertising demand is weak and most networks run repeat shows. The rate-card price is just a baseline price. The actual price that an advertiser pays depends on the mix of commercial slots that it purchases. Certain shows and certain weeks of the year are heavily in demand. For example, such shows as Friends and ER are in high demand, while other shows with lower historical ratings are not

Figure 1: Before the up-front market begins, NBC forecasts its show ratings and the market demand. Based on these estimates, it sets the rates for all the programs in the broadcast schedule. During the up-front market, NBC sells about 60 to 70 percent of its inventory. Demands are periodically re-estimated during the up-front market, and the information is used to update the rates and revise sales strategy.
very desirable to advertisers. Similarly, periods in November and early December are in great demand because of the Christmas advertising rush, whereas the period immediately after Christmas is not in high demand. The price that an advertiser pays depends on the mix of slots that it purchases. Advertisers who are interested in time slots in high demand pay more for a specific commercial slot than those who also buy time slots that are not in high demand. Each advertiser therefore either pays a premium or gets a discount off the rate card prices based on the mix of shows and time periods for which it purchases time slots. This premium (discount) is expressed as a percentage over (under) the rate-card price and is called the percent-to-minimum (PTM).

During the up-front market, clients approach NBC with plan requests. The sales process (Figure 2) begins with an NBC account executive (AE) receiving sales requests from an agency. An AE typically works with several agencies. The sales management prioritizes the current sales requests from all agencies in the order of their importance to NBC. Clients that usually pay high premiums and spend a lot of money with NBC are given high priority. Sales planners work on the sales requests in order of their priority, developing a detailed schedule of commercial slots to be assigned to a client that meets both the client’s and the network’s requirements.

The sales management checks the plan to ensure that it meets all of NBC’s requirements. Management tries
to ensure that the plan does not give more high-premium inventory to the client than is necessary to win the contract. The plan is prepared to meet an overall budget target specified by the client. NBC tries to minimize the amount of premium inventory it includes in any given plan. If management does not approve the plan, it sends it back to the planning department for revisions. Once the plan is approved internally, the AE sends it to the agency and follows up with a phone call. The agency may reject the plan if it feels that it does not meet all its requirements. If that happens, the AE asks the planner for revisions. Once the agency and NBC are satisfied with the proposal, they sign a deal. If they cannot reach agreement, the deal is lost.

Periodically, during the up-front market, NBC managers re-estimate the demand based on new market knowledge gained from the sales made to date, the proposals under negotiation, and the impending demand estimates. They then revise the rate card and set aside inventory for the future.

Project History

In January 1996, GE Corporate Research and Development (CRD) awarded the lead author (Bollapragada) $50,000 in funding to develop a project in the general area of yield management for NBC. This funding translated to roughly 50 person days of effort. With the help of the CRD management, he then visited the NBC network sales department in New York City to get it interested in what operations research could do for its business. After several meetings with the sales personnel, he developed a good understanding of NBC’s sales processes. Working with the first author from NBC (Scholes), he learned that the current method for generating sales plans was a major source of delays and suboptimality in generating revenue. At that time complex sales plans were generated manually. This time-consuming approach made it difficult to respond to customers quickly. It also placed enormous pressure on sales planners and inevitably led to suboptimal results. A system that would remove this planning bottleneck would give NBC a major competitive advantage by enabling it

- to create plans that more accurately met customers’ requirements,
- to respond to customers more quickly,
- to make the most profitable use of its limited inventory of advertising slots, and
- to reduce rework.

In addition, it would also relieve some of the intense pressure on the sales and planning staffs during the up-front market period and enable them to work more productively and efficiently.

After analyzing the problem carefully, the lead author realized that the sales-plan-generation problem could be modeled as an integer program. He had proposed to develop a system for NBC that would automatically and optimally generate sales plans. However, the sales department was skeptical because of the complexity of the problem. The sales managers were also unaware of operations research and management science (OR/MS) techniques and how they could be employed to solve this problem.

Using the initial funding, the lead author decided to demonstrate how OR/MS techniques could be used to improve NBC’s sales processes. He put together a high-level approximate linear programming model for sales planning. This model would give guidelines on generating a good sales plan by specifying the number of commercial slots that should be allotted to a client by show and by quarter. He implemented this model using AMPL with CPLEX as the solver and directed the development of a user interface in Microsoft Excel using VBA. CRD implemented this simple application in three months, well before the start of the 1996 up-front market in the month of May. The system ran on a standalone PC and was used by the manager of sales planning during the 1996 up-front market. When a customer request arrived, the manager would enter it into this system and obtain guidelines on plan generation as a printout. He would then send the request along with the planning guidelines to a planner. The planner used the guidelines to develop a detailed plan. This reduced the time needed to generate a sales plan by

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The method for generating sales plans was a major source of delays.
one hour to about three hours per plan. This small system proved to be a major success during the 1996 up-front market.

The success of this Excel-based system opened several doors for CRD to implement OR/MS work at NBC. NBC funded us to develop a full-scale sales-optimization system, which we completed before the 1998 up-front market. Since then we have made two major upgrades to this system, released in 1999 and 2000. NBC also asked us to estimate demands by show in the program schedule and by week in the broadcast year during the up-front market. This information is invaluable to the sales managers during the up-front market as they revise prices and adjust the sales strategy. In April 1999, we installed a system called the inventory snapshot system for estimating show- and week-level demands. CRD was also funded to develop a system called ISCI (Industry Standard Commercial Identification) rotator, which was put into use in May 2000.

CRD and the information technology (IT) organization at NBC jointly developed these systems. CRD developed the optimization algorithms and implemented the client-server systems. The IT organization at NBC interfaced the applications with the NBC mainframe systems, developing routines to transfer data back and forth between the applications and the mainframe computer, and handled testing and maintenance.

CRD spent about $750,000 in developing these applications. NBC paid about 90 percent of CRD’s expenses. In addition, we estimate that NBC spent roughly $200,000 for their development and maintenance costs. Most of the development of these applications was done between 1997 and 1999.

The Sales-Plan-Generation Problem

NBC prepares sales plans to meet clients’ requirements. Clients generally communicate them by using the sales-request forms supplied by NBC (Figure 3). The sales request includes the client’s budget, that is, the amount the client is willing to spend on a package or the number of people the client wants to reach in its primary demographic category. The client also specifies how the budget is to be distributed over the four quarters in the broadcast year. The request may also specify how the commercials in the plan have to be distributed over the various shows in the program schedule, the weeks in the year, and the unit lengths of the commercials. The clients may specify these distribution or weighting criteria as a fraction of all equivalent 30-second commercials in the sales plan or as a percentage of the total dollars or impressions achieved in the plan. A typical request specifies weighting requirements using some or all of these criteria: the program mix, the weekly weighting, and the unit mix.

The client organization generally specifies the shows in the program schedule that it is interested in and the fraction of the total number of commercials in the schedule to be included on each show. Clients normally specify these requirements as fractions of the total number of equivalent 30-second commercials in the plan. In addition, clients may also specify how the commercials in the schedule are distributed by week in each quarter.

Clients may purchase commercials of varying lengths in their schedules. The most popular unit lengths are 30-seconds and 15-seconds. Sixty-second, 120-second, and 10-second units are not uncommon. A client that requests more than one unit length normally specifies the percentage of commercials of each length desired. (The Appendix contains a comprehensive mathematical representation of the client requirements as specified in a sales request.)

In response to the sales request, the NBC sales department prepares a sales plan that meets all the client requirements. A sales plan or a proposal consists of a complete schedule of commercials to be aired for the client, the cost of the package, audience estimates, and options to cut back or expand the package (Figure 4). The goal of our project was to develop a system to minimize the time it took to produce the sales plan, to improve accuracy in meeting client requirements, and to minimize the amount of premium inventory assigned to a plan. We called this system the automatic sales-plan-generation system (APG). More than 40 people use APG. Sales planners use it to generate
Figure 3: A typical client request consists of the client budget, product category and demographic information, plan weighting by week, program mix requirements, and the commercials slot lengths desired by the client. Clients generally communicate these requirements using a form supplied by NBC.

plans. Middle managers use it to ensure that as little premium inventory as possible is allocated to each plan. Upper managers review the reports it produces to revise sales strategy. At the core of the system is an algorithm we developed for generating plans.

**Model Formulation**

We formulated the sales-planning problem as a goal program. The budget on a client request (also NBC’s revenue from the sales contract) is fixed. Therefore it is optimal for NBC to use as little premium inventory as possible in meeting each client’s requests. Not all inventories are equally valuable. *Friends* and *ER*, for example, are in very high demand. Similarly, certain weeks in the year, especially during the sweeps periods, are highly desired by the advertisers. To quantify the value of inventory on show $s$ in week $w$, we define parameters $R_{sw}$. Management ranks the shows and weeks in the year by their importance. We calculate the parameters $R_{sw}$ using these rankings and the availability of inventory during various weeks. Any premium inventory saved can be used to attract additional class-A customers who pay high rates to be on the best shows. We therefore minimize the amount of premium inventory that is assigned to a plan.

We express the requirements specified in the client request as goal constraints. Sometimes NBC cannot meet all the goals because it lacks sufficient inventory. We associate penalty factors for not meeting the goals. The penalties are linearly proportional to the magnitude of deviation from the goals. Not all goals are equally important. We choose the penalty factors to vary based on the importance of the client requirements. We determined the actual penalty factors by
trial and error by solving plans under tight inventory constraints, which make it impossible to meet all client requirements. In our formulation, we also minimize the total penalty cost incurred on the sales plan to ensure that all client requirements are met when feasible.

Our formulation of the sales-planning problem follows:

Minimize
the amount of premium inventory assigned to a plan and the total penalty incurred in meeting goals
Subject to
Inventory constraints:
airtime availability constraints, and product conflict constraints;
Client requirement constraints:
budget constraints,
show-mix constraints,
weekly weighting constraints, and unit-mix constraints.

The decision variables are the numbers of commercials of each spot length the client requested that are to be placed in the shows and weeks included in the sales plan. (We give a detailed mathematical formulation of the problem in the Appendix.)

The inventory constraints are the hard constraints in the model. The airtime-availability constraints ensure that sufficient inventory is available on a given airing of a show to schedule a commercial for a client. The inventory available on an airing of a show changes constantly during the up-front market. We have to connect to the NBC mainframe computer to obtain the real-time inventory data.
The product-conflict constraints make sure that commercials for two competing products are not aired during the same commercial break. For example, commercials for both Coke and Pepsi should not air in the same commercial break. To model the product-conflict constraints, we query the database to obtain the total number of commercial breaks in a show airing and the number of competing commercials already placed on that show. We use this information to calculate the maximum number of commercials that we can allocate to the client on a given show airing.

The client-requirement constraints model the criteria specified in the client request. Because of the discrete nature of the problem, it may be infeasible to generate a plan that meets all requirements exactly. For example, it may not be feasible to allot exactly 10 percent of the commercial slots on a particular show to the plan while meeting all other requirements. However, it is sufficient to meet these requirements within a certain range. When clients ask that 10 percent of commercials be on a certain show, they generally accept a percentage between 9.7 and 10.3 percent. The ranges of the allowed intervals vary with the requirement types. The sales planner is allowed to control the widths of these intervals from the user interface.

In some situations, it may still be infeasible to meet the client requirements within these intervals because of insufficient inventory. We therefore associate non-negative slack variables with each of these constraints. We associate linear penalty costs with the slack variables to ensure that the client requirements are always met when feasible. Not all requirements are equally important. We choose the magnitudes of the penalty costs to vary to reflect this.

The objective is to meet the client’s requirements in a manner that allows for the best use of NBC’s inventory. The objective function includes a term, which represents the total value of inventory assigned to the sales plan as measured using the parameters $R_{sw}$. The objective function also includes terms that measure the penalties incurred in not meeting the client requirements.

The Sales-Plan-Generation Algorithm

The goal-programming formulation results in an integer program that is too large to solve in a reasonable amount of time using existing math program solvers, such as CPLEX. We therefore developed a customized algorithm for solving the problem (Figure 5). The algorithm involves solving the linear relaxation of the problem and intelligently rounding the solution to get an initial approximate solution. We then improve this solution using a search algorithm (Figure 6). Our search algorithm explores the feasible region while maintaining and updating a list of recent (tabu) steps and remembering the best solution obtained thus far.

In our algorithm, using the data entered by the user, we formulate a linear program that is a linear relaxation of the goal program presented in the Appendix. The linear program is solved using the CPLEX linear program solver. The results are rounded using a heuristic-rounding algorithm. The integer solution thus obtained is improved using a search algorithm.

In our rounding algorithm, we sort the variables in the descending order of the size of their fractional parts in the linear programming solution. We obtain the starting integer solution by truncating the variables. We then revisit the variables that had nonzero fractional values in the linear program solution one at a time in the sorted order, deciding whether to increment the value of a variable by one. We increment a variable if doing so does not violate any of the upper bounds on client requirements. Otherwise, we do not.
Our search algorithm improves the initial integer solution by evaluating a fixed number of solutions in the feasible region. The algorithm explores the feasible region taking one step at a time and always saving the best solution obtained so far.

We improve the initial solution obtained from rounding the solution of the linear relaxation using a customized search algorithm (Figure 6). The algorithm searches through the solution space by taking one step at a time. At each step a decision is made whether to add or delete a single unit in a show airing included in the plan. The resulting plan is evaluated after this step is executed. This solution is compared with the best solution on hand. If the current solution is better than the best solution, the best solution is replaced with the current one. The algorithm maintains a list of the most recent steps taken in a list of fixed length. We use this list to ensure that we do not reverse a step that was taken recently. The process is repeated for a fixed number of steps. We use the best solution on record as the final solution.

Our algorithm on average takes less than two minutes to execute. A user normally takes about five to 10 minutes to enter all the data required for the algorithm. Once the system comes up with a plan, it displays it, and the user can make changes. The complete process for generating a sales plan now takes between 15 and 30 minutes. This process formerly took between three and five hours. The amount of rework has been greatly reduced. Most plans now meet all client requirements and management objectives in the first pass. For a few small plans that we compared, the solutions were within two percent of the optimal.

Demand Prediction

The NBC TV network has around 250 customers, about 20 percent of whom account for 80 percent of the network’s revenues. NBC’s demand-prediction problem is quite different from those of businesses in the airline, car-rental, and hotel industries that use yield-management techniques. The airline and car-rental industries have millions of customers. Little information is available about individual customers. Therefore, these industries use demand models that are statistical in nature. In contrast, NBC has detailed knowledge of its customers. It has tools that record and update estimates of the amount of money each customer is going to spend with NBC and its competitors prior to the upfront season. However, NBC does not know how these dollars will translate to demands for individual shows during the various weeks in the year. This information would be useful for setting prices at the show level.

We developed a system that uses the estimated customer dollars and the historical buying patterns of the clients to estimate the demand at the show and week levels. The buying patterns of most of the major clients do not vary much from year to year. For example, most motion picture production companies spend heavily on Thursday nights during the Christmas and summer movie seasons. However, the shows on the program schedule change from one year to the next. To enable
us to predict demand, we map shows from one year to the next. We assign proper weighting factors when a single show is replaced by multiple shows of shorter lengths and vice versa. For example, several shows on Saturday during prime time (between 8:00 pm and 11:00 pm) 2000 were replaced by a single broadcast of the XFL football show in 2001.

Our inventory snapshot system comes up with estimates of the quantities of inventory that NBC can expect to sell during the up-front market on every airing of a show. It does this as follows. For each client, sales personnel enter an estimated budget. When the client requirements are received, the budget on the request replaces this estimate. Similarly, prior to receiving a sales request from the client, we estimate the program mix and weekly weighting percentages by mapping the previous year percentages for the client to the current year programs and weeks. We then have enough information (estimated or actual) for each client to generate a detailed sales plan. We use our planning algorithm to generate sales plans for all the clients. These plans contain estimated inventory purchases for the clients for the entire broadcast year. We then roll up these plans to estimate the total amount of inventory NBC can expect to sell on each airing of the shows. The system produces several inventory reports based on these estimates.

The inventory reports show the pressure on inventory by show and by week and the availability by product category. Shows and weeks with higher than average demand are highlighted. During the up-front market, the system is used several times a day to revise the demand estimates based on the sales plans under negotiation and the contracts already signed. NBC’s managers use this information to adjust prices for shows based on their expected sell-out levels. They also use information to decide whether to pursue certain clients aggressively or to save inventory for later sale.

**ISCI Rotator**

Major advertisers, such as Proctor and Gamble and General Motors, buy hundreds of time slots for a broadcast season. They decide on the actual commercials to be aired in these slots later. During the broadcast year, they send the videotapes of the commercials to be aired in the slots purchased. Each tape contains a single commercial and has a code written on it for identification. These codes are called Industry Standard Commercial Identification (ISCI) codes. The client gives instructions on the number of times each tape has to be aired. Normally two airings of a commercial have to be separated by as large an interval as possible. TV network personnel schedule the commercials by ISCI codes following the client’s instructions. In the past, the scheduling was done manually, following a cumbersome process that took several hours per day. The process was also error prone, causing lost revenues.

We developed an algorithm for automatically scheduling commercials. Our algorithm maximizes the time between any two airings of a given commercial while meeting all user requirements. The inputs to the algorithm are the total number of available slots, the number of different commercials to be aired in these slots, and the number of times each commercial has to be aired. The user is allowed to manually schedule any number of commercials prior to using the algorithm to automatically schedule in the rest of the time slots. We implemented the algorithm in a software application that NBC has used since April 3, 2000. The software system greatly reduced the time taken to schedule commercials and eliminated errors that occurred during scheduling.

The ISCI rotator problem is equivalent to placing balls of different colors in slots such that the distance between any two adjacent balls of the same color is as large as possible. The natural formulation of this problem has a nonlinear objective function with linear constraints (Appendix). This problem is NP-complete and has not been addressed in the OR literature. We developed a heuristic greedy algorithm that is implemented in our application. We are currently working on a more sophisticated optimal algorithm. Our results are encouraging, and we intend to publish them in the near future.

**System Implementation**

We have implemented the algorithms used for sales planning and demand prediction in a client server
application, written in Visual Basic. The application relies on two databases. The central database stores such information as the program schedule, rate cards, and audience estimates. In addition, the system communicates with NBC’s mainframe computer to get real-time on-air inventory and to upload completed plans. The CPLEX linear program solver is used in solving the linear relaxation of the goal program. The rounding algorithm and the search algorithm are coded in Visual Basic.

The central Sybase database, the server running the CPLEX software, and NBC’s mainframe computer are all located in Cincinnati, Ohio. Over 40 users located in New York, New York, Burbank, California, Chicago, Illinois and Detroit, Michigan use the system, communicating with the central resources in Cincinnati. The client application runs on Windows NT machines.

The application software was developed collaboratively by GE Corporate Research and Development center (CRD) and an NBC software team. We chose tools to be compatible with other NBC systems, so that we could easily install the application at NBC. While the actual algorithm code and user interface were developed at CRD, NBC provided software for the application to connect to the two databases and to maintain system security. We followed rigorous software engineering and quality methodologies in developing the systems.

We designed the Visual Basic application using object-oriented methodologies. We defined classes for the modeling information needed to fill out a sales plan. We included third-party spreadsheet controls in the application, which allow developers to take advantage of formatting and formula features in the controls rather than developing new methods. The controls also provide a common look and feel to the various spreadsheet forms used within the application. This Visual Basic add-in also provides methods for common spreadsheet functions, including cut, paste, clear, and print.

We wanted to make the application easy to use. A large amount of data is needed to generate a plan. To help users fill in that data, we introduced features to reuse data from prior plans, clear all data, and apply equally weighted factors, where applicable. Furthermore, we implemented a common look and feel among screens, especially for navigation. Buttons allow users to move logically to the next or prior form, but to enter data on forms in random order, users can choose menu items to select data-entry order.

Upon starting the system, the user logs on to both the central database and the NBC mainframe. As the user selects a plan for analysis or reporting, all relevant data is downloaded from the central database; keeping all data in memory improves performance and allows the user to make changes and try various what-if scenarios. When the user is ready to solve a plan, all relevant data is reviewed to maintain consistency of the entire data set as user inputs are modified. Data can be saved at any time to the database. During the save operation, an entire plan’s worth of data is saved to maintain data consistency. Sales planners and managers can revisit the data later to review plans and make changes.

To make the system robust, we designed the algorithm to work even if the CPLEX server or the mainframe computer is not available. If the CPLEX server is not available, the algorithm can still generate fairly good plans. If the mainframe is not available, we use default inventory data to generate plans that can be fixed later. A variety of reports are available within the application. Screen data can be printed out for the current plan under analysis and for the final plan. A variety of more general reporting features are also available for generating plan reports, plan requests, and inventory snapshots.

Impact Summary
We installed the first complete version of the sales-plan-generation system in April 1998. Over 40 sales personnel from offices in New York, Los Angeles, Chicago, and Detroit use it. After its successful use in the 1998 up-front market, we developed the next version of the system to incorporate enhancements requested by several users and installed it in March 1999. We introduced another set of improvements in 2000. We
implemented the inventory-snapshot system for estimating the show-level demand in 1999, and it has been in use since. Over these years, these systems have provided many benefits:

—By generating optimal plans, they have saved millions of dollars of good inventory for NBC while meeting all the customer requirements.
—They have increased revenues through better inventory management and pricing.
—They have reduced the time needed to produce a sales plan from three to four hours to about 20 minutes, enabling the sales force to focus on selling to customers.
—They have helped NBC to respond quickly to agencies and secure a greater share of the available money in the market. Being first in the market made NBC better able to set the terms.
—They helped NBC sales managers to resolve deals more quickly than in the past and better read the market resulting in a more accurate prediction of the up-front outcome.
—They improved service to the agencies by minimizing errors, rework, and response time.
—They decreased rework on plans by more than 80 percent.
—The ISCI rotator system greatly improved productivity in scheduling commercials and eliminated errors.

These systems increased NBC’s revenues by at least $50 million a year. From May 1996 to June 2000, NBC used these systems to generate plans and manage inventory worth more than $9 billion. The net gains it obtained using these systems between 1996 and 2000 are over $200 million. Most of the revenue gains came from the following three sources.

—By generating optimal plans, NBC ensures that it uses as little premium inventory as possible in meeting client requirements, thus saving valuable inventory for later sale. It can use this inventory to attract more class-A customers, who generally pay a premium to be on the best shows. The gains obtained as a result are over $15 million a year.
—By turning out plans quickly, NBC can target the best customers. The amount of money class-A customers spend on NBC increased after these systems were put in place, generating well over $20 million a year in additional revenues.
—Because it can predict demand by show and by week, management can revise the rate card readily. Quicker resolution of deals also gives management a better read of the market. We estimate that these improved pricing and sales decisions generate over $15 million in revenues annually.

These systems have become an integral part of the sales processes at NBC. The APG and inventory-snapshot systems are now critical for the up-front market; it is almost impossible to go through an up-front market without them.

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Appendix. The Sales-Plan-Generation Problem

We use the following general parameters:

- $S$ = the set of all shows.
- $W$ = the set of all weeks.
- $Q$ = the set of quarters.
- $Q_W$ = the set of all weeks in quarter $q$.
- $R_{sw}$ = the rank of show $s$ in week $w$.
- $P_{sw}$ = rate-card price for a 30-second commercial on show $s$ in week $w$.
- $I_{sw}$ = number of 30-second equivalent spots available to sell on show $s$ in week $w$.
- $A_{sw}$ = number of people in the client’s demographic that are expected to watch show $s$ in week $w$.
- $X_{sw}$ = maximum number of commercials that can be aired on show $s$ in week $w$ for the client without violating product-conflict constraints.

These are the plan goals:

- $L$ = the set of all allowable commercial lengths requested by the client. The lengths of the commercials are expressed as multiples of 30-second units.
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\[ B_1, B_2 \] are the lower and upper bounds on the total plan budget in rate-card dollars.

\[ B_{q1}, B_{q2} \] are the lower and upper bounds on plan budget in quarter \( q \).

\( \alpha_{q1}, \alpha_{q2} \) are the lower and upper bounds on the fraction of commercials that are to be aired on show \( s \).

\( \beta_{q1}, \beta_{q2} \) are the lower and upper bounds on the fraction of the total impressions in the quarter to be realized in week \( w \).

\( \gamma_{q1}, \gamma_{q2} \) are the lower and upper bounds on the fraction of the total units in the plan to be of length \( l \).

It may not be possible to meet all the requirements of a client because of inventory constraints. We therefore introduce the following slack variables that measure the deviations in meeting the client requirements.

\[ \sigma^b_{q1}, \sigma^b_{q2} \] are the slack variables associated with the lower and upper bounds on the budget in quarter \( q \).

\[ \sigma^w_{q1}, \sigma^w_{q2} \] are the slack variables associated with the lower and upper bounds on the fraction of commercials that are to be aired on show \( s \).

\[ \sigma^u_{q1}, \sigma^u_{q2} \] are the slack variables associated with the lower and upper bounds on the fraction of the total impressions in the quarter to be realized in week \( w \).

\[ \sigma^l_{q1}, \sigma^l_{q2} \] are the slack variables associated with the lower and upper bounds on the fraction of the total units on the plan to be of length \( l \).

The slack variables are all nonnegative. We introduce penalties for not meeting the client goals. The penalty incurred in not meeting a goal is proportional to the slack variable associated with that goal constraint. Because it is more important to meet some requirements than others, we choose the penalty parameters based on the importance of the goals. The penalty parameters we use are as follows:

\( \pi^b \) is the linear penalty associated with not meeting quarterly budget requirements.

\( \pi^w \) is the linear penalty associated with not meeting the showmix requirements.

\( \pi^l \) is the linear penalty associated with not meeting the unit-mix requirements.

The decision variables are the following:

\( x_{s,w} \) is the number of commercials of length \( l \), on show \( s \) in week \( w \) assigned to the plan.

We use the following dependent variables to make the problem formulation more readable:

\( h = \sum_{s \in S} \sum_{w \in W} \sum_{l \in L} \pi_{s,w} x_{s,w} \), the sum of rate card dollars for all units assigned to the plan.

\( b_q = \sum_{s \in S} \sum_{w \in W} \sum_{l \in L} \pi_{s,w} x_{s,w} \), the total rate card dollars for all units in quarter \( q \).

\( g = \sum_{s \in S} \sum_{w \in W} \sum_{l \in L} A_{s,w} x_{s,w} \), the total audience for the plan.

\( g_q = \sum_{s \in S} \sum_{w \in W} \sum_{l \in L} A_{s,w} x_{s,w} \), the total rate audience for quarter \( q \).

\( u = \sum_{s \in S} \sum_{w \in W} \sum_{l \in L} x_{s,w} \), the total 30-second equivalent units assigned to the plan.

\( w_q = \sum_{s \in S} \sum_{w \in W} \sum_{l \in L} l_{s,w} \), the total 30-second equivalent units in quarter \( q \).

**Problem Formulation**

Minimize

\[
\sum_{s \in S} \sum_{w \in W} \sum_{l \in L} \pi_{s,w} x_{s,w} + \sum_{q \in Q} \left( \pi^b u_q + \sum_{r \in R} \pi^w s_{r,q} + \sum_{q' \in Q} \pi^l s_{l,q} + \sum_{g \in G} \pi^u s_{u,g} + \sum_{l \in L} \pi^l s_{l,q} \right)
\]

subject to the following constraints:

\[
\sum_{q \in Q} b_q - b = 0,
\]

\[
\sum_{q \in Q} g_q - g = 0,
\]

\[
\sum_{q \in Q} u_q - u = 0,
\]

\[
\sum_{s \in S} \sum_{w \in W} \sum_{l \in L} \pi_{s,w} x_{s,w} - b_q = 0 \quad \forall q \in Q,
\]

\[
\sum_{s \in S} \sum_{w \in W} \sum_{l \in L} A_{s,w} x_{s,w} - g_q = 0 \quad \forall q \in Q,
\]

\[
\sum_{s \in S} \sum_{w \in W} \sum_{l \in L} x_{s,w} - u_q = 0 \quad \forall q \in Q.
\]

**Budget constraints:**

\[ b \geq B_1, \]

\[ b \geq B_2, \]

\[ b_q + \sigma^b_{q1} \geq B_{q1} \quad \forall q \in Q, \]

\[ b_q - \sigma^b_{q2} \leq B_{q2} \quad \forall q \in Q. \]

**Show-mix constraints:**

\[ \sum_{w \in W} \sum_{l \in L} x_{s,w} - \alpha_{q1} u + \sigma^w_{q1} \geq 0 \quad \forall s \in S, \]

\[ \sum_{w \in W} \sum_{l \in L} x_{s,w} - \alpha_{q2} u - \sigma^w_{q2} \leq 0 \quad \forall s \in S. \]

**Weekly weighting constraints:**

\[ \sum_{w \in W} \sum_{l \in L} A_{s,w} x_{s,w} - \beta_{q1} g + \sigma^w_{q1} \geq 0 \quad \forall w \in W_q, \forall q \in Q, \]

\[ \sum_{w \in W} \sum_{l \in L} A_{s,w} x_{s,w} - \beta_{q2} g - \sigma^w_{q2} \leq 0 \quad \forall w \in W_q, \forall q \in Q. \]

**Unit-mix constraints:**

\[ \sum_{w \in W} \sum_{l \in L} x_{s,w} - \gamma_{q1} u + \sigma^l_{q1} \geq 0 \quad \forall l \in L, \]

\[ \sum_{w \in W} \sum_{l \in L} x_{s,w} - \gamma_{q2} u - \sigma^l_{q2} \leq 0 \quad \forall l \in L. \]

**Inventory constraints:**

\[ \sum_{l \in L} b_{s,w} \leq l_{s,w} \quad \forall s \in S, \forall w \in W \text{ air time availability constraints}, \]
\[ \sum_{n_{ij}} x_{ij} \leq x_{sw} \quad \forall s \in S, \forall w \in W \] product conflict constraints.

The ISCI Rotator Problem

This problem is equivalent to placing balls of different colors in slots such that any two balls of the same color are placed as far apart as possible. We have \( N \) balls, of which \( n_1 \) are of color 1, \( n_2 \) are of color 2, and so on. Assign \( N \) balls to \( N \) slots such that the balls of any one color are as equally spaced as possible. In the ISCI (Industry Standard Commercial Identification) rotator problem, commercial airings are equivalent to balls. The colors are the set of unique ISCI codes.

Notation:
- \( i \) is the index on balls of a given color.
- \( j \) is the index on colors, \( j = 1, 2, \ldots, J \).
- \( k \) is the index on slots, \( k = 1, 2, \ldots, N \).
- \( n_j \) is the number of balls of color \( j \).
- \( N \) is the total number of available slots = \( \sum_j n_j \).
- \( q_j = N/n_j \) is the theoretical distance between two adjacent balls of color \( j \).

Decision variable:
- \( Y_{ijk} \) is 1, if ball \( i \) of type \( j \) is assigned to slot \( k \), otherwise.
- \( X_{ij} \) is the position (or slot number) of the \( i \)th ball of color \( j \) (equals \( k \) for which \( Y_{ijk} = 1 \)).

Problem Formulation

\[
\begin{align*}
\text{Minimize} & \quad \sum_{j=1}^{J} \sum_{i=1}^{n_j-1} |X_{i+1,j} - X_{ij} - q_j| \\
\text{subject to:} & \\
& \sum_{j=1}^{J} \sum_{k=1}^{N} Y_{ijk} = n_j, \quad j = 1, 2, \ldots, J, \\
& X_{ij} = \sum_{k=1}^{N} kY_{ijk}, \quad j = 1, \ldots, J; \quad i = 1, \ldots, n_j, \\
& \sum_{j=1}^{J} \sum_{i=1}^{n_j} Y_{ijk} = 1, \quad k = 1, 2, \ldots, N, \\
& X_{ij} < X_{i+1,j}, \quad j = 1, \ldots, J; \quad i = 1, \ldots, n_j - 1, \\
& 1 \leq x_{ij} \leq N
\end{align*}
\]

\( Y_{ijk} \) binary.

Heuristic Algorithm:

1. Sort data in descending order of \( n_j \).
2. Start with highest \( n_j \). Let \( a_j \) be the fractional part of \( q_j \). Place the first ball of this color in the first available slot. Distribute the remaining balls of this color as follows.
   \( a_j < 0.5 \): Let \( S \) be the slot that is \( q_j + 1 \) slots away from the current slot containing a ball of color \( j \). If an \( S \) is already occupied, try \( S - 1 \). If \( S - 1 \) is also occupied then try \( S + 1, S - 2, S + 2 \) and so on.
   \( a_j > 0.5 \): Let \( S \) be the slot that is \( q_j - 1 \) slots away from the current slot containing a ball of color \( j \). If an \( S \) is already occupied, try \( S + 1 \). If \( S + 1 \) is also occupied then try \( S - 1, S + 2, S - 2 \) and so on.
3. Repeat Step 2 for the next highest \( n_j \) and so on.