PRODUCTION FUNCTION AND LABOR MARKET: A SUMMARY

I. PRODUCTION FUNCTION

1. What determines the level of real GDP (Y)? To answer this question, it is necessary to look at the production function—a graph, table, or equation that shows the maximum amount of output that can be produced from a given set of inputs and a given state of technology. For our purposes, we shall assume there are only two inputs—labor (L) and capital (K). Capital consists of an economy's stock of buildings, machines, tools, transportation networks, etc.

2. Let L = aggregate level of employment; K = Capital; and A = a measure of technology known as total factor productivity. An increase in A means that the inputs are more productive—that is, the economy can produce more output with a given amount of inputs. The production function can be written in functional form as \( Y = A* f(L,K) \). At this stage, we can at least state the following relationships. An increase in aggregate employment (L) will increase Y, other things constant. Intuitively, this simply means that if more people are employed then an economy can produce more goods and services. Likewise, an increase in K will increase Y, other things constant. Finally, as mentioned above, an increase A will increase Y, other things constant. Holding K and A constant, we can show a graph of the production function with Y on the vertical axis and L on the horizontal axis (See Figure 1). Notice that a change in L is represented as a movement along the production function. On the other hand, a change in K or A cause shifts in the production function. This last point will be elaborated on in the discussion below.

3. Holding K and A constant, Y is determined by L. In other words, an increase (decrease) in L results in an increase (decrease) in Y. If L determines Y (holding K and A constant), then what determines L? To answer this question, it is necessary to look at the aggregate labor market (that is, the aggregate supply and demand for labor).

II. THE MARGINAL PRODUCT OF LABOR & LABOR DEMAND

1. We shall first look at the demand side. To understand the demand side, we use the following production function example. The numbers in the table are computed by assuming that the production function can be described by the equation \( Y = A*K^{0.4}*L^{0.6} \).

<table>
<thead>
<tr>
<th>L</th>
<th>Y</th>
<th>MP</th>
<th>VMP</th>
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<tbody>
<tr>
<td>0</td>
<td>0</td>
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</tr>
<tr>
<td>1</td>
<td>31</td>
<td>31</td>
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<td>2</td>
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<td>3</td>
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<tr>
<td>5</td>
<td>81.5</td>
<td>10.2</td>
<td>$10.2</td>
</tr>
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MP = marginal product of labor = \( \Delta Y/\Delta L \) = the change in Y/the change in L.

VMP is the value of the marginal product, which equals P*MP. The symbol P represents the price of the output and is assumed to be $1 per unit for this example. These tables warrant the following comments:

(i) Table 1 assumes that A is fixed at 20 and K is fixed at 3.

Notice that as employment increases, output increases but at a diminishing rate. The third column measures the benefit of hiring an additional unit of labor and is known as the marginal product of labor, which is defined as the change in output divided by the change in labor. In money terms this benefit is measured by the value of the marginal product.

TABLE 1: A=20 & K=3

TABLE 2: A=40 & K=3

TABLE 3: A=20 & K=4
(=P*MP), which is given in the fourth column (it is assumed that price equals $1).

(ii) Notice that in Table 2 we still assume K is 3, but we let A double to 40. An increase in A is interpreted as an improved state of technology, or another way of stating the same thing, total factor productivity has increased. If you compare Table 1 and Table 2, you should notice that at any given amount of labor, output has doubled--labor is more productive. On a graph, this shows up as an upward shift in the production function. It is equally important to note that the marginal product of labor has also doubled at any given quantity of labor. This also shows up as an upward shift in the MP curve. Conclusion: An increase (decrease) in A causes an upward (downward) shift in the production function and marginal product of labor curve. See Figures 2a and 2b. (Note: It is logically possible to have an upward or downward shift in the production function without the MP of labor curve shifting. For our purposes, however, we shall always assume that an upward shift in the production function is always accompanied by an upward shift in the MP curve.)

(iii) In Table 3, A = 20 (as in Table 1), but K = 4. If you compare Tables 1 and 3, you should notice that the increase in K has made labor more productive--at any given L, output has increased. Thus, an increase in K-like an increase in A-causes an upward shift in the production function and MP of labor curve.

2. It is important to discuss what might cause changes in A because of the importance this variable plays in explaining business cycles in certain classical macroeconomic models. Changes in A are frequently referred to as "supply shocks" or "productivity shocks" or "technology shocks". As the examples below will illustrate, the term "technology" is used in a very broad sense in this model (it effectively refers to anything which causes A to change). This is the terminology I shall employ in describing these changes. What might cause changes in A (supply shocks)? Some examples include:
a. Weather or other changes in the natural environment. Examples include droughts, major floods, and severe winters. In general, we would expect these to be temporary supply shocks.
b. Inventions or innovations in management techniques that improve efficiency, such as the adoption of the assembly-line technique, or microcomputers, or new crop-rotation schemes. In general, we would expect these to be permanent supply shocks.
c. Changes in government regulations such as antipollution laws, that affect the technologies or production methods used. Government policies that lead to increased flexibility or mobility in labor and capital markets improve efficiency and thereby increase total factor productivity. These may be temporary or permanent supply shocks.
d. Changes in the supplies of inputs other than L or K, such as the oil shocks of the 1970s, or the opening up of western lands for settlement in the 19th-century US. These shocks may be temporary or permanent.

3. In our discussion of the labor market, we shall be assuming that (a) all labor is alike, (b) labor markets are perfectly competitive--no single firm or worker can significantly influence the wage rate, and (c) that firms are profit maximizers. Thus, firms will hire an additional unit of labor as long as the additional benefits of hiring one more worker exceeds the additional costs of hiring one more worker. The additional benefits of hiring one more worker is measured by the VMP in nominal terms, or by MP in real terms. The cost of hiring an additional unit of labor is measured by the nominal wage rate (W) or, in real terms, by the real wage rate (W/P). A firm maximizes its profit by hiring labor up to the point where P*MP = W or, if we rearrange this equation, where MP = (W/P). For example, if W is $16, then the firm can maximize its profits by hiring 2 units of labor (since that is where VMP=W=$16 or alternatively where MP=(W/P) = 16. NOTE: Because a firm hires labor up to point where (W/P) = MP, the MP curve tells us the quantity of labor demanded at any given real wage rate. IN OTHER WORDS, THE MP CURVE IS THE SAME AS THE DEMAND CURVE. Therefore, the variables which shift the MP curve (i.e., changes in K and A) are the same variables which shift the demand for labor curve.
III. SUPPLY OF LABOR
1. We shall assume that an increase in the real wage rate leads to an increase in the quantity of labor supplied, other things constant. In other words, the labor supply curve has a positive slope.
2. What causes the aggregate labor supply curve to shift? Some of the important factors are:
   a. Changes in wealth. (By wealth, I mean an individual's financial and physical assets--stocks, bonds, savings accounts, real estate, etc.) In general, an increase in wealth will increase an individual's demand for leisure. This implies a decrease in the supply of labor (supply curve shifts to the left).
   b. Changes in the expected future real wage rate. An increase in the real wage rate that a worker expects to receive in the future makes that person effectively wealthier and thus reduces the amount of labor the amount of labor supplied at the current real wage.
   c. Changes in working-age population. Examples include changes in immigration laws or changes in birth rates.
   d. Changes in the labor-force participation rate. For example, the large increase in the number of women entering the labor force over the last few decades.
   e. Changes in the marginal income tax rate. Changes in the marginal income tax rates have theoretically ambiguous effects on labor supply. On the one hand, a cut in the marginal rate means that the benefits of an additional hour of work has increased (or stated another way, the cost of an additional hour of leisure--the forgone wage--has risen). This first effect implies an increase in labor supply. On the other hand, a cut in the marginal tax rate makes individuals effectively wealthier. Feeling wealthier, people desire to consume more leisure and work less. That is, this effect implies a decrease in labor supply. I shall be assuming that this wealth effect is small enough to ignore. We may therefore conclude that a cut in the marginal income tax rate will increase labor supply.

IV. LABOR MARKET
1. With the supply side added to the model, we have the labor market. Full-employment is defined as aggregate supply of labor ($L^s$) = aggregate demand for labor ($L^d$). Note: full-employment does not imply a zero unemployment rate in this model. There can still be frictional or structural unemployment at $L^s=L^d$.
2. Classical theory believes that the real wage rate adjusts rapidly to equate supply and demand. Any surpluses or shortages in the labor market are quickly eliminated by adjustments in the real wage rate. This is a very important point. It means that classical theorists believe that the labor market is nearly always at full employment ($L^s = L^d$).
3. If we combine the labor market with the production function, we can determine the aggregate level of employment, the real wage rate, and the level of output--for a given $K$ and $A$. See Figure 3.

V. CHANGES IN LABOR MARKET EQUILIBRIUM: SHIFTS IN DEMAND AND/OR SUPPLY
1. Suppose the economy is hit by an adverse supply shock (a drop in $A$), perhaps due to a spell of bad weather. What will happen to the real wage rate, the level of employment, and output? First, we must be a little more precise and ask whether the supply shock is temporary or permanent. A permanent supply shock tends to influence workers' expectations of future real wages (which shifts the current labor supply curve), whereas a temporary supply shock would not. Let us assume that the supply shock is temporary. In this case, the fall in the MP of labor would shift the labor demand curve to the left, causing the real wage rate and the level of employment to fall. The production function also shifts downward, and combined with the lower level of employment, implies a lower level of output. Figure 4a.
2. How would a cut in the marginal income tax rate affect the labor market? The cut in the tax rate would increase labor supply, driving down the pre-tax real wage rate (the after-tax real wage rate, not directly shown on the graph, would increase), and increasing the level of employment. The increased level of employment would cause output to increase. See Figure 4b.
FIGURE 3

Production Function

Y = A₁f(K,L)

Y = A₂f(K,L)

Labor Market

(W/P)

L₁

W/P₁

L₁ = MPL₁

L₂

W/P₂

L₂ = MPL₂

FIGURE 4a.

Production Function

Y = A₁f(K,L)

Y = A₂f(K,L)

Labor Market

(W/P)

L₁

W/P₁

L₁ = MPL₁

L₂

W/P₂

L₂ = MPL₂
To summarize our model to this point:

**LABOR MARKET**

1. Labor Demand Function: \[ L^d (W/P; \ A, \ K) \]  
   (-) (+) (+)

2. Labor Supply Function: \[ L^s (W/P; \ wealth, \ (W/P)_e, \ WAPOP, \ LFPR, \ t) \]  
   (+) (-) (-) (+) (+) (-)

WAPOP = Working-Age Population  
LFPR = Labor Force Participation Rate  
t = income-tax rate

**PRODUCTION FUNCTION**  
(+) (+) (+)

1. Production Function: \[ Y = A f(K, L) \]  
2. Some examples of factors which can lead to changes in \( A \) are: (a) weather changes, (b) inventions/innovations, (c)
changes in government regulations, (d) changes in the availability of input OTHER THAN labor or capital.

MONEY MARKET

I. THE DEMAND FOR MONEY
1. The demand for money is defined as the desire to hold money as a store of value. That is, the demand for money is the desire to hold part of one's wealth in the form of money. Wealth can be held in many forms other money: it can be held in other financial assets (which we generically call "bonds"), or in non-financial assets (real estate or other physical assets), or even in the form of human capital. Therefore, the decision to hold money in one's portfolio will weigh the tradeoffs between holding money versus holding other assets.
2. Why hold money at all? In general, it pays a very low rate of return (currency pays a zero nominal return). There must be some benefit to holding money, but what are these benefits? The answer is money's liquidity. Money is the most liquid of all assets. The liquidity of an asset is the ease and quickness with which it can be exchanged for goods, services or other assets.
3. What variables affect the demand for money?
   a. Price level (P). The higher the price level, the more dollars people require to conduct transactions (that is, the greater the need for liquidity) and thus the more dollars people will want to hold.
   b. Real income (Y). An increase in real income generally leads to an increase the number of transactions that individuals or businesses conduct. The greater the number of transactions, the greater the demand for liquidity, and therefore the greater the demand for money.
   c. Expected real rate of interest (re). An increase in the expected real rate of interest on alternative assets raises the cost of holding money, and therefore reduces the demand for money.
   d. Expected inflation rate. An increase in the expected inflation rate raises the cost of holding money. The value of money is fixed in nominal terms, and therefore an expected increase in the inflation rate implies an expected increase in the rate at which the purchasing power of money falls.
   e. Payment technologies (the technologies for making and receiving payments). For example, the introduction of credit cards allows people to reduce their average holdings of money.
4. The demand for money can be written as a function of these variables as follows:
   \[ M_d = f(P, Y, re, \pi_e) \]

II. MONEY SUPPLY
1. We shall be assuming that the money supply is completely determined by the Federal Reserve System (the Fed). This will be discussed in detail later on.

III. MONEY MARKET
1. To use the supply and demand framework for money, we must define what we mean by the price of money. We define the price of money as \( (1/P) \), where \( P \) is the price level. Recall that the price level is an index showing the number of units of money required to purchase an average commodity bundle. The price of money--the inverse of the price level--is the number of average commodity bundles required to purchase a unit of money. Thus, an increase in the price level (that is, a decrease in the price of money) causes the demand for money to increase. We can therefore illustrate the demand curve for money as a negatively-slopped curve when we put the price of money on the vertical axis.
2. What causes the demand for money curve to shift? Recall in our earlier discussion we mentioned several variables that affect the demand for money: the price level, real income, real rate of interest, expected inflation rate, and payments technology. Changes in the price level (i.e.,the price of money) cause movements along the demand curve, not shifts. However, a change in any of these other variables would show up as shift in the money demand curve. In other words, the demand for money curve shifts when there are changes in real income, expected real rate of interest, expected inflation rate, and payment technologies.
3. As mentioned above, we shall assume that the money supply is completely determined by the Fed. This implies that the money supply curve will be completely vertical.
4. Money market equilibrium exists when \( M^s = M^d \). When this is not the case, the price level will adjust to bring the
money market back to equilibrium.
5. What happens when the Fed increases the money supply (holding other things constant)? Initially, $M^s > M^d$. That is, there is an excess supply of money. Individuals discover they are holding more money than they desire to hold. They attempt to reduce excess holdings of money by purchasing goods and services, which will cause the price level to rise. The increase in the price level will increase the demand for money until it again equals the supply of money.

\[
\frac{1}{P} \quad \text{Money Market}
\]

\[
\begin{array}{c}
M^s \\
M^d
\end{array}
\]

\[
M^s, M^d
\]

THE CLASSICAL MODEL
1. If we combine the labor market and production function with the loanable funds market and the money market we have the classical model.
2. We will be using this model in class to discuss the effects on the economy of the following changes: a. Changes in $A$ (or $K$), b. Changes in labor supply, c. Changes in the money supply, and d. Changes in $G$ and/or $T$. We shall also use this model to see how well classical business cycle theory explains the business cycle facts.
3. The classical model can be summarized as follows:

**LOANABLE FUNDS MARKET**

\[
(+) (+) (-)
\]
1. Supply of loanable = $S(r; Y, Y^e, \text{fiscal policy variables})$.
2. A change in the expected real rate of interest is represented as a movement along the supply curve, whereas changes in $Y, Y^e$, or fiscal policy variables ($G$ and/or $T$) are represented as shifts in the supply of loanable funds curve.
3. Demand for loanable funds = $I(r; EFPI)$
4. A change in $r$ is represented as a movement along the demand curve, whereas a change in expected future profitability of investment is represented as a shift in the $I$ curve.
5. Equilibrium condition: $S = I$. Rapid adjustments in $r_e$ ensure that this market clears rapidly.

**LABOR MARKET**

\[
(-) (+) (+)
\]
1. $L^d(W/P; \text{changes in } K, \text{changes in } A)$
2. $L^s(W/P; \text{wealth, } (W/P)^e, \text{working-age population, labor-force participation rate, marginal-income-tax rate})$
3. Equilibrium condition: $L^d = L^s$.
4. Changes in the real wage rate are represented as movements along the supply and/or demand curve. A change in any of the other variables listed after the semicolon cause shifts in their respective curves).
5. The labor market clears rapidly in the classical model.

**PRODUCTION FUNCTION**

\[
(+) (+), (+)
\]
1. $Y = A \cdot f(K, L)$.
2. Changes in $L$ are represented as movements along the production function, and changes in $K$ and/or $A$ are presented as shifts in the production function.

**MONEY MARKET**

1. $M^s$ is determined by the Fed.
2. $M^d(P, Y, r, \pi^e)$ note: the price of money is defined as $(1/P)$.
3. Equilibrium condition: $M^s = M^d$. 

4. A change in P (i.e., a change in 1/P) is represented as a movement along the money demand curve. Changes in the other listed variables are represented as shift variables. The classical model in graphical terms can be seen on the next page.

CLASSICAL MACROECONOMIC MODEL

SYMBOLe KEY:
r (or r_e) = expected real rate of interest
Y = current real output = current real income = current real GDP
Y^e = expected future real income
I = real investment spending
S = real national saving
G = real government spending
T = real tax revenue
A = a measure of technology known as total factor productivity
K = capital stock (buildings, machines, tools, infrastructure, . . .)
L = labor (aggregate level of employment)
W = nominal wage rate
P = price level
(W/P) = real wage rate
(W/P)' = expected future real wage rate
M* = money supply
M^d = money demand
(1/P) = price of money (value of money)
MP = marginal product of labor

1. Temporary Negative Supply Shock

1. Labor Market: The decrease in A causes MP to fall (“labor is less productive”) which means that the labor demand curve shifts left. There is no shift in the labor supply curve since the decrease in A is temporary. Equilibrium W/P and L decline.

2. Production Function: The decrease in A causes the production function to shift downwards. This shift downward coupled with the decline in L results in a decline in real output.

3. Loanable Funds Market: The decline in real income (Y) causes a reduction in saving (S shifts to the left). This leads to an increase in r. The higher cost of borrowing causes I to fall (movement along the I curve).

4. Money Market: The decline in Y lowers desired transactions which causes money demand to fall. In addition, the higher r raises the opportunity cost of holding money, and this also causes money demand to fall. The decline in money demand results in a decline in 1/P (or an increase in P).
II. An increase in G

1. **Loanable Funds Market**: The increase in G, holding T constant, implies an increase in government borrowing. The more the government borrows, the less the amount of funds available for private borrowers—thus, S falls. The decline in S (leftward shift in S curve) causes r to rise. The higher r means the cost of borrowing is greater and therefore I declines (movement along I curve).

2. **Money Market**: The higher r raises the cost of holding money and thus lowers money demand. The drop in money demand causes 1/P to fall (that is, P to rise).

3. **Labor Market**: For a given W, the rise in P would tend to lower the real wage rate—W/P. The lower real wage rate would lead to a shortage of labor. The classical model assumes the labor market clears rapidly, and therefore the shortage will cause W to be bid up rapidly. As W rises, the real wage rate moves back to its original level. In the classical model—since the labor market clears rapidly—this means that the real wage rate does not change in the short run. Likewise, there is no change in L.

4. **Production Function**: Since there is no change in L (and no shift in the production function) there is no change in Y.
Note on crowding-out effect: Given that \( Y = C + I + G \), and given that we have now seen that \( Y \) does not change in the classical model, it follows that an increase in \( G \) will be matched by a decrease in \( C+I \) (private spending). This is the crowding-out effect. There are more resources employed in the government sector producing government goods, but fewer resources in the private sector producing private goods.

The channel by which the crowding-effect works in this model is through \( r \). The increase in \( G \), as noted above, causes \( r \) to rise which causes \( I \) to fall. In addition, the higher \( r \) will cause private saving (movement up along \( S_2 \) curve) to rise. For a given \( Y \), the increase in private saving implies a drop in \( C \).

### III. An increase in \( M \).

1. **Money Market**: The increase in money supply will create an excess supply of money at the original equilibrium \( 1/P \). The public attempts to reduce their excess holdings of money by buying goods which causes \( P \) to rise (or \( 1/P \) to fall).

2. **Labor Market**: For a given \( W \), the rise in \( P \) would tend to lower the real wage rate—\( W/P \). The lower real wage rate would lead to a shortage of labor. The classical model assumes the labor market clears rapidly, and therefore the shortage will cause \( W \) to be bid up rapidly. As \( W \) rises, the real wage rate moves back to its original level. In the
classical model—since the labor market clears rapidly—this means that the real wage rate does not change in the short run. Likewise, there is no change in L. Note: Notice that in the labor market the real wage rate remains unchanged, but the NOMINAL wage rate rises.

3. **Production Function**: Since there is no change in L (and no shift in the production function) there is no change in Y.

4. **Loanable Funds Market**: No change.

Thus, the change in the money supply does not affect any real variables. It only leads to a change in nominal variables. Money is said to be neutral.