Chapter 5

Discounted Cash Flow Valuation

Key Concepts and Skills
• Be able to compute the future value of multiple cash flows
• Be able to compute the present value of multiple cash flows
• Be able to compute loan payments
• Be able to find the interest rate on a loan
• Understand how loans are amortized, or “paid off”
• Understand how interest rates are quoted

Chapter Outline
• Future and Present Values of Multiple Cash Flows
• Valuing Level Cash Flows: Annuities and Perpetuities
• Comparing Rates: The Effect of Compounding Periods
• Loan Types and Loan Amortization

Multiple Cash Flows – FV Example 5.1
• Find the value at year 3 of each cash flow and add them together.
  – Today (year 0): FV = $7,000(1.08)^3 = $8,817.98
  – Year 1: FV = $4,000(1.08)^2 = $4,665.60
  – Year 2: FV = $4,000(1.08) = $4,320
  – Year 3: value = $4,000
  – Total value in 3 years = $8,817.98 + 4,665.60 + 4,320 + 4,000 = $21,803.58
• Value at year 4 = $21,803.58(1.08) = $23,547.87

Multiple Cash Flows – FV Example 2
• Suppose you invest $500 in a mutual fund today and $600 in one year. If the fund pays 9% annually, how much will you have in two years?
  ▪ FV = $500(1.09)^2 + $600(1.09) = $1,248.05

Example 2 Continued
• How much will you have in 5 years if you make no further deposits?
  ▪ First way:
    ▪ FV = $500(1.09)^5 + $600(1.09)^4 = $1,616.26
  ▪ Second way – use value at year 2:
    ▪ FV = $1,248.05(1.09)^3 = $1,616.26
Multiple Cash Flows – FV Example 3

• Suppose you plan to deposit $100 into an account in one year and $300 into the account in three years. How much will be in the account in five years if the interest rate is 8%?
  \[- FV = $100(1.08)^1 + $300(1.08)^2 = $136.05 + $349.92 = $485.97 \]

Example 3 Time Line

Multiple Cash Flows – PV Example 5.3

• Find the PV of each cash flow and add them
  – Year 1 CF: $200 / (1.12)^1 = $178.57
  – Year 2 CF: $400 / (1.12)^2 = $318.88
  – Year 3 CF: $600 / (1.12)^3 = $427.07
  – Year 4 CF: $800 / (1.12)^4 = $508.41
  – Total PV = $178.57 + 318.88 + 427.07 + 508.41 = $1,432.93

Example 5.3 Time Line

Multiple Cash Flows – PV Another Example

• You are considering an investment that will pay you $1,000 in one year, $2,000 in two years, and $3,000 in three years. If you want to earn 10% on your money, how much would you be willing to pay?
  \[- PV = $1,000 / (1.1)^1 = $909.09 \]
  \[- PV = $2,000 / (1.1)^2 = $1,652.89 \]
  \[- PV = $3,000 / (1.1)^3 = $2,253.94 \]
  \[- PV = $909.09 + 1,652.89 + 2,253.94 = $4,815.92 \]

Decisions, Decisions

• Your broker calls you and tells you that he has this great investment opportunity. If you invest $100 today, you will receive $40 in one year and $75 in two years. If you require a 15% return on investments of this risk, should you take the investment?
  \[- PV = $40/(1.15)^1 + $75/(1.15)^2 = $91.49 \]
  \[- No! The broker is charging more than you would be willing to pay. \]
Saving For Retirement
• You are offered the opportunity to put some money away for retirement. You will receive five annual payments of $25,000 each beginning in 40 years. How much would you be willing to invest today if you desire an interest rate of 12%?
  \[ PV = \frac{25,000}{(1.12)^{40}} + \frac{25,000}{(1.12)^{41}} + \frac{25,000}{(1.12)^{42}} + \frac{25,000}{(1.12)^{43}} + \frac{25,000}{(1.12)^{44}} = \$1,084.71 \]

Quick Quiz: Part 1
• Suppose you are looking at the following possible cash flows: Year 1 CF = $100; Years 2 and 3 CFs = $200; Years 4 and 5 CFs = $300. The required discount rate is 7%.
  • What is the value of the cash flows at year 5?
  • What is the value of the cash flows today?
  • What is the value of the cash flows at year 3?

Annuities and Perpetuities Defined
• Annuity – finite series of equal payments that occur at regular intervals
  – If the first payment occurs at the end of the period, it is called an ordinary annuity
  – If the first payment occurs at the beginning of the period, it is called an annuity due
• Perpetuity – infinite series of equal payments

Annuities and Perpetuities – Basic Formulas
• Perpetuity: \[ PV = \frac{C}{r} \]
• Annuities:
  \[ PV = \frac{C}{r} \left( \frac{1}{1 - \left(\frac{1}{1 + r}\right)^n} \right) \]
  \[ FV = C \left( \frac{(1 + r)^n - 1}{r} \right) \]

Annuity – Example 5.5
• You borrow money TODAY so you need to compute the present value.
  • Formula:
  \[ PV = \frac{C}{r} \left( \frac{1}{1 - \left(\frac{1}{1 + r}\right)^n} \right) = 23,999.54 \]
Annuity – Sweepstakes Example

• Suppose you win the Publishers Clearinghouse $10 million sweepstakes. The money is paid in equal annual installments of $333,333.33 over 30 years. If the appropriate discount rate is 5%, how much is the sweepstakes actually worth today?
  \[ PV = \frac{333,333.33 \times [1 - 1/1.05^{30}]}{.05} = 5,124,150.29 \]

Buying a House

• You are ready to buy a house and you have $20,000 for a down payment and closing costs. Closing costs are estimated to be 4% of the loan value. You have an annual salary of $36,000 and the bank is willing to allow your monthly mortgage payment to be equal to 28% of your monthly income. The interest rate on the loan is 6% per year with monthly compounding (.5% per month) for a 30-year fixed rate loan. How much money will the bank loan you? How much can you offer for the house?

Buying a House - Continued

• Bank loan
  \[ \text{Monthly income} = \frac{36,000}{12} = 3,000 \]
  \[ \text{Maximum payment} = .28(3,000) = 840 \]
  \[ PV = \frac{840 \times [1 - 1/1.005^{360}]}{.005} = 140,105 \]

• Total Price
  \[ \text{Closing costs} = .04(140,105) = 5,604 \]
  \[ \text{Down payment} = 20,000 - 5,604 = 14,396 \]
  \[ \text{Total Price} = 140,105 + 14,396 = 154,501 \]

Quick Quiz: Part 2

• You know the payment amount for a loan and you want to know how much was borrowed. Do you compute a present value or a future value?
• You want to receive $5,000 per month in retirement. If you can earn .75% per month and you expect to need the income for 25 years, how much do you need to have in your account at retirement?