I. Probability and Expected Value

In all that we have done so far, we've assumed that choices are being made under conditions of certainty -- prices are known, income is known, and so forth. But many situations and decisions involve an important element of uncertainty. For example, when you buy a lottery ticket, you are exchanging a certainty (the cost of the ticket) for a gamble (a chance to win the lottery). When you drive a car, you are also taking a gamble that your car might get destroyed (or you might get killed). The purpose of this lesson is to introduce a structure for dealing with choice under conditions of uncertainty.

We want to talk about people's preferences over risky situations, which we will refer to as gambles. A gamble is a risky situation with a probability for each possible outcome, such that all the probabilities add up to 1. For example, a coin toss is a gamble that assigns a probability 1/2 to heads, 1/2 to tails.

To be more precise, we should note that a probability is a number from 0 to 1 that expresses the likelihood of an event occurring, or the frequency with which that event would occur if we repeated the gamble many times.

The expected value of a gamble is the average outcome you would get if the gamble took place many times. For example, suppose you have a 75% chance of winning a dollar, and a 25% of winning nothing. The expected value of this gamble is 75 cents; if you took this gamble many times, on average you would win 75 cents.

Example: You have a house worth $80,000. There is a 10% chance it will burn down, leaving only a land value of $10,000. The expected value of this risky situation is .9($80,000) + .1($10,000) = $73,000.

A fair gamble is one whose price is equal to its expected (monetary) value. For example, suppose you can buy a ticket for a lottery that will give you $100,000 with 5% probability (and nothing otherwise). The expected value of the lottery ticket is $5,000. If the ticket price is $5000, the lottery is a “fair” gamble. For a lower price, the gamble is better than fair; for a higher price, it's worse than fair (unfair).

An aside: Notice that most lotteries you see, such as the state lottery, are not fair. Why? Because a certain amount of money is not returned to any of the contestants, as it's taken for purposes of running the game, funding government, etc. Suppose there are n contestants, each of whom pays a ticket price of p, and each of whom has a 1/n chance of winning the total payoff, which is np - R (the total payments by customers minus the “rake”). The expected payoff is p - R/n, which is obviously less than p.

II. Preferences and Attitudes Toward Risk
It was once thought that people would always accept a fair (or better than fair) gamble, and reject an unfair gamble. But this is clearly untrue, as demonstrated by the number of people who buy lottery tickets, gamble in casinos where the house has the advantage, etc. These people apparently like risk enough to accept an unfair gamble.

On the other hand, many people also avoid gambles even when they're fair. For example, which would you prefer -- a 50% chance of a million dollars, or $500,000 (the expected value) with certainty? I'd take the latter, as would most people. Yet the expected value of both options is the same. People who would choose the expected value over the gamble are called “risk averse.” People who would choose the gamble are called “risk loving.” People who are indifferent are called “risk neutral.”

Because of the fact that people do not simply value gambles according to their expected value, economists have developed the notion of “expected utility” to represent people’s preferences over risky situations. We will use our preference symbols from consumer theory (preferred to, less preferred than, indifferent) to show a person’s preferences with regard to gambles. For example, say that G is the gamble above (50% chance of a million dollars). For those of you who preferred $500,000, we would say $500,000 \succ G. This would be reversed for a risk lover, with indifference for a risk neutral person.

More generally, we say the following: If E(G) is the expected value of the gamble G, then you’re risk averse if E(G) \succ G, risk loving if E(G) \prec G, and risk neutral if E(G) \sim G.

III. The Certainty Equivalent

For any gamble, we can find some level of wealth that -- if received with certainty -- would make you exactly as happy as the gamble. For instance, consider the gamble where you have a 50% chance of a million dollars. For most of you, you’d rather have $500,000 than the gamble. But how about $400,000? How about $300,000? Sooner or later, I’ll find a number small enough that you’d prefer to take the gamble. Then I could come up with a somewhat higher number that would induce you take the cash. Going up and down in a smaller and smaller interval, I could eventually find an amount of money that would make you exactly indifferent between the money and the gamble. That would be the certainty equivalent.

In general, a person’s certainty equivalent of a gamble G is the amount of money w' such that the person is indifferent between the gamble and the money: w' \sim G.

This gives us another way of characterizing attitudes toward risk. If w' = E(G), then you’re risk neutral. If w' < E(G), then you’re risk averse. And if w' > E(G), then you’re risk loving.

IV. Insurance
In general, we expect people who are risk averse to be willing to buy insurance. Insurance, put simply, is an exchange in which you make a payment in order to get rid of a gamble -- that is, to avoid or reduce a risk.

“Fair” insurance is insurance whose premium (policy price) is equal to the expected loss for whatever is being insured. The expected loss is the probability of the accident, multiplied by the amount the insurance company must pay if the accident occurs.

Example: Suppose you have a 10% chance of having an accident, for which the policy will pay $20,000. Then the fair premium would be $2000.

Why is this “fair”? Because on average, the insurance company will have to pay $2000 per policy. The company will pay nothing 90% of the time, and pay $20,000 10% of the time. The company’s expected loss is .1($20,000) = $2000, which is the same as the fair premium. As a result, the company will break even if it sells many policies at the fair premium. Any lower premium will cause the firm to make losses, and therefore the fair premium is also the minimum premium.

Notice that we’re assuming that any insurance company sells many policies. As a result, the company converts individual probabilities of accidents into an aggregate frequency of accidents. While any single customer has a 10% chance of having an accident, the insurance company knows with (near) certainty that 10% of its customers will have an accident. For this reason, insurance companies are usually treated as risk neutral.

The expected loss (which is equal to the fair premium) can be interpreted as the marginal cost of selling an additional insurance policy. When an insurance company sells one more policy, it acquires a potential liability equal to the expected loss. In the example above, selling one more insurance policy, there is a 10% chance the company will have to pay $20,000. So selling this policy increases the company’s costs by an expected $2000.

Recall that under perfect competition, firms set price equal to marginal cost (P = MC). Since the expected loss is the marginal cost of an insurance policy, that means the “fair” premium is actually the perfectly competitive premium. If a perfectly competitive firm is making profits by charging above the fair premium, other firms can enter the insurance industry and undercut the premium to gain customers. In general, we use the terms “fair premium,” “perfectly competitive premium,” and “minimum premium” interchangeably.

Of course, there are costs of running an insurance company besides making pay-outs: processing costs, employee wages, etc. Some of these costs are fixed, while others vary with the number of insurance policies sold. So even in a perfectly competitive insurance market, premiums would not really be “fair” by this definition above.

The next question is, how much will customers actually pay? We know risk averters will pay at least the fair premium -- but how much more than that will they pay?
The maximum premium is the premium that’s just high enough to make the customer indifferent between buying and not buying the policy. For now, let’s assume we’re talking about full insurance, which is insurance that pays the full amount of the loss in the event of an accident. If you have an initial wealth of \( w_0 \), and you buy a policy with premium \( \delta \), you get \( w_0 - \delta \) with certainty after buying the policy. (Why? Because if any accident occurs, the insurance company pays you the full amount of the loss, so you’re exactly as well off as if the accident hadn’t occurred.) If you don’t buy the policy, then you stay in your original risky situation, the gamble \( G \).

For you to be indifferent between buying and not buying the insurance policy, it must be true that \( (w_0 - \delta) \sim G \). But we already know something else that you consider exactly equivalent to the gamble: the certainty equivalent. By definition, \( w' \sim G \). Therefore, it must be the case that \( (w_0 - \delta) = w' \). Thus, your maximum premium is given by \( \delta(\text{max}) = (w_0 - w') \). In other words, the maximum premium is a premium so high that, if you paid it, you’d be left with your certainty equivalent.

Example: Your initial wealth is $90,000, but there is a 10% chance your house will burn down, leaving you with wealth of only $40,000. Thus, you’re exposed to a gamble: a 90% chance of $90,000 and a 10% chance of $40,000. The expected value of this gamble is $85,000. But you’re risk averse, so let’s suppose that your certainty equivalent is $82,000. In that case, the maximum insurance premium is $90,000 - $82,000 = $8000. For insurance to fully cover this situation, you’d pay any insurance premium less than $8000, and reject any higher premium.

What we’ve done here is to find a range of possible values for an insurance premium. The minimum value is \( \delta(\text{min}) \), the “fair” premium, because any lower premium would cause the insurance company to make losses. The maximum premium is \( \delta(\text{max}) \), because any higher premium would be refused by the customer. The actual premium will fall somewhere in between. For a perfectly competitive insurance company (without any administrative costs), the premium would be \( \delta(\text{min}) \), but otherwise it will be somewhat higher.

Example: In the example above, the minimum premium is $5000 and the maximum premium is $8000. The actual premium will be in between.

Example: You have a wealth of $30,000. There is a 5% chance that you’ll contract a disease that will cost $10,000 to cure. So you face a gamble: 95% chance of $30,000 and a 5% chance of $20,000, for an expected value of $29,500. Since you’re risk averse, let’s suppose your certainty equivalent is $29,100. So the maximum premium you’ll pay is $30,000 - $29,100 = $900. For full insurance, the expected loss is .05($10,000) = $500, and that’s the minimum premium. So the actual premium will be between $500 and $900.

V. Less than Full Insurance
Less-than-full insurance is insurance that pays out less than the whole monetary loss. There are various ways in which an insurance policy can be less-than-full. (a) It could have a cap on the total pay-out. (b) It could require a deductible that you must pay, and then the insurance company will pay the rest of the loss. (c) It could require a copayment that you must make each time you make an insurance claim.

If you buy less-than-full insurance, you’ll still face a gamble, because you’ll still have lower wealth if the accident happens than if it doesn’t. But it will be a less risky gamble than the one you had without insurance. You wouldn’t be willing to pay a premium as high as the one you’d pay for full insurance, but you’d probably be willing to pay a somewhat lower premium.

It’s not as easy to find the maximum premium in this case, and we won’t go through the procedure. But finding the minimum premium is very easy. Just multiply the probability of the accident by the amount the insurance company will actually have to pay out if the accident occurs.

There are various reasons why it sometimes makes sense to have less-than-full insurance, some of which we’ll discuss later.

VI. Adverse Selection

Adverse selection is a problem that arises when an insurer can't tell the difference between two groups of people with different risks and thus has to charge the same premium for both groups. (It can also happen if the insurance company is forced to charge the same premium for both groups even though the company can tell the difference.) The problem is that an average premium (between the premiums for the two different risk groups) might be too high for the lower-risk group, causing some members of the lower-risk group to go uninsured.

Example: Suppose that insurers in a competitive market can't tell the difference between smokers and non-smokers. Smokers have a 20% chance of getting lung cancer, whereas non-smokers have only a 10% chance. Assume smokers are exactly one-half of the population. As a result, a person randomly chosen from the population as a whole has a 15% of getting cancer (from the insurance company’s perspective). The cost of treating lung cancer is $20,000.

The average fair premium for the whole population is .15($20000) = $3000. The fair premium for the smokers alone would be .2($20000) = $4000, and the fair premium for the non-smokers alone would be .1($20,000) = $2000. Now, we know that risk-averse people will accept any fair (or better) insurance policy, and for the smokers, the average fair premium is better than fair. So they will definitely choose to buy insurance at the average fair premium.

But what about the non-smokers? The average fair premium is higher than the fair premium for these customers. Because they are risk averse, they’ll be willing
to pay more than the fair premium of $2000, but how much more? Anyone with a certainty equivalent lower than $17000 (i.e., $20,000 - $3000) will buy the insurance. But anyone with a certainty equivalent higher than $17,000 would find herself worse off with the insurance than without it. As a result, some non-smokers will choose not to get insured.

That’s bad for the non-smokers who don’t get insured, but it can get even worse. If some non-smokers don’t buy insurance, and the insurance companies know this is true, then any person buying a policy is more likely to be a smoker than a non-smoker. If, say, only half of the non-smokers buy, then 2 out of 3 policy buyers will be smokers. And that means the insurance company will have to charge more than the $3000 average fair premium to break even. But if the premium rises, then even more non-smokers will decide not to buy – and that means the population of insurance buyers will be even more skewed toward smokers, leading to yet another premium hike… In the extreme, it could turn out that no one but smokers get insured. This is the problem of adverse selection.

Adverse selection does not *always* produce the extreme result of only the riskiest people getting insured. The more general result is just that policy buyers, as a group, tend to be a riskier group than the population as a whole, and some of the lower-risk people in the population go uninsured. This is unfortunate for the people who go uninsured, but it’s also unfortunate for the insurance companies: they would like to be able to offer insurance to lower risk people, but their inability to distinguish high-risk from low-risk people means they will attract more high-risk people.

How do insurance companies deal with this problem? The most common method is to distinguish between different risk groups. Many health insurance companies require a test to find out which customers are smokers, and they charge higher premiums to those who are. Similarly, auto insurance companies charge higher rates for teenagers and men than they do for adults and women, because the former groups tend to be riskier than the latter. Insurance companies may also require tests (physicals, driving tests, etc.) to try to assess the risk of specific individuals.

There are actually many examples of adverse selection outside the insurance industry. The most famous example is Akerlof's model of used car markets. The idea is that there are two kinds of used cars, “peaches” (good cars) and “lemons” (bad cars). Potential sellers know what kind of cars they have, but potential buyers don't. So potential buyers will pay a price corresponding to the expected car value -- somewhere midway between the price for a known peach and the price for a known lemon. But this price may be too low to induce the owners of peaches to put their cars up for sale. As a result, the market is dominated by lemons, and as people learn this, the used-car price drops further -- making the owners of peaches even less willing to bring their cars to market. Again, real-world markets have various means of dealing with this problem, such as warranties.

Application of adverse selection: mandatory benefits. Every state has passed various regulations requiring health insurance companies to include certain
benefits in their health insurance policies (such as abortion, contraception, acupuncture, mental illness, etc.). Now, some people are more likely to use these policies than others, but it’s hard for the insurance company to tell the difference. For example, it’s hard to distinguish someone who’s abstinent from someone who’s sexually active. The insurance companies will have to raise premiums to cover the added benefits. At the higher premium, the people more likely to use the benefits will probably continue to buy, but some people who are less likely to use the benefits will choose not to. This will skew the population of insurance buyers toward those most likely to use the benefits, causing the insurance companies to raise premiums further. As a result, a number of people who would otherwise have bought health insurance end up going uninsured.

VII. Moral Hazard

Moral hazard is a term used to describe the fact that people tend to engage in riskier behavior when they are insured against losses resulting from their behavior. For instance, drivers may take more risks on the road when they know the insurance company will pay for auto repairs. Moral hazard is a problem if this fact causes someone not to be able to insure against a risk that would be insurable if moral hazard didn’t exist.

Example: Suppose you have a driver with a car worth $20,000. It will be totaled with probability $p = .05$. But if the driver is fully insured, he will drive more recklessly so that $p=1$. Uninsured, the driver faces a gamble with a 95% chance of $20,000 and a 5% chance of $0$. The expected value of this gamble is $19,000. Because the driver is risk averse, his certainty equivalent for this gamble is $18,500, so the maximum premium he’ll pay is $1500. But from the insurance company’s perspective, the chance of an accident is 15%, so the expected loss (minimum insurance premium) is .1($20,000) = $2000. Notice that this premium is more than the driver is willing to pay, so no full insurance policy gets sold.

The best of all possible worlds would be if the driver were fully insured but continued to drive carefully, as though he were not insured. But once he's insured, he'll drive recklessly -- he's already got the policy, so his behavior lacks personal consequences. If he could, he would (prior to buying insurance) agree to constrain his future self to driving carefully.

How can this problem be dealt with? One solution is to charge higher premiums for people who have been in wrecks (or in the case of other types of insurance, people who have made claims before). This is an incentive to act carefully even when insured, because reckless behavior could lead to an increase in one's future premiums (and possibly ending up uninsured).

Another solution is to have less-than-full insurance, in the form of deductibles or copayments. Suppose that a potential loss of $10,000 is enough to induce the driver above to drive carefully. Then the insurance company could offer a policy to insure a loss up to $10,000, with a minimum premium of .05($10,000) = $500. Because it's better
than no insurance at all, the driver will buy the policy. This is a second-best solution, though, because it's not as good as full insurance with careful behavior.

There are numerous other examples of the moral hazard problem. Examples include: (1) FSLIC and the S&L debacle; (2) flood insurance and the placement of homes in flood zones; and (3) health insurance for behavior-related health problems (e.g., injuries from dangerous recreations like bungee-jumping).