I. The Concept of Profit Maximization

In the theory of the consumer, we assumed that consumers act to maximize their utility. The equivalent assumption in the theory of the firm is that firms act to maximize their profits.

Profit is defined as total revenue minus total cost.

\[ \Pi = TR - TC \]

(We use \( \Pi \) to stand for profit because we use \( P \) for something else: price.)

Total revenue simply means the total amount of money that the firm receives from sales of its product or other sources.

Total cost means the cost of all factors of production. But – and this is crucial – we have to think in terms of opportunity cost, not just explicit monetary payments. If the owner of the business also works there, we must include the value of his time. If the firm owns machines or land, we must include the payments those factors could have earned if the firm had chosen to rent them out instead of using them.

If only explicit monetary costs are considered, we get accounting profit. But to find economic profit, we need to take into account the opportunity cost, implicit or explicit of all resources employed.

Just as the consumer faced constraints (income and prices), the firm also faces constraints, but of a somewhat different form. The main constraints faced by the firm are:

- technology, as summarized in the cost curves of the last section;
- the prices of factors of production, also taken into account by the cost curves; and
- the demand for its product.

II. Demand Curve Facing the Firm

In previous discussion, we’ve talked about the market demand curve, which tells how much of a good consumers will buy at each price from producers all together. The market demand curve is the sum of individual demand curves, which we derived from consumers’ preferences and constraints.

But unless the firm is a monopolist, it does not face the entire market demand, but only a portion of it. The firm’s demand curve tells how much consumers will buy at each price from a particular firm.
The shape of the firm’s demand curve is related to the degree of competition in the market. Loosely speaking, more competition causes the firm’s demand curve to be more elastic (flatter), because consumers can respond to price increases by shifting their purchases to other firms. Less competition, on the other hand, implies a more inelastic (steeper) demand curve. Perfect competition and monopoly turn out to be the extreme ends of the spectrum:

- a perfectly competitive firm faces a perfectly horizontal demand curve;
- a monopoly faces the whole market demand curve.

**III. Total and Marginal Revenue**

Total revenue (TR) is the total amount of money the firm collects in sales. Thus,

\[ \text{TR} = pq \]

if \( p \) is the price and \( q \) is the quantity the firm sells. Notice that I’m using a small \( q \), because this is just one firm (\( Q \) is reserved for the market as a whole).

If the firm faces a downward-sloping demand curve, picking a quantity \( q \) automatically implies picking a price \( p \). Why? Because the firm must be operating on the demand curve. Any chosen price corresponds to a specific quantity that consumers will buy at that price, and any chosen quantity corresponds to a specific price that will induce consumers to buy the chosen quantity. To sell more, the firm must lower its price.

Graphically, TR is represented the rectangle created by \( p \) and \( q \).

Marginal revenue is the change in total revenue from increasing quantity by one unit. That is, \( \text{MR} = \Delta \text{TR}/\Delta q \), where the change in \( q \) is usually one.

Now, you might think that MR must be equal to the price, \( p \), because that’s how much you get paid for selling one more unit. But this is not true in general. Why not? Because if you face a downward-sloping demand curve, you have to lower your price to sell more. So if you increase your quantity, you’re also lowering your price for all the previous units of the good.

Example: Suppose you’re currently selling 10 units at $20 each. To sell an 11\text{th} unit, you’ll have to lower the price to $19. So, you gain $19 from the additional unit. But you also lose $1 each on the previous ten units that you could have sold at $20 each. So your marginal revenue from the 11\text{th} unit is not $19, but rather $9. You can see this from looking at the total revenue: before, \( \text{TR} = 20(10) = 200 \), but now \( \text{TR} = 19(11) = 209 \). Thus, TR rose by only $9.

This analysis leads to the following general conclusion: that MR is always below the demand curve. Why? At any quantity, the demand curve tells us the price corresponding
to that quantity. But we’ve just shown that the MR must be less than the price, and hence below the demand curve. The only place MR and the demand curve are equal is the point where the cross the vertical axis.

In general, any time a firm lowers its price, there are two effects: the “people buy more units” effect, and the “people pay less per unit” effect. Which effect is larger determines whether MR is positive or negative.

<table>
<thead>
<tr>
<th>elastic demand</th>
<th>1%Δ in price → more than 1%Δ in quantity</th>
<th>“people buy more” outweighs “people pay less”</th>
<th>MR &gt; 0 (above horizontal axis)</th>
</tr>
</thead>
<tbody>
<tr>
<td>unit elastic demand</td>
<td>1%Δ in price → 1%Δ in quantity</td>
<td>effects exactly cancel out</td>
<td>MR = 0 (crossing horizontal axis)</td>
</tr>
<tr>
<td>inelastic demand</td>
<td>1%Δ in price → less than 1%Δ in quantity</td>
<td>“people pay less” outweighs “people pay more”</td>
<td>MR &lt; 0 (below horizontal axis)</td>
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When the demand curve is a straight line, the MR always has a slope exactly twice that of the demand curve, so that it crosses the horizontal axis exactly half-way between the origin and the point where the demand curve does crosses the axis. Why? Because it turns out that a linear demand curve is always unit elastic exactly half-way down its length, and therefore MR crosses the horizontal axis at a quantity exactly half-way down the demand curve.

So, if I give you a demand curve P = b – mq, what’s the MR? MR = b – 2mq.

IV. Marginal Revenue for the Perfectly Competitive Firm

Above, I said MR is always below the firm’s demand curve. But there is an important exception to this rule: the perfectly competitive firm.

In a future lecture, we’ll talk about the main features of a perfectly competitive market. But for now, just remember that the key feature of a perfectly competitive firm is that it takes the price of the good (as determined by the market) as given. Increasing its output will not drive down the price, nor will cutting back its quantity raise the price. We call this the assumption of price-taking. The assumption of price-taking behavior can be represented graphically by having the firm face a horizontal demand curve. (Note: This is not the whole market demand curve – it is the firm’s demand curve.) It is horizontal at the market price p*.

You can think of the firm’s demand curve as having a very, very slight downward slope to see that if the firm raises its price even slightly, all of its customers immediately leave and go to other firms. This indicates the very high degree of competition in the market.

The practical effect of this assumption is that a firm can always sell another unit at the market price p* without affecting the price of its own previous units. As a result, when
the firm decides to raise its quantity, there is only one effect: the “people buy more” effect. There is no “people pay less.” When the firm sells one more unit of the good, the added (marginal) revenue is just the price of the good, $p^*$. As a result, the PC firm’s MR is exactly the same as its firm demand curve: $MR = p^*$.

Note that this is just a special case of the general model of profit-maximization. Recall that we can find MR by taking the firm demand curve and doubling its slope. But in this case, the slope of the demand is zero, so the slope of the MR is also zero.

V. Profit Maximization

The firm uses MR and MC to decide how much to produce.

- Suppose increasing output by one unit will bring in more additional revenues than it costs to produce. That is, $MR > MC$. Then it makes sense to produce the unit, because doing so will create more added benefit than added cost.
- On the other hand, suppose your last unit cost more to produce than it brought in additional revenue. Then $MR < MC$. And it makes sense to cut back your production (not produce that unit after all), because the last unit created more added cost than added benefit.
- So the only point at which you’re satisfied is where $MR = MC$. This is the general rule for profit maximization, which we’ll use with any firm, whether a perfect competitor, monopolist, or anything in between.

Graphically, we find the quantity $q^*$ where the MR and MC curves cross. What price corresponds to this quantity? We find it by extending $q^*$ up to the demand curve to get $p^*$. No other price will work: a higher price will reduce sales below $q^*$, and a lower price will increase sales above $q^*$.

How much profit does the firm actually make? To find this out, we need more information about the firm’s costs. We can get all the information we need from ATC.
Recall that ATC = TC/Q. Therefore, TC = ATC × Q. Graphically, we can see this as the rectangle created by q* and ATC* on the ATC curve.

Profit is defined as the difference between TR and TC. TR, as we saw before, is the rectangle p*q*. TC is the rectangle ATC*q*. So the difference in the areas of these rectangles is profit, as shown below.

As drawn, the firm is making profits, but it needn’t be such a happy outcome. If the ATC curve had been higher, we could have gotten at ATC* greater than the price. When this happens, the firm is making negative profits, also known as losses. This can happen even though the firm is doing the best it can, because any choice other than q* would make it lose even more (with the possible exception of shutting down entirely – something we’ll take up later).

VI. Profit Maximization for the Perfectly Competitive Firm

Just like other firms, a perfectly competitive firm sets MC = MR to maximize profits. But now, MR = p*, so the firm sets MC = p*. In other words, as long as the price of a unit exceeds the cost of producing it, produce the unit.
Example: Suppose \( MC = 2 + .25q \), and \( p^* = 4 \). Then
\[
4 = 2 + .25q \\
2 = .25q \\
q^* = 8
\]

Example: Suppose \( MC = q^2 - 8q + 17 \), and \( p^* = 10 \). Then
\[
10 = q^2 - 8q + 17 \\
0 = q^2 - 8q + 7 \\
0 = (q - 7)(q - 1) \\
q^* = 7 \text{ or } q = 1
\]

The correct answer for this example is \( q^* = 7 \), not \( q = 1 \). Why did we get two answers, and how did we choose? It’s because this MC is a parabola, and it crosses the firm demand curve twice – once from above, and once from below. But it turns out that the lower quantity doesn’t maximize profits; in fact, it minimizes profits. Notice that increasing quantity above that point means getting higher MR than MC, so it makes sense to do so.

Just as in the general theory of profit-maximization, we need to use ATC to find out the perfectly competitive firm’s actual profits or losses.
VII. Applications of MC = MR

The MC = MR rule is a very versatile one, which can be applied to many other decisions by firms other than how much to produce. It can be applied to advertising: increase the number of times you run your TV commercial as long as the added revenue from running it one more time outweighs the added cost of running it one more time. Or, it can be applied to choose hours of operation. For example, suppose that you run a palm-reading business. Because labor costs are fixed in this business, MC per hour of business is constant (a simplification). But each added hour in the evening will bring in fewer and fewer customers than the last, which means MR is declining. Where they are equal is the optimal number of hours of operation.

This is just one of the many ways in which MR = MC rule can be used to make business decisions other than choosing the profit-maximizing quantity.

Also, notice that average cost did not play any role in the choice of output. All that matters is the marginal effect on profits of increasing or decreasing your output. If a firm fails to follow the MC = MR rule and uses average cost instead, it will lose profits.

Example: The case of Continental Airlines. An airline has to decide how many flights to run during any given period of time. In the early 1960s and before, airlines typically made this decision by asking whether the additional revenue from a flight (the MR) was greater than the per-flight cost of a flight. In other words, they used the rule \( MR = TC/q \). But Continental broke from convention and started running flights even when the added revenues were below average cost. The other airlines said Continental was crazy – but Continental made money hand over fist. Eventually, the other airlines followed suit. What made Continental’s strategy superior? The per-flight cost consists of variable costs, including jet fuel and pilot salaries, and those are very relevant to the decision about whether to run another flight. But the per-flight cost also includes expenditures like rental of terminal space, general and administrative costs, and so on. These costs do not change with the number of flights, and therefore are irrelevant to that choice.
In order to put MC-MR thinking to its greatest use, it’s important not to fall into the trap of thinking about fixed and variable inputs as necessarily connected to the time frame (short run or long run). Remember that whether an input is fixed or variable depend on whether it changes with respect to a particular decision, and there are many different decisions. One way of thinking about them is in terms of “the meeting you’re at” – what kind of decision is being considered? Some inputs are fixed with respect to some decisions, while they are variable with respect to others. For whatever kind of decision you’re making, only the inputs that are variable with respect to that decision are relevant.

Example: Airlines again. When the airline is deciding whether to run an additional flight, the costs of G&A, reservation systems, etc., are fixed. The costs of jet fuel, pilots’ wages, meals, flight attendants’ wages, etc., are variable. But now suppose the airline has already decided to run a particular flight, and now they’re deciding how many seats to sell (that is, what price to charge per seat). For this decision, the costs of jet fuel and pilots’ wages are fixed, while only the costs of meals and flight attendants’ wages (assuming more customers require more flight attendants) are variable.

Example: If you’re deciding how many hours your employees should work, their wages are variable costs, but their fixed benefits packages (e.g., health insurance) are fixed costs. If you’re deciding whether to hire a new employee, both wages and benefits are variable costs.

VIII. The Shut-Down Decision

There is one exception to the rule that average costs are irrelevant in deciding how much to produce: the decision about whether to produce at all.

You should think about the MC = MR rule as the rule you should follow if you’ve decided to produce at all. But it might make sense to shut down the firm instead. How should the firm make this choice? You might think that the firm should shut down if it’s making losses, but that’s not quite true. Why? Because even if the firm shuts down, it will still have to pay its fixed costs (in the short run). So what really matters is whether the firm’s revenues cover its variable costs. If the answer is yes, the firm should stay open even though it’s making losses – because its revenues still partially offset the fixed costs. But if the answer is no, the firm should shut down – because the choice to produce is just adding even more losses on top of the fixed costs.

In other words, it must be the case the firm’s total revenues (TR) exceed the firm’s total variable costs (TVC).

\[ TR \geq TVC \]
\[ p \times q \geq AVC \times q \]
\[ p \geq AVC \]
This shows that the firm should stay open if the price (at its optimally chosen quantity) is greater or equal to its average variable cost. But if the price is below the average variable cost, the firm should shut down.

Note that this is a short-run shut-down decision. In the long run, there are no fixed costs, only variable costs. That means the firm can avoid making any losses at all by shutting down, so the firm should shut down any time it’s making economic losses -- unless there are changes the firm can make to its scale that allow it to escape those losses. This may be possible because the firm can shift to a different SRATC curve via a different capital choice.