In questions 1 through 4 write the letter from (a) to (d) in the space provided. No partial credit will be awarded.

1. Calculate the following indefinite integral: \( \int (5x^4 - x^3 + 8x - 1) \, dx \).

   (a) \( 20x^4 - 3x^2 + 8 \)
   (b) \( x^5 - \frac{1}{4}x^4 + 4x^2 - x + C \)
   (c) \( 20x^4 - 3x^2 + 8 + C \)
   (d) \( x^5 - 4x^4 + 8x^2 - x + C \)

   Answer: (B)

2. An object is moving along a line with an acceleration of \( a = t \) cm/sec\(^2\) at time \( t \) in seconds, and with an initial velocity of \( v_0 = v(0) = 3 \) cm/sec. Find the velocity after 2 seconds.

   (a) \( v(2) = 3 \) cm/sec
   (b) \( v(2) = 4 \) cm/sec
   (c) \( v(2) = 5 \) cm/sec
   (d) \( v(2) = 6 \) cm/sec

   Answer: (C)

   If \( a = t \), then \( v = \int t \, dt = \frac{t^2}{2} + C \). Then \( 3 = v(0) = 0^2/2 + C \), thus \( v(t) = \frac{t^2}{2} + 3 \), and \( v(2) = \frac{2^2}{2} = 3 = 5 \) cm/sec.

3. Suppose that \( \sum_{i=1}^{10} a_i = 10 \) and \( \sum_{i=1}^{10} b_i = 30 \). What is the value of \( \sum_{i=1}^{10} (2a_i + b_i - 4) \)?

   (a) 46
   (b) 10
   (c) 66
   (d) 30

   Answer: (B)

   \( \sum_{i=1}^{10} (2a_i + b_i - 4) = 2 \sum_{i=1}^{10} a_i + \sum_{i=1}^{10} b_i - 4 \sum_{i=1}^{10} 1 = 2(10) + 30 - 4(10) = 10 \).

4. What is the solution to the differential equation \( \frac{dy}{dx} = x^{-3} + 2 \) with initial condition \( y = 3 \) at \( x = 1 \)?

   (a) \( -3x^{-4} + C \)
   (b) \( -\frac{1}{2}x^{-2} + 2x + \frac{3}{2} \)
   (c) \( -3x^{-4} + 6 \)
   (d) \( -\frac{1}{4}x^{-4} + 2x + \frac{5}{4} \)

   Answer: (B)

   \( y = \int (x^{-3} + 2) \, dx = -\frac{1}{2}x^{-2} + 2x + C \) and \( 3 = y(1) = -\frac{1}{2} + 2 + C = \frac{3}{2} + C \), thus \( C = \frac{3}{2} \).
5. Calculate the sum of the areas of the circumscribed rectangles for the function \( y = x^2 + x \) on the interval \([0, 2]\) with \( n = 5 \) equal intervals (or rectangles).

\[
\Delta x = \frac{5}{2} = 0.4, \text{ thus } x_0 = 0, x_1 = 0.4, x_2 = 0.8, x_3 = 1.2, x_4 = 1.6, \text{ and } x_5 = 2. \text{ Thus }
\]

\[
\text{Area} = \sum_{i=1}^{5} f(x_i)\Delta x
= f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x + f(x_5)\Delta x
= 0.4(0.4^2 + 0.4 + 0.8^2 + 0.8 + 1.2^2 + 1.2 + 1.6^2 + 1.6 + 2^2 + 2)
= 0.4(14.8) = 5.92.
\]

6. Consider the function \( f(x) = 3x^4 - 8x^3 + 5 \). Give the following information. (Justify all your answers.)

(a) Compute \( f'(x) \) and \( f''(x) \).
\[
\begin{align*}
 f'(x) &= 12x^3 - 24x^2 \\
 f''(x) &= 36x^2 - 48x
\end{align*}
\]

(b) What are the critical points of \( f(x) \)?
There are no endpoints or singular points since \( f' \) is a polynomial. For stationary points we solve \( f'(x) = 0 \).

\[
\begin{align*}
12x^3 - 24x^2 &= 0 \implies x = 0
\end{align*}
\]

\[
\begin{align*}
12x^2(x - 2) &= 0 \implies x = 0 \text{ or } x = 2.
\end{align*}
\]

The only critical values are \( x = 0 \) and \( x = 2 \).

(c) Make the corresponding table to determine the intervals where the function is increasing or decreasing.

<table>
<thead>
<tr>
<th>Interval</th>
<th>((-\infty, 0))</th>
<th>((0, 2))</th>
<th>((2, \infty))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Value</td>
<td>-1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>( f'(x) )</td>
<td>-36</td>
<td>-12</td>
<td>108</td>
</tr>
<tr>
<td>conclusion</td>
<td>(\downarrow)</td>
<td>(\downarrow)</td>
<td>(\uparrow)</td>
</tr>
</tbody>
</table>
(d) Indicate the values of $x$ for which there is a local maximum (if any), and also the values of $x$ for which there is a local minimum (if any).

Based on (c), there is a minimum at $x = 2$ and there are no local maximums.

(e) What are the possible points of inflection of $f(x)$? (values of $x$ where $f''(x)$ is undefined or zero)

$f''(x)$ is always defined (it is a polynomial) and if we set $f''(x) = 0$ we get

\[36x^2 - 48x = 0 \implies 12x(3x - 4) = 0 \implies x = 0 \text{ or } x = 4/3.\]

The only possible inflection points are at $x = 0$ or $x = 4/3$.

(f) Make the corresponding table to determine the intervals where the function is concave up or concave down.

<table>
<thead>
<tr>
<th>Interval</th>
<th>$(-\infty, 0)$</th>
<th>$(0, \frac{4}{3})$</th>
<th>$(\frac{4}{3}, \infty)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Value</td>
<td>$-1$</td>
<td>$1$</td>
<td>$2$</td>
</tr>
<tr>
<td>$f''(x)$</td>
<td>$84$</td>
<td>$-12$</td>
<td>$48$</td>
</tr>
<tr>
<td>Conclusion</td>
<td>$\cup$</td>
<td>$\cap$</td>
<td>$\cup$</td>
</tr>
</tbody>
</table>

(g) Indicate the values of $x$ for which there is an inflection point (if any)

Based on (f), there are inflection points at $x = 0$ and $x = 4/3$ since there was a change in concavity there.

7. Find the following indefinite integrals:

(a) $\int 3x^2 \sqrt{x^3 + 7} \, dx$.

Let $u = x^3 + 7$, then $du = 3x^2 \, dx$ and

\[\int 3x^2 \sqrt{x^3 + 7} \, dx = \int \sqrt{u} \, du = \int u^{1/2} \, du = \frac{2}{3} u^{3/2} + C = \frac{2}{3} (x^3 + 7)^{3/2} + C.\]

(b) $\int \frac{\cos x}{\sin x + 3} \, dx$. Hint: use $u = \sin x + 3$.

Let $u = \sin x + 3$, then $du = \cos x \, dx$ and

\[\int \frac{\cos x}{\sin x + 3} \, dx = \int \frac{1}{u} \, du = \ln |u| + C = \ln |\sin x + 3| + C\]

8. A farmer wishes to fence off two identical adjoining rectangular pens, each with 900 square feet of area, as shown in the figure. Find the dimensions $x$ and $y$ such that the least amount of fence is required. Make sure you justify why the dimensions give the required minimum.
Since the area of each pen is 900 then \( xy = 900 \). The amount of fencing needed is \( F = 4x + 3y \), we can substitute \( y = \frac{900}{x} \) to get
\[
F(x) = 4x + \frac{2700}{x}.
\]
The feasible domain of \( F \) is \((0, \infty)\). (Note \( x \) can’t be zero). The derivative of \( F \) is
\[
F'(x) = 4 - 2700x^{-2} = 4 - \frac{2700}{x^2}.
\]
There are no endpoints or singular points \((x = 0 \text{ is not in the domain of } F)\), to get the stationary values we set \( F'(x) = 0 \).
\[
4 - \frac{2700}{x^2} = 0 \implies 4 = \frac{2700}{x^2} \implies x^2 = \frac{2700}{4} = 675 \implies x = \pm \sqrt{675} = \pm 15\sqrt{3}.
\]
The negative option is out of the domain so \( x = 15\sqrt{3} \) is the only critical value. Now we need to see if it is a max or a min (or neither). In class we saw three different ways to do this, here we only present the 1st derivative test:

<table>
<thead>
<tr>
<th>Interval</th>
<th>(0, 15\sqrt{3})</th>
<th>(15\sqrt{3}, \infty)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Value ( x )</td>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>( f'(x) )</td>
<td>-2696</td>
<td>1</td>
</tr>
<tr>
<td>conclusion</td>
<td>_</td>
<td>_</td>
</tr>
</tbody>
</table>

Since \( x = 15\sqrt{3} \) is the only local minimum then it is the global minimum. In this case \( y = \frac{900}{x} = \frac{900}{15\sqrt{3}} = 20\sqrt{3} \). So the required dimensions are \( x = 15\sqrt{3} \approx 25.981 \) feet, and \( y = 20\sqrt{3} \approx 34.641 \) feet.

9. The domain of a differentiable function \( f(x) \) is the interval \([-6, 8]\). Sketch the graph of a function \( y = f(x) \) making use of the properties below. Label any local maximums or minimums and points of inflection.

- \( f'(x) > 0 \) in the interval \((-1, 3)\).
- \( f'(x) < 0 \) in the intervals \((-6, -1) \text{ and } (3, 8)\).
- \( f''(x) > 0 \) in the intervals \((-3, 1) \text{ and } (5, 8)\).
- \( f''(x) < 0 \) in the intervals \((-6, -3) \text{ and } (1, 5)\).

There are many correct solutions, but they all share the general shape of the graph.