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<table>
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<th>Problem</th>
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1. Suppose you conduct a significance test for a population proportion using $\alpha=10\%$ and your $p$-value is .184. Which of the following should be your conclusion?
   a. Reject $H_0$
   b. Reject $H_A$
   c. Do not to reject $H_0$
   d. Do not to reject $H_A$

2. When are $p$-values negative?
   a. when the test statistic is negative.
   b. when the sample statistic is smaller than the hypothesized value of the parameter
   c. when we fail to reject the null hypothesis
   d. never

3. You have measured the systolic blood pressure of a random sample of 25 employees of a company. A 95% confidence interval for the mean systolic blood pressure for the employees is computed to be (122,138). Which of the following statements gives a valid interpretation of this interval?
   a. About 95% of the sample of employees have a systolic blood pressure between 122 and 138.
   b. About 95% of the employees in the company have a systolic blood pressure between 122 and 138.
   c. If the sampling procedure were repeated many times, then approximately 95% of the resulting confidence intervals would contain the mean systolic blood pressure for employees in the company.
   d. If the sampling procedure were repeated many times, then approximately 95% of the sample means would be between 122 and 138.
   e. The probability that the sample mean falls between 122 and 138 is equal to 0.95.

4. The average growth of a certain variety of pine tree is 10.1 inches in three years. A biologist claims that a new variety will have a greater three-year growth. A random sample of 45 of the new variety has an average three-year growth of 10.8 inches and a standard deviation of 2.1 inches. The appropriate null and alternate hypotheses to test the biologist’s claim are:
   a. $H_0: \mu = 10.1$ against $H_A: \mu > 10.1$
   b. $H_0: \mu = 10.1$ against $H_A: \mu \neq 10.1$
   c. $H_0: \mu = 10.8$ against $H_A: \mu > 10.8$
   d. $H_0: \mu = 10.1$ against $H_A: \mu < 10.1$
   e. $H_0: \mu = 10.8$ against $H_A: \mu \neq 10.8$

5. What is statistical inference on $\mu$?
   a. Drawing conclusions about a sample mean based on information contained in a population.
   b. Drawing conclusions about a population mean based on information contained in a sample.
   c. Drawing conclusions about a sample mean based on the measurements in that sample.
   d. Drawing conclusions about the population proportion based on information contained in the sample.

6. The notation for the sample proportion, the population proportion, and the claimed population proportion respectively are
   a. $p, p_0, \hat{p}$
   b. $\hat{p}, p, p_0$
   c. $p_0, p, \hat{p}$
   d. $p, \hat{p}, p_0$
   e. $p_0, \hat{p}, p$
7. The 90% confidence interval for a population mean is (1.2, 5.2)
   a. Then the population mean is 3.2, and the margin of error is 2.
   b. Then the sample mean is 2, and the margin of error is 3.2.
   c. Then the sample mean is 3.2, and the margin of error is 2.
   d. Then the sample mean is 1.2, and the margin of error is 5.2.

8. Researchers at a University conducted a study in which 67 students were weighed in September of their freshman year and again in April of their freshman year. The two samples are
   a. independent
   b. paired

9. An appropriate 95% confidence interval for $\mu$ has been calculated as (-0.73, 1.92) based on n=15 observations from a population with a normal distribution. Suppose we wish to test $H_0: \mu = 0$ versus $H_a: \mu \neq 0$. Based on this confidence interval,
   a. we should reject $H_0$ at the $\alpha = 0.10$ level of significance.
   b. we should not reject $H_0$ at the $\alpha = 0.10$ level of significance.
   c. we should reject $H_0$ at the $\alpha = 0.05$ level of significance.
   d. we should not reject $H_0$ at the $\alpha = 0.05$ level of significance.

10. Which one of these statements is FALSE?
   a. Increasing the sample size will decrease the margin of error.
   b. Increasing the confidence level will decrease the margin of error.
   c. It is usually unrealistic to assume that we know the population standard deviation.
   d. Confidence intervals are not valid if the sample is not chosen randomly.

11. Briefly explain the difference between the goals of a confidence interval and a hypothesis test.

    Goal of a CI: to estimate an unknown population parameter
    Goal of a HT: to test a claimed population parameter.

12. The braking distances of a simple random sample of 50 hybrid cars were collected, and their mean and standard deviation were calculated. Based on the sample results, a 95% confidence interval was calculated: (132.6ft, 139.4ft).

   a. In this study, what is the parameter we want to estimate? Denote this quantity by a symbol and explain what the symbol stands for in this problem.

      We want to estimate a population mean, $\mu$. In this example, we want to estimate the mean braking distances of ALL hybrid cars.

   b. Interpret the calculated confidence interval in context.

      We are 95% confident that the mean breaking distance of ALL hybrid cars is between 132.6 ft and 139.4 ft.
c. What is exactly in the middle of the given confidence interval? Circle ALL the correct answers:

- point estimate
- sample proportion
- sample mean
- population proportion
- population mean
- 95%
- 136ft
- margin of error
- \[
\frac{132.6 + 139.4}{2} = 136
\]

\[\text{margin of error}\]

\[\frac{136 - 132.6}{2} = 1.7\]

\[\frac{139.4 - 132.6}{2} = 3.4\]

\[E = 136 - 132.6 = 3.4\]

\[E = \frac{(139.4 - 132.6)}{2} = 3.4\]

d. What is the margin of error of the given confidence interval?

\[E = 136 - 132.6 = 3.4 \quad \text{OR} \quad E = \frac{(139.4 - 132.6)}{2} = 3.4\]

e. What could be done to reduce the margin of error? List two ways to achieve this goal.

- Either increase the sample size, or lower the confidence level.

f. True or False?

- If they had taken 100 samples of 50 randomly selected hybrid cars and had calculated a 95% confidence interval from each sample, probably about 95 of these confidence intervals would not contain the population parameter, and about 5% would contain it.

13. A magazine article titled *Are You Ready for an Interview?* claims that 50% of all senior executives say that the most common job interview mistake is to have little or no knowledge of the company. To check this claim, Accountemps (a company specialized in temporary professional accounting and finance positions) conducted a survey of 155 randomly selected senior executives, and found that 66 of them said just that: the most common job interview mistake is to have little or no knowledge of the company. Based on these survey results, can you conclude that the magazine’s article published a false claim? (That is not 50%?)

a. Specify the null and alternative hypotheses for this test, using the correct symbols and numbers.

- Null: \[p = 0.50\]
- Alternative: \[p \neq 0.50\]

b. Check the conditions for a hypothesis test.

- Random sample, checked.

\[np_0 = 155(0.50) > 10\]
\[n(1 - p_0) = 155(1 - 0.5) > 10\]

c. Determine the value of the test statistic, and the p-value.

- Test statistic: \[z = -1.85\]
- p-value = 0.065
d. Which one of the following statements is true at the 5% level of significance?
   (i) the results are significant, and so we can reject the null hypothesis.
   (ii) the results are not significant, and so we can reject the null hypothesis.
   (iii) the results are significant, and so we cannot reject the null hypothesis.
   (iv) the results are not significant, and so we cannot reject the null hypothesis.

e. State your conclusion in context.

   Since the p-value is greater than 5%, we cannot reject the null hypothesis. That is, at the 5% significance level, we can't reject the magazine’s claim that 50% of all senior executives say that the most common job interview mistake is to have little or no knowledge of the company.

f. Calculate the 95% confidence interval.

   (0.348, 0.504)

g. Based on the confidence interval you calculated in part (f), would you reject the null hypothesis or not? Explain your answer. Did you arrive at the same conclusion as in part (e)?

   Since the claimed value, 0.50 is inside the 95% confidence interval, we cannot reject the null hypothesis at the 5% level. Same conclusion as in part (e).

14. During routine screening, a doctor notices that 22% of her adult patients show higher than normal levels of glucose in their blood—a possible warning signal for diabetes. Hearing this, some medical researchers decide to conduct a large-scale study, planning to estimate the proportion to within 3% with 90% confidence. How many randomly selected adults must they test?

   $n = \left( \frac{z^*}{m} \right)^2 \hat{p}(1 - \hat{p}) = \left( \frac{1.645}{0.03} \right)^2 0.22(1 - 0.22) = 515.95$

   They need to select at least 516 adults.

15. During an angiogram, heart problems can be examined via a small tube (catheter) threaded into the heart from a vein in the patient’s leg. It’s important that the company that manufactures the catheter maintain a diameter of 2.00mm. Each day, quality control personnel make several measurements to check the diameters of the catheters. If they discover a problem, they will stop the manufacturing process until it is corrected.

a. State the null and alternative hypotheses in symbols.

   $H_0: \mu = 2 \text{ mm} \quad H_a: \mu \neq 2 \text{ mm}$

b. A random sample of 55 catheters yielded the following results.

   **T Confidence Intervals**
   
<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>StDev</th>
<th>95.0 % CI</th>
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<tbody>
<tr>
<td>DIAMETER</td>
<td>55</td>
<td>2.06</td>
<td>0.19</td>
<td>(2.0086, 2.1114)</td>
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</table>

   **T-Test of the Mean**
If you were one of the quality control personnel, what would be your recommendation? In your discussion use both the results from the T-Test, and from the T-Confidence Interval. You must explain your decision, and write your conclusion in context.

Since the p-value is less than 5%, and the claimed value, 2mm is not in the 95% confidence interval, we have enough evidence to reject the null hypothesis. That is, we can reject the claim that the mean diameter of the catheters is 2mm. If I were one of the quality control personnel, I would stop the manufacturing process until the problems are corrected.

c. Is the confidence interval valid if the distribution from which the 55 measurements were taken is not normally distributed? Explain briefly.

Yes, the confidence interval is still valid because the sample size is greater than 30.

16. A company institutes an exercise break for its workers to see if it will improve job satisfaction, as measured by a questionnaire that assesses workers’ satisfaction where a higher score means higher satisfaction. Scores for ten randomly selected workers before and after the implementation of the exercise program are shown below.

<table>
<thead>
<tr>
<th>Worker</th>
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<th>After</th>
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<tr>
<td>1</td>
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<td>33</td>
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<tr>
<td>2</td>
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Here are the summaries of two possible analyses:

**Paired t-Test** of \( \mu(\text{before-after}) = 0 \) vs. \( \mu(\text{before-after}) < 0 \)
- t-Statistic = -2.627
- \( p = 0.014 \)

**2-Sample t-Test** of \( \mu(\text{before}) - \mu(\text{after}) = 0 \) vs. \( \mu(\text{before}) - \mu(\text{after}) < 0 \)
- t-Statistic = -1.284
- \( p = 0.108 \)

a. Which of these tests is correct for these data? Explain.

It is a paired t-test because the “before” / “after” scores are recorded for each worker.
b. Using the test you selected, state your conclusion in context.

p-value = 0.014. Since the p-value is smaller than 5%, we can reject the null hypothesis. That is, we can reject the claim that there is no difference between the satisfaction scores before and after the exercise program. We can conclude that there is a difference between the scores.

17. A manufacturer claims that the calling range (in feet) of its 2.4 GHz cordless phone is greater than that of its leading competitor. You perform a study using 14 randomly selected phones from the manufacturer and 16 randomly selected phones from its competitor. At the 5% significance level, can you support the manufacturer’s claim? Assume the populations are normally distributed. State the claims, and write your conclusion in context based on the given p-value.

p-value: 0.046

Hypotheses:

\[ H_0 : \mu_{\text{manufacturer}} = \mu_{\text{competitor}} \]
\[ H_1 : \mu_{\text{manufacturer}} > \mu_{\text{competitor}} \]

Conclusion in context:

Since p-value < 5%, we can reject the null hypothesis. We can reject the claim that the mean calling ranges (in feet) are the same for both phones.

\[ \hat{p} = \frac{x}{n} \]
\[ \hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \]
\[ z = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}} \]
\[ \bar{x} \pm z^* \frac{s}{\sqrt{n}} \]
\[ t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \]

\[ n = \left( \frac{z^* \sigma}{E} \right)^2 \]
\[ n = \left( \frac{z^*}{E} \right)^2 \hat{p}(1-\hat{p}) \]

\[ z^* \text{ for } 90\% \text{ confidence: } 1.645 \]
\[ z^* \text{ for } 95\% \text{ confidence: } 1.96 \]
\[ z^* \text{ for } 99\% \text{ confidence: } 2.576 \]