Differentiability versus Derivative

Derivative

velocity, slope of the tangent

\[ f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} \]

Function is differentiable if and only if:

- There is a function with derivative
  \[ f(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} \]
  and differential is
  \[ df(x) = f'(x) \, dx \]
  \[ m = f'(x), \, h = dx \]

Differential

Approximating the function

\[ f(x+h) \approx f(x) + mh + O(h) \]

The best prediction is when

\[ f(x+h) - f(x) = mh + O(h) \]

Differential is the principal linear part at the increment of the function, this is the mathematical way to say that differential is the best linear approximation to the function.

If linear approximation is not enough, consider quadratic, cubic, etc...

\[ f(x+h) = f(x) + mh + \frac{m}{2} h^2 + \frac{m}{3!} h^3 + \ldots + \frac{f^{(n)}(x)}{n!} h^n + O(h^n) \]

Taylor Expansion