1. Three positive particles of charges 11 \( \mu \text{C} \) are located at the corners of an equilateral triangle of side 15.0 cm. Calculate the magnitude and direction of the net force on each of the particles.

Because all the charges and their separations are equal, we find the magnitude of the individual forces:

\[
F = \frac{kQ^2}{L^2} = \frac{(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(11.0 \times 10^{-6} \text{ C})^2}{(0.150 \text{ m})^2} = 48.4 \text{ N}.
\]

The directions of the forces are determined from the signs of the charges and are indicated on the diagram. For the forces on the top charge, we see that the horizontal components will cancel. For the net force, we have

\[
F = F_1 \cos 30° + F_1 \cos 30° = 2F_1 \cos 30° = 2(48.4 \text{ N}) \cos 30° = 83.8 \text{ N} \uparrow, \text{ or away from the center of the triangle}.
\]

From the symmetry each of the other forces will have the same magnitude and a direction away from the center:

The net force on each charge is 83.8 N away from the center of the triangle.

Note that the sum for the three charges is zero.

2. An alternative charge of 6.00 mC and -6.5 mC are placed at each corner of a square 1.00 m on a side as it is shown in Figure. Determine the magnitude and direction of the force on each charge.

Because the magnitudes of the charges and the distances have not changed, we have the same magnitudes of the individual forces on the charge at the upper right corner:

\[
F_1 = F_2 = k\frac{Q^2}{L^2} = 3.24 \times 10^5 \text{ N},
\]

\[
F_3 = k\frac{Q^2}{2L^2} = 1.62 \times 10^5 \text{ N}.
\]

The directions of the forces are determined from the signs of the charges and are indicated on the diagram. For the forces on the top charge, we see that the net force will be along the diagonal. For the net force, we have

\[
F = F_1 \cos 45° + F_2 \cos 45° + F_3
\]

\[
= -2(3.24 \times 10^5 \text{ N}) \cos 45° + 1.62 \times 10^5 \text{ N}
\]

\[
= -2.96 \times 10^5 \text{ N} \downarrow, \text{ or toward the center of the square}.
\]

From the symmetry, each of the other forces will have the same magnitude and a direction toward the center:

The net force on each charge is 2.96 \( \times 10^5 \) \( \text{ N} \) toward the center of the square.

Note that the sum for the three charges is zero.

3. A +5.7 \( \mu \text{C} \) and -3.5 \( \mu \text{C} \) charge are placed 25 cm apart. Where can be a third charge be placed so that it experiences no net force?

If we place a positive charge, it will be repelled by the positive charge and attracted by the negative charge. Thus the third charge must be placed along the line of the charges, but not between them. For the net force to be zero, the magnitudes of the individual forces must be equal:

\[
F = k\frac{Q_1r_1^2}{x^2} = k\frac{Q_2r_2^2}{x^2}, \text{ or } \frac{Q_1}{(L+x)^2} = \frac{Q_2}{x^2};
\]

(5.7 \( \mu \text{C} \))(0.25 \text{ m} + x)^2 = (3.5 \( \mu \text{C} \))x^2, which gives \( x = 0.91 \text{ m}, -0.11 \text{ m} \).

The negative result corresponds to the position between the charges where the magnitudes and the directions are the same. Thus the third charge should be placed 0.91 m beyond the negative charge.

Note that we would have the same analysis if we used a negative charge.
4. A +30 µC charge is placed 32 cm from an identical +30.0 µC charge. How much work would be required to move a -0.50 µC test charge from point midway between them to a point 10 cm closer to either of the charges?

We find the electric potentials of the stationary charges at the initial and final points:

\[ V_a = k\left[\frac{Q_1}{r_{1a}} + \frac{Q_2}{r_{2a}}\right] \]
\[ = \left(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left[\frac{\left[30 \times 10^{-6} \text{ C}\right]}{(0.16 \text{ m})} + \frac{\left[30 \times 10^{-6} \text{ C}\right]}{(0.16 \text{ m})}\right] = 3.38 \times 10^6 \text{ V}. \]

\[ V_b = k\left[\frac{Q_1}{r_{1b}} + \frac{Q_2}{r_{2b}}\right] \]
\[ = \left(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left[\frac{\left[30 \times 10^{-6} \text{ C}\right]}{(0.26 \text{ m})} + \frac{\left[30 \times 10^{-6} \text{ C}\right]}{(0.06 \text{ m})}\right] = 5.54 \times 10^6 \text{ V}. \]

Because there is no change in kinetic energy, we have

\[ W_{a\rightarrow b} = \frac{1}{2} \frac{q^2}{r} = \frac{1}{2} kQ^2 \]
\[ = \left(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left[\frac{\left[30 \times 10^{-6} \text{ C}\right]}{(1.0 \times 10^{-10} \text{ m})}\right] = 6.9 \times 10^{-18} \text{ J} = 43 \text{ eV}. \]

5. How much work would be required to bring three electrons must be done to bring three electrons from great distance apart to within 1.0 \times 10^{-10} \text{ m} from one another?

We find the potential energy of the system of charges by adding the work required to bring the three electrons in from infinity successively. Because there is no potential before the electrons are brought in, for the first electron we have

\[ W_1 = (-e) V_0 = 0. \]

When we bring in the second electron, there will be a potential from the first:

\[ W_2 = (-e) V_1 = (-e)k(-e)/r_{12} = ke^2/d. \]

When we bring in the third electron, there will be a potential from the first two:

\[ W_3 = (-e) V_2 = (-e)\left\{\frac{k(-e)e}{r_{12}} + \frac{k(-e)e}{r_{23}}\right\} = 2ke^2/d. \]

The total work required is

\[ W = W_1 + W_2 + W_3 = (ke^2/d) + (2ke^2/d) = 3ke^2/d \]
\[ = \left(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(1.60 \times 10^{-19} \text{ C}\right)^2/\left(1.0 \times 10^{-10} \text{ m}\right) = 6.9 \times 10^{-18} \text{ J} = 43 \text{ eV}. \]

6. Electron starts from rest 72.5 cm from a fixed point charge with \(Q=0.125\mu\text{C}\). How fast will the electron be moving when it is very far away?

When the electron is far away, the potential from the fixed charge is zero. Because energy is conserved, we have

\[ \frac{1}{2}mv^2 + \frac{q^2}{r} = 0; \]

\[ \frac{1}{2}mv^2 = e\left(kQ/r\right) \]
\[ \frac{1}{2} (9.11 \times 10^{-31} \text{ kg})v^2 = (1.60 \times 10^{-19} \text{ C})(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(0.125 \times 10^{-6} \text{ C})(0.725 \text{ m}), \]

which gives \( v = 2.33 \times 10^7 \text{ m/s} \).

Three charges are at the corners of an equilateral triangle (side \(l\)) as shown in Figure. Determine the potential at the midpoint of each of the sides.

The distances from the midpoint of a side to the three charges are \(\sqrt{2}, \sqrt{2}, \text{ and } \sqrt{3} \text{ cos } 30^\circ\).

At point \(a\), we have

\[ V_a = k\left[\frac{\left(-Q\right)}{(\sqrt{2})} + \frac{\left(+Q\right)}{(\sqrt{2})} + \left(-3Q\right)/\left(\sqrt{3} \text{ cos } 30^\circ\right)\right] \]
\[ = \left(kQ/\sqrt{2}\right)/(-2) + (+2) + (-3/\sqrt{3} \text{ cos } 30^\circ) = -3.5 kQ/\sqrt{2}. \]

At point \(b\), we have

\[ V_b = k\left[\frac{\left(+Q\right)}{(\sqrt{2})} + \frac{\left(-3Q\right)}{(\sqrt{2})} + \left(-Q\right)/\left(\sqrt{3} \text{ cos } 30^\circ\right)\right] \]
\[ = \left(kQ/\sqrt{2}\right)/(+2) + (-6) + (-1/\sqrt{3} \text{ cos } 30^\circ) = -5.2 kQ/\sqrt{2}. \]

At point \(c\), we have

\[ V_c = k\left[\frac{\left(-3Q\right)}{(\sqrt{2})} + \frac{\left(-Q\right)}{(\sqrt{2})} + \left(+Q\right)/\left(\sqrt{3} \text{ cos } 30^\circ\right)\right] \]
\[ = \left(kQ/\sqrt{2}\right)/(-6) + (-2) + (+1/\sqrt{3} \text{ cos } 30^\circ) = -6.8 kQ/\sqrt{2}. \]