Problem 1. Let \((X,F,\mu)\) be a probability space. Let \(A_1, A_2, \ldots\) be a sequence of subsets of \(X\) belonging to \(F\).

(a) Show that if \(A_1 \supset A_2 \supset A_3 \supset \cdots\), then
\[\mu\left(\bigcap_{n=1}^{\infty} A_n\right) = \lim_{n \to \infty} \mu(A_n).\]

(b) Show that if \(A_1 \subset A_2 \subset A_3 \subset \cdots\), then
\[\mu\left(\bigcup_{n=1}^{\infty} A_n\right) = \lim_{n \to \infty} \mu(A_n).\]

Problem 2. Let \(X\) be a probability space and \(A_n\) measurable sets. Show that the probability of \(\liminf_n A_n^c\) is 0 if and only if the probability of \(\limsup_n A_n\) is 1.

Problem 3. Let \((X,F,\mu)\) be a probability space, and let \(A_1, A_2, \ldots\) be in \(F\). Show that
\[\mu(\liminf_n A_n) \leq \liminf_n \mu(A_n) \leq \limsup_n \mu(A_n) \leq \mu(\limsup_n A_n).\]

Problem 4. Let \((X,F,\mu)\) be a probability space. Show that if \(A_1, A_2, \cdots, A_n\) are independent sets from \(F\), then the sets \(A_1^c, A_2, \cdots, A_n\) are also independent.